Iterative modeling & control design
Tying together the threads

Modern model-based control needs two things

A local performance objective using the nominal model
A robustness measure involving the rough model error

Modeling with closed-loop data tells us about the model mismatch

Performance-based control design comes with a performance expectation

\[
\left\| \begin{pmatrix} (1 + P(e^{j\omega})C(e^{j\omega}))^{-1} & (1 + P(e^{j\omega})C(e^{j\omega}))^{-1} C(e^{j\omega}) \\ P(e^{j\omega}) (1 + P(e^{j\omega})C(e^{j\omega}))^{-1} & P(e^{j\omega}) (1 + P(e^{j\omega})C(e^{j\omega}))^{-1} C(e^{j\omega}) \end{pmatrix} \right\|_{\infty} \leq \gamma
\]

\[
\left| \frac{P(e^{j\omega}) - \hat{P}(e^{j\omega}, \theta)}{\hat{P}(e^{j\omega}, \theta)} \times \frac{C(e^{j\omega})\hat{P}(e^{j\omega}, \theta)}{1 + C(e^{j\omega})\hat{P}(e^{j\omega}, \theta)} \right| < 1
\]

\[
\left| H(e^{j\omega}) \times \frac{P(e^{j\omega}) - \hat{P}(e^{j\omega}, \theta)}{\hat{P}(e^{j\omega}, \theta)} \times \frac{C(e^{j\omega})\hat{P}(e^{j\omega}, \theta)}{1 + C(e^{j\omega})\hat{P}(e^{j\omega}, \theta)} \times \frac{1}{1 + C(e^{j\omega})P(e^{j\omega})} \right| < \epsilon
\]
Key principles

Model-based control performance depends on nominal model and model quality

Nominal model and model quality depend on the controller operating when the data is collected

Can we match up these two issues?

Can we set up the problem so that the successive controllers cause successive model to become more appropriate?

Try to link all the frequency-domain formulae to share a common objective

But first some clues ...
First-cut controller $C_0(z)$

Design experimental reference signal $r(t)$, collect data

Fit a process model with data selection and filtering to reflect objectives

Design a Linear Quadratic Gaussian controller with Loop Transfer Recovery (LQG/LTR) - a level of computed robustness

Expect the best but challenge the assumptions

Run the controller validation test

The control design comes with expected performance measures

Spectra of controlled signals

Deviation from expected tells us about model quality for that controller
PID performance vs $C_0(z)$

We expected much lower variance, less spectral coloring.
Aha!

Let's look at the spectrum of the torque from the LQG/LTR controlled system.

According to my calculations

\[ \hat{\Phi}_t(\omega) = \left| \frac{\hat{H}(e^{j\omega})}{1 + \hat{P}(e^{j\omega})C_0(e^{j\omega})} \right|^2 \]

But it really is measured to be

\[ \Phi_t(\omega) = \left| \frac{H(e^{j\omega})}{1 + P(e^{j\omega})C_0(e^{j\omega})} \right|^2 \]

Frequency-weight the next LQG control design criterion to accommodate this

\[ J_{LQG} = \frac{1}{N} \sum_{k=1}^{N} \left\{ [F(z) t_k]^2 + \lambda [F(z) s_k]^2 \right\} \]

\[ = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ |F(e^{j\omega})|^2 \Phi_t(\omega) + \lambda |F(e^{j\omega})|^2 \Phi_s(\omega) \right\} d\omega \]

Aha! Choose frequency-weighting

\[ F(z) = \frac{H(z)}{1 + P(z)C_0(z)} \times \frac{1 + \hat{P}(z)C_0(z)}{\hat{H}(z)} \]

Use this to design \( C_1(z) \) using the same model.
A sequence of control adjustments

Model
Control design
Refined control using data-based frequency-weighting
•
•

Re-model the process with data from latest controlled data
Redesign controller
Refine controller
•
•
•
•
Sugar mill controllers

