Simultaneous IDA-Passivity-based control of a Wound Rotor Synchronous Motor

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Abstract—This paper presents a new nonlinear passivity-based controller for a wound rotor synchronous machine, acting as a motor drive. From the standard dq-model the control objectives are stated, and the Port-controlled Hamiltonian model is also obtained. A simple power flow study allows to state the control goals in terms of reactive power compensation and ohmic losses reduction. Starting from the Hamiltonian structure, the Simultaneous Interconnection and Damping Assignment (SIDA-PBC) technique is used to develop the control action. The desired robustness of the control action is also taken into account in the design procedure. This results in a globally asymptotically stabilizing controller, which is validated via numerical simulations.

I. INTRODUCTION

The wound rotor synchronous machine (WRSM) is used for generation and also for drive applications [15]. In the generation case the field voltage is used for regulating the stator voltage, while in motor applications this variable can be used to compensate the power factor of the machine [16]. Several techniques are proposed for controlling the WRSM. Linear techniques are the most used in the industry [9][17], but decoupling methods [8], widely employed for asynchronous machines, have also been extended to the synchronous case, and advanced nonlinear controllers have been applied to this class of machines as well [10].

Passivity-based control (PBC) is a technique that can be used to design controllers for a large kind of systems. Control of a rather general class of electrical machines using PBC methods has been proposed in [11], and the specific cases for synchronous generators and drives can be found also in [5] and [7], respectively. Recently, a new technique based on the PBC properties called Interconnection and Damping Assignment (IDA-PBC) has been proposed in [12]. Using the IDA-PBC approach many electrical machines have been controlled [14], in particular induction machines [1] and permanent magnet synchronous ones [13]. The simultaneous IDA-PBC methodology was proposed in [2], were the induction machine was studied and controlled. The SIDA-PBC technique offers more degrees of freedom than IDA-PBC, and allows to solve more complex interconnected systems and to design output feedback, as opposed to state feedback, controllers (see examples in [2] and [4]).

The main goal of this work is to design a control algorithm for a wound rotor synchronous drive machine, based on the Simultaneous IDA-PBC technique. The paper is organized as follows. In Section II the wound rotor synchronous machine model is introduced and its control goals are described. Section III presents the SIDA-PBC technique and then the control law is obtained. The simulation results are included in Section IV and, finally, conclusions are stated in Section V.

II. THE WRSM MODEL AND CONTROL GOALS

In this Section we present the dynamical model of the WRSM. From the well-known dynamical equations we also propose a port-Hamiltonian model which allows to describe in a compact form and with a nice physically interpretation the system dynamics. Finally, we compute, in terms of the fixed point values, the active and reactive powers flowing through the stator side of the machine and define the control objectives.

A. dq-model of the WRSM

The state space model in dq coordinates of a wound rotor synchronous machine with a field winding (and no damper windings) is given by [3]
\[
\begin{align*}
\dot{\lambda}_d &= -R_s i_d + n_p \omega L_s i_q + v_d \\
\dot{\lambda}_q &= -n_p \omega i_d i_d - R_s i_q - n_p \omega M i_F + v_q \\
\dot{\lambda}_F &= -R_F i_F + v_F
\end{align*}
\]
where \( \lambda^T = (\lambda_d, \lambda_q, \lambda_F) \in \mathbb{R}^3 \) and \( i^T = (i_d, i_q, i_F) \in \mathbb{R}^3 \) are the fluxes and currents, respectively, \( \omega \) is the mechanical speed, \( n_p \) is the number of pole pairs, \( R_s \) and \( R_F \) are the ohmic resistances of the stator \( dq \) and rotor field windings, and \( L_s, L_F \) and \( M \) are the leakage and mutual inductances.

The dynamical model has to be completed with the mechanical equation
\[
J_m \frac{d\omega}{dt} = n_p M i_F i_q - B_r \omega + \tau_L.
\]
Here \( J_m \) is the rotor inertia, \( \tau_L \) is a generic mechanical torque (negative in case of braking), and \( B_r \) is a viscous mechanical damping coefficient.

