Modeling of an Industrial Vibrating Double-Deck Screen of a Urea Granulation Circuit

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Ivana M. Cotabarren, José Rossit, Juliana Piña, Verónica Bucalá
Chem. Eng. Department, Universidad Nacional del Sur, Bahía Blanca, Argentina

Introduction

Urea granulation is a complex operation that cannot be carried out in a single device; it is rather achieved by a combination of process units with specific functions constituting a granulation circuit (Figure 1). The main unit is the granulator where small urea particles, known as seeds (generally out of specification product), are continuously introduced and sprayed with a concentrated solution of the fertilizer (melt). The seeds grow through deposition of the melt droplets onto the solids surface, followed by urea solidification and water evaporation. The granules that leave this size enlargement unit are cooled down and subsequently classified by double-deck screens into product, oversize and undersize streams. The product is transported to storage facilities, while the oversize fraction is fed to double-roll crushers for size reduction. The crushed oversize particles are then combined with the undersize granules and returned to the granulator as seeds.

Many authors found that the operation of the screening and crushing units has a decisive influence on the recycle stream and hence on the circuit stability. In view of this and the recognized important role of plant simulation to predict and optimize the granulation circuit operation, reliable models for all the process units should be available. Recently, a validated mathematical model for the industrial double-roll crusher of a urea granulation circuit has been reported.

The fertilizer industry requires, as aforementioned, a classification step that is usually performed by double-deck vibrating screens. These devices have been extensively studied by numerous authors in the context of the mineral processing industry. Models in the literature can be classified as phenomenological, empirical and numerical; being based on theory of the screening process, empirical data and computer solutions of Newtonian mechanics, respectively.

Within the phenomenological models two different approaches, the kinetic and probabilistic, have been used to represent these screening operations. The probabilistic approach is based on the probability of a particle passing through the aperture of the screen. On the other hand, the kinetic approach defines the screening performance as a rate process that varies with the distance along the screen and depends on the amount and particle size distribution of the material being processed. The empirical models aim to predict the quantity of undersize that can pass the screen. Among these, some are based on a theoretical capacity which is affected by a set of correction factors accounting for the effect of oversize, half-size, and near-size material; whether the screen is a top or lower deck on a multi-deck unit; type of aperture; material density, etc. Due to their simplicity, the empirical models are preferred for the simulation of industrial screens.
fact, many commercial simulators such as Aspen Plus, Modsim, and Moly-Cop Tools have empirical models implemented in their routines.

![Typical urea granulation circuit.](image)

In this contribution, a fitted model for a large-scale double-deck vibrating screen of an industrial urea granulation plant is presented. After thorough examination of several empirical, probabilistic and kinetic models, it was concluded that the model proposed by Karra is the most suitable to describe the screening operation in terms of the available industrial data. This empirical approach results of great simplicity and high efficiency since it manages to predict all the experimental data (approximately 940 points) by just fitting three parameters. Besides, it is usually recommended for predicting the performance of industrial vibrating screens and has demonstrated to be overwhelming respect to other empiric models to predict the urea granules classification.

In order to estimate the screen performance, all the flowrates around the equipment and their corresponding particle size distributions must be known. It is of common knowledge that significant errors may occur when attempting to measure solid streams flowrates. In fact, the only reliable and available experimental mass flow for this work was that corresponding to the product stream. The remaining and unmeasured individual mass flows were determined through a data reconciliation procedure, which simultaneously allows modifying the most unreliable measured variables to satisfy the material mass balances for every size interval. Indeed, the experimental results confirm the need of using solid streams data reconciliation to have industrial data consistency.

**Data reconciliation**

In the present work, the screen classification parameters were fitted using industrial data from a plant of high capacity. The experimental data were collected from two large-scale double-deck vibrating screens of identical characteristics. They operate with a small angle of inclination and with fixed vibration. Samples by duplicate, of the feed ($F$) and of the oversize ($O$), product ($P$) and undersize ($U$) streams of each of the two screens (A and B) were collected and granulometrically analyzed (Figure 2).
Data reconciliation was formulated as a constrain optimization problem and solved by means of the Athena Visual Studio Software. The goal of this optimization problem was to minimize the total difference between measured and estimated mass fractions for each size interval, weighted by the variance of the measurements using the least square method. Defining \( F, O, P \) and \( U \) as the feed, oversize, product and undersize mass flows and \( X_{F,i}, X_{O,i}, X_{P,i}, X_{U,i} \) as the mass fractions for the size class \( i \) of the feed, oversize, product and undersize streams related to screens A and B; the objective function was expressed as:

\[
f_0 = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( X_{ji} - \overline{X_{ji}} \right)^2 / W_{ji}
\]  

where \( N \) was the number of size intervals (i.e. 13) and \( M \) was the number of streams around the screens with different particle size distributions (i.e. 7: \( O_A, P_A, U_A, O_B, P_B, U_B \) and \( F \), since \( F_A \) and \( F_B \) have the same mass fractions); \( \overline{X_{ji}} \) represented the average value of duplicate measurements; \( X_{ji} \) corresponded to the estimated value; and \( W_{ji} \) was the variance associated to each \( \overline{X_{ji}} \).

