A Probabilistic Approach to Process Identification and Control

A Case Study in Pulp Bleaching Focused on the Stationary Probability Density Function

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Outline

• Stochastic Processes
• Probabilistic Framework
• Perspective for Identification and Control
• Bleaching Reactor
• Identification
• Controller Design
• Conclusions
Practical realities of the processing and manufacturing industries:

- Nonlinear processes,
- (Non-Gaussian) random disturbances – prevents true equilibrium,
- Quality specifications
Quantifying Performance

– quadratic loss functional about setpoint; nonquadratic loss functional for quality specification
Process Time Series, Dispersion

– time series and histogram for process data
Mathematical Framework

\[ x_{t+1} = f(x_t, u_t) + w_t, \quad u_t = k(x_t) \]

\[ w_t \sim p_w(w_t) \]

\[ p(x_{t+1}) = \int_{\mathcal{D}_x} p_w(x_{t+1} - f(x_t, k(x_t))) p(x_t) \, dx_t \quad (\star) \]

Model Identification

\[ \hat{x}_{t+1} = \hat{f}_a(x_t, u_t) \rightarrow e_t = \hat{x}_t - x_t \]

\[ \rightarrow J = \frac{1}{N} \sum_{t=1}^{N} e_{t+1}^T e_{t+1} \]

PEM – L. Ljung (1999)

\[ \int_{\mathcal{D}} \left( \hat{f}_a(x, u) - f(x, u) \right)^T \left( \hat{f}_a(x, u) - f(x, u) \right) p(x, u) \, dx \, du + \Sigma \]
Model Error

– exact and approximate discrete-time process feedbacks
Control Design

\[ J = E[\ell(x_t, u_t)] \]

\[ J = \int_{D_x} \ell(x_t, k(x_t)) p(x_t) \, dx_t \]

P.R. Latour (1996), T.J. Harris (1992)
Control Design

- parameterize control law: \( u_t = k(x_t) \approx \hat{k}(x_t; a) \)
- parameterize stationary PDF: \( p(x) \approx \hat{p}(x; c) \)
- find parameters to optimize objective
- implicit constraint on parameters from (*)
PDF-Shaping

- in previous design, shape of PDF comes out of optimization
- instead of using objective function, pick a good PDF
- find corresponding controller parameters

The Bleaching Reactor

– bleach mixed with pulp, mixture flows through reactor, lignin removed

• PDE’s for lignin and bleach
• boundary conditions, initial conditions
• include convection, dispersion and reaction terms

\[
\frac{\partial C}{\partial t} = -v \frac{\partial C}{\partial z} + D \frac{\partial^2 C}{\partial z^2} - kC^3 L^3
\]
Time Series
Model Building from Time Series

– tried different orders, nonlinearities
– plots of model errors
– improvements in range of 28 – 29%
inlet lignin concentration is measured feedforward disturbance variable, needs probabilistic description
Control

\[ J = E \left[ \ell \left( L_{k+b}^{\text{out}} \right) + \frac{(C_{k}^{\text{in}} - 40)^2}{4000} \right] \]

where:

\[ \ell \left( L_{k+b}^{\text{out}} \right) = \begin{cases} 
1.4 - L_{k+b}^{\text{out}} & L_{k+b}^{\text{out}} \leq 1.4 \\
5 + \left( L_{k+b}^{\text{out}} - 1.4 \right)^2 & L_{k+b}^{\text{out}} > 1.4 
\end{cases} \]

– one design based on removing input loss from expectation, one design based on full, unconditional expectation
Control Results

– Summations of errors are $J = 3221$ and $J = 2497$. 
Discussion

- very different behaviours
- first bounded, ‘gives up’ sometimes, tends to be myopic
- second strategy more aggressive, keeps lignin concentration lower
- 22% decrease in the cost for the second control law
Summary

- demonstrated use of probabilistic concepts to benefit identification and control in a process control setting
- ideas based on probabilistic concepts lead to improved experimental designs for the generation of identification data – improved process models
- use of unconditional expectation in regulatory controller design leads to better long term performance
Future Work

Probabilistic techniques for processing and manufacturing industries is an open area of research:

- continued interested in shaping distribution of identification data
- impact of random feedforward variables on PDF of process can be analyzed in general
- investigating full output feedback case
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Questions?