Fluxes and currents are related by
\[
\lambda = Li
\]

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where
\[
L = \begin{pmatrix} L_s & 0 & M \\ 0 & L_s & 0 \\ M & 0 & L_F \end{pmatrix}.
\]

### B. Port-Hamiltonian model of the WRSM

Hamiltonian modeling uses the state dependent energy functions to characterize the dynamics of the different subsystems, and connects them using a Dirac structure, which embodies the power preserving network of relations established by the corresponding physical laws. The result is a mathematical model with an specific structure, called port-controlled Hamiltonian system (PCHS) [18], which lends itself to a natural, physics-based analysis and control design.

Explicit PCHS have the form
\[
\begin{aligned}
\dot{x} &= (J(x) - R(x))\partial_x H(x) + g(x)u \\
y &= g^T(x)\partial_x H(x)
\end{aligned}
\tag{5}
\]

where \(x \in \mathbb{R}^n\) is the vector state, \(u, y \in \mathbb{R}^m\) are the port variables, and \(H(x): \mathbb{R}^n \rightarrow \mathbb{R}\) is the Hamiltonian function, representing the energy function of the system. The \(\partial_x\) (or \(\partial\), if no confusion arises) operator defines the gradient of a function of \(x\), and in what follows we will take it as a column vector. \(J(x) \in \mathbb{R}^{n \times n}\) is the interconnection matrix, which is skew-symmetric \((J(x) = -J(x)^T)\), representing the internal energy flow in the system, and \(R(x) \in \mathbb{R}^{n \times n}\), the dissipation matrix, symmetric and, in physical systems, semi-positive definite \((R(x) = R^T \geq 0)\), which accounts for the internal losses of the system. Finally, \(g(x) \in \mathbb{R}^{n \times m}\) is an interconnection matrix describing the port connection of the system to the outside world. It yields the flow of energy to/from the system through the port variables, \(u\) and \(y\).

Equations (1), (2), (3) and (4) can be given a port-Hamiltonian form with energy variables, fluxes \((\lambda_d, \lambda_q, \lambda_F)\) and momentum \((p = J_m\omega)\). Then, the interconnection and dissipation matrices are, respectively
\[
J(x) = \begin{pmatrix} 0 & n_pL_s\omega & 0 & 0 \\ -n_pL_s\omega & 0 & 0 & -n_pMi_F \\ 0 & 0 & 0 & 0 \\ 0 & n_pMi_F & 0 & 0 \end{pmatrix},
\]
\[
R = \begin{pmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_F & 0 \\ 0 & 0 & 0 & B_r \end{pmatrix},
\]

with Hamiltonian (energy) function
\[
H(x) = \frac{1}{2}x^T L^{-1}x + \frac{1}{2}J_m \omega^2,
\]

and
\[
g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
\]

where the external inputs are \(u = (v_d, v_q, v_F, \tau_L)\).

### C. Fixed points and power flow study

In this subsection we compute the electrical power, in steady-state, flowing through the stator side of the machine in order to compensate the power factor. All this study consider that the three-phase system is sinusoidal and balanced.

Fixed points are solutions to
\[
\begin{aligned}
0 &= -R_s i_d^* + n_p\omega L_s i_q^* + v_d \\
0 &= -n_p\omega L_s i_d^* - R_s i_d^* - n_p\omega Mi_F^* + v_q \\
0 &= -R_F i_F^* + v_F \\
0 &= n_p Mi_F^* i_q^* - B_r\omega^* + \tau_L
\end{aligned}
\tag{6}
\]

and using the fixed points (6) and (7), we obtain
\[
P_s^* = R_s^2 (i_d^{*2} + i_q^{*2}) + n_p\omega^* Mi_F^* i_q^*,
\]

taking into account \(\tau_c = n_p Mi_F i_q\), we recover the power balance equation (in steady-state)
\[
P_s^* = \underbrace{R_s^2(i_d^{*2} + i_q^{*2})}_{\text{Electrical losses}} + \underbrace{\tau_c \omega^*}_{\text{Mechanical power}}
\tag{9}
\]

The same study for the reactive power,
\[
Q_s = v_d i_q - v_q i_d,
\]

together with (6) and (7), yields
\[
Q_s^* = -n_p\omega^* (L_s (i_d^{*2} + i_q^{*2}) + Mi_F^* i_q^*).
\tag{10}
\]

Notice that \(i_F^*\) can compensate the power factor (or \(Q_s = 0\)) with an appropriate value in the third term of (10).

### D. Control objectives

Following the results presented above, we can summarize the control goals as follows: to regulate the mechanical speed, \(\omega\), and to compensate the power factor, \(i_q^* = 0\). To achieve these objectives we have three control inputs, \(v_d, v_q\) and \(v_F\). There is still one degree of freedom which allows to regulate \(i_d = i_d^*\) to minimize the loses (see equation (9)).

Unfortunately, both control goals (reactive power and minimization losses) cannot be achieved simultaneously. Equation (10) shows that the term which compensates the reactive power requires a nonzero \(i_d\) value. For this reason, the control goal \(i_d^* = 0\) is replaced by the objective of getting an small value of the d-current component.

### III. CONTROL DESIGN

Figure 1 shows the proposed control scheme. As explained above the control objectives are to regulate \(\omega\), \(i_d\) and to compensate the reactive power. The SIDA-PBC controller directly regulates \(\omega\) and \(i_d\), while \(Q_s\) is controlled through the rotor current \(i_F\) by means of setting \(Q_s^* = 0\) in equation (10),\(^2\) implying that
\[
i_F^* = \frac{L_s}{Mi_d^*} (i_d^{*2} + i_q^{*2}),
\tag{11}
\]

\(^2\)The \(i_q\) is used instead of \(i_d^*\) to improve the robustness (the \(i_q^*\) value is strongly dependent on the parameters).
The equilibrium assignment of the desired energy function as

\( F(x) = \begin{pmatrix} F_{11}(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{24}(x) \\ 0 & 0 & F_{33}(x) & 0 \\ 0 & F_{42}(x) & F_{43}(x) & F_{44}(x) \end{pmatrix} \)

where inequality (13) must be accomplished.

Decoupling control actions and feedback outputs is possible if the \( F \) matrix contains only one nonzero term in each row. The three first rows of \( F(x) \) have nonzero \( F_{11}, F_{24} \) and \( F_{33} \) elements, which relate \( v_d, v_q \) and \( v_F \) to the errors in \( i_d, \omega \) and \( i_F \), respectively. This is a critical point to improve the robustness of the resulting controller, because in this way only the fixed points of the outputs appear in the control action. Effectively, as it is shown later, the fixed point \( i^*_d \), whose computation requires solving the fixed points equations, (6)–(8), with strong dependence on the parameters, do not appear in the control law.

From the three first rows of (12) we obtain the control actions

\[
\begin{align*}
v_d &= R_k i_d - n_p \omega L_s i_q + F_{1} k_d (i_d - i^*_d) \\
v_q &= n_p \omega L_s i_d + R_s i_q + n_p \omega M_i F + F_{24} k_\omega (\omega - \omega^*) \\
v_F &= R_F i_F + F_{33} \gamma_F (i_F - i^*_F),
\end{align*}
\]

while from the fourth row,

\[
\begin{align*}
n_p M_i F i_q - B_r \omega + \tau_L - F_{42} \gamma_q (i_q - i^*_q) - F_{43} \gamma_F (i_F - i^*_F) - F_{44} k_\omega (\omega - \omega^*) &= 0.
\end{align*}
\]

To solve equation (14) we propose

\[
\begin{align*}
F_{42} &= \frac{1}{\gamma_q} n_p M_i F \\
F_{43} &= \frac{1}{\gamma_F} n_p M_i q \\
F_{44} &= -\frac{1}{k_\omega} B_r
\end{align*}
\]

where the steady-state solution of (4)

\[
\tau_L = -n_p M_i F i_q + B_r \omega^*
\]

has been also used. In order to simplify the solution, now we assign

\[
\begin{align*}
F_{11} &= -1 \\
F_{24} &= -F_{32}.
\end{align*}
\]

With the last choice, it is clear that (13) holds if \( F_{33} < 0 \) and

\[
-2(F_{33} + F_{44}) > F_{43}^2,
\]

or substituting,

\[
-2 \left( F_{33} - \frac{1}{k_\omega} B_r \right) > \left( \frac{1}{\gamma_F} n_p M_i q \right)^2.
\]
Finally, with
\[ F_{33} = -\frac{1}{4} n_p^2 M^2 i_q^2 \]
the previous equation reduces to
\[ \frac{1}{k} \frac{1}{\omega} B_r > \frac{1}{\gamma F}. \]

In order to simplify the controller the following parameters are assigned
\[ \gamma_q = n_p M i\omega, \]
\[ \gamma F = k_F \frac{4}{n_p^2 M^2}, \]
and the control actions are finally
\[ v_d = R_s i_d - n_p \omega L_s i_q - k_d (i_d - i_d^*), \]
\[ v_q = n_p \omega L_s i_d + R_s i_q + n_p \omega M i\omega F - k_d (\omega - \omega^*), \]
\[ v F = R_F i_F - k_F i_F^2 (i_F - i_F^*), \]
while the restriction can be expressed as
\[ k\omega < B_r \frac{(n_p M)^4}{16} k_F^2. \]

From the point of view of energy efficiency, it is desirable for an electrical machine to have a mechanical damping as close to zero as possible, and many control design strategies assume that \( B_r = 0 \). In our case, mechanical losses play a fundamental role, because the \( k\omega \) value is bounded from above by \( B_r \), and this seems to limit the range of available \( k\omega \). However, even if \( B_r \) is very small, it cannot be strictly zero and, taking into account that \( k_F \) is a free tuning gain, the open set for which the closed loop system is stable can be enlarged.

**IV. SIMULATIONS**

In this section we present some simulations using the designed controller. The WRS parameters are: \( L_s = 1 \text{mH}, R_s = 0.0303 \Omega, M = 1.5 \text{mH}, L_F = 8.3 \text{mH}, R_F = 0.0539 \Omega, n_p = 2, J_m = 0.01525 \text{kg\cdotm}^2, B_r = 0.05 \text{N\cdotm\cdots}, \) and \( \tau_L = 0 \text{N\cdotm} \). The control parameters are selected as: \( k_d = 1, k_F = 10 \) and \( k\omega = 0.001 \).

The first numerical experiment is performed increasing the desired speed from \( \omega^* = 100 \text{rad\cdots}^{-1} \) to \( \omega^* = 150 \text{rad\cdots}^{-1} \) at \( t = 2 \text{s} \). As pointed out in Section II, the desired d-current must have a small value in order to reduce the electrical losses; in our case, it is fixed at \( i_d^* = -0.1 \text{A} \). Figure 2 shows that the system is perfectly regulated under changes of the desired outputs.

Figure 3 shows the results of a second test. In this case the external torque is suddenly changed at \( t = 2 \text{s} \) from zero to \( \tau \omega = -0.8 \text{N\cdotm} \). Even under this change, the system achieves the control goals due to the fact that some robustness has been built-in in the design of the controller.

**V. CONCLUSIONS**

The SIDA-PBC technique has been applied to control a wound rotor synchronous machine for a motor drive application. The SIDA-PBC matching equation has been solved using the algebraic approach and the desired robustness of the resulting controller has been taken into account. The obtained controller is globally asymptotically stable and assures stability for a large range of control gain values. The presented method also allows to decouple the outputs, improving the robustness and facilitating the gain tuning.

Future research includes a dynamical extension keeping the Hamiltonian structure to improve the robustness (basically on the electrical parameters \( R_s, L_s, M, \) and \( R_F \)) and the behavior. This can be easily done for the \( i_d \) and \( i_F \) currents (due to the fact that they are passive outputs), but a more complicated task is to design a dynamical extension for the speed loop. Experimental validation with a real plant using the presented control law will be also considered in the future.

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Fig. 3. Simulation results: Mechanical speed, reactive power and $i_d$ current, for a change of the external torque.

REFERENCES