To completely formulate the data reconciliation, the objective function \( f_0 \) was subjected to the following restrictions:

\[
F_A X_{F,i} = O_A X_{O,i} + P_A X_{P,i} + U_A X_{U,i}
\]  

\[
F_B X_{F,i} = O_B X_{O,i} + P_B X_{P,i} + U_B X_{U,i}
\]  

\[
\sum_{i=1}^{13} X_{ji} = 1 \quad j = 1..7
\]  

\[
F_A / (1 - \lambda) = F_B / \lambda
\]  

\[
P_A + P_B = P
\]
where $\lambda$ was the fraction of the total feed derived by a diverter to screen B (Figure 2) and $P$ corresponded to the measured product flowrate, which was not a variable to be reconciled. The parameter $\lambda$ was fitted during data reconciliation to estimate the actual flowrate being fed to screens A and B during the experimental tests.

Figure 3 condenses the correspondence between the reconciled and experimental mass fractions for each particle size and solid stream. The observed differences are as expected, considering the well known uncertainties when sampling and analyzing solid streams. From the 1638 total experimental points (Test 1: 13 size classes x 7 streams x 10 samples; Test 2: 13 size classes x 7 streams x 8 samples), the 92% is included within the 20% deviation line, and 81% within the 10% one.

Figure 3. Reconciled vs. experimental mass fractions of all the size classes.

The feed flow split parameter ($\lambda$) varied from 0.43 to 0.56, being its average value 0.48. These results indicated that the diverter was almost capable to split the feed evenly, in good agreement with its initial position selected to perform the experiments. The feed flowrates ($F$) estimated through the reconciliation procedure were in good agreement with the values calculated from the energy balance of the cooler (see Figure 1) located downstream the granulation unit. The reconciliation step was essential to have consistent industrial experimental data to fit the screen models.

Screen Model

An ideal classification step would separate perfectly the solid stream, i.e. the particles bigger than the mesh aperture would be retained while the smaller ones should pass through the screen. However, in practice, separation processes are not perfect and the probabilities of reaching the screen and passing through it are not the same for different particle sizes.

The most realistic measurement of screen performance is provided by the partition curves. To represent a non-ideal classification operation, like the actual screening process, Karra\textsuperscript{1} used oversize partition coefficients for each size interval. For
each class size \( i \), the oversize partition coefficient (\( T_i \)) is defined as the amount of oversize within interval \( i \) divided by the amount of material of that size in the feed.

\[
T_i = \frac{O X_{Oi}}{F X_{Fi}}
\]  

(7)

Once all \( T_i \) are known, the particle size distributions of the oversize and undersize streams can be calculated through simple mass balances for the solids belonging to each size class \( i \).

\[
O X_{Oi} = T_i F X_{Fi}
\]  

(8)

\[
U X_{Ui} = (1 - T_i) U X_{Ui}
\]  

(9)

The oversize partition values (\( T_i \)) are usually plotted against the corresponding geometric mean particle size:

\[
d_i = \sqrt{d_1d_2}
\]  

(10)

where \( d_1 \) and \( d_2 \) are the lower and upper limits for size interval \( i \), respectively.

In order to develop an empirical model capable of describing the screening performance, a mathematical formulation of the partition curve is needed. Such formulation requires determining the functionality with respect to the \( d_{50} \) parameter. This cut size (median) is defined as the size corresponding to the 50% partition curve, and thus represents the size at which particles have equal chance of staying on the screen or passing through. The cut size is a measure of the screening efficiency; \( d_{50} \) values close to the mesh size indicate high efficiencies. Nevertheless, the cut size is always less than the aperture.\(^{17}\) It has been established that process parameters such as particle to aperture ratio, composite nature of the feed, deck location, feed rate and bulk density, affect the \( d_{50} \) significantly.\(^{11}\) This dependence was also observed analyzing the available experimental data.

Karra\(^{11}\) proposed the following \( d_{50} \) correlation to represent industrial screening data:

\[
d_{50} = h \left( \frac{U^T}{SK} \right)^a
\]  

(11)

where \( h \) is the screen aperture (mm), \( U^T \) are the theoretical undersize tons per hour fed to the screen (fed material whose size is smaller than the aperture size); \( S \) is the screen surface (m\(^2\)); \( K \) is a product of factors that correct the screen basic capacity to represent the deviation from the standard conditions and adjustment to the specific conditions and \( a \) is a fitting parameter.

The variable \( K \) is expressed as follows:

\[
K = A B C D E F D G
\]  

(12)

where \( A \