- PID controller $C_{-1}(z)$
- Model with PID derived data $\hat{P}_0(z)$
  - LQG/LTR un-frequency-weighted control $C_0(z)$
  - Frequency-weighted controller adjustment $C_1(z)$
  - Frequency-weighted controller adjustment $C'_1(z)$
- New model identified with $C_1(z) \hat{P}_1(z)$
  - LQG/LTR un-frequency-weighted control $C_2(z)$
  - Frequency-weighted controller adjustment $C'_2(z)$
- Stop at controller $C_2(z)$
Ultimate performance

A2 Torque LQG Controller 2'

mean 1.01 MNm
std 0.03

03-Nov 16:00
04-Nov 04:00

A2 Chute Height LQG Control 2'

mean 39.4 %
std 6.1

03-Nov 16:00
04-Nov 04:00
Some conclusions

There is much to be gained from examining the modeling and controller design as a joint problem.

Modeling is fairly expensive

So it makes sense to re-use the model via controller tuning.

The closed-loop data are really informative about controller performance

which, in turn, is informative about model quality

Iterative solution is necessary

It is possible to tune the controller without a model at all

Iterative Feedback Tuning

Based on gradient calculation from data.
Iterative feedback tuning

Controller is parametrized by a set of numbers $\rho$  \[ C_\rho(z) \]

Try to optimize the choice of $\rho$ using experimental data

Estimate the gradient of the performance criterion

Gradient is calculated by filtering closed-loop signal through filters such as

\[
\frac{\partial C_\rho(z)}{\partial \rho} = F'(z)
\]

Adjust controller using gradient

\[
\rho_{k+1} = \rho_k - \alpha \frac{\partial J_{LQG}}{\partial \rho}
\]
IFT performance - distillation column

Control Error Over 1 Day - Initial Tuning

Control Error Over 1 Day - After 6 Iterations

From: Hjalmarsson, Gevers, Gunarsson, Lequin, CSM, 1998
Many reports of applications success
Local minimizers, model-free adaptation

APC04 Vancouver
Joint Modeling & Control Design
Caution in controller tuning

Care is needed in moving between controllers

Guarantees of stability and performance would be helpful

But we need to look at more than the performance alone
Vinnicombe’s $\nu$-gap metric

$$\delta_\nu(C_1, C_2) = \max_\omega \left| (1 + |C_1(e^{j\omega})|^2)^{1/2} (C_1(e^{j\omega}) - C_2(e^{j\omega})) (1 + |C_2(e^{j\omega})|^2)^{1/2} \right|$$

$$\delta_\nu(C_1, C_2) \in [0, 1]$$

Frequency-domain distance measure between systems/controllers

There is an additional phase condition

If $P$ stabilizes $P$ define the stability margin

$$b_{P,C_1} = \left[ \max_\omega \left| \begin{pmatrix} P(1 + C_1 P)^{-1} C_1 & P(1 + C_1 P)^{-1} \\ (1 + C_1 P)^{-1} C_1 & (1 + C_1 P)^{-1} \end{pmatrix} \right| \right]^{-1}$$

$$b_{P,C_1} \in [0, 1]$$

Then we have the following guarantee

$$\arcsin b_{P,C_2} \geq \arcsin b_{P,C_1} - \arcsin \delta_\nu(C_1, C_2)$$
What went wrong?

![Graph showing the relationship between two variables with a noticeable anomaly near a specific point, labeled as \( b_{P,C_{fb}} \).]
Conclusions

There is an emerging group of frequency domain formulae and expressions which assist in dealing with linking

Control stability and performance robustness

Model approximation bias and variance

Controller adjustment using gradient methods

Stability and performance guarantees

Underlying much of this is the analysis of closed-loop experimental data prior to modeling and after controller implementation

It makes sense to view the eventual control design as providing guidance to the testing and validation of models

Likewise the margin analysis yields limits to controller tuning

Closed-loop data and vigorous tests are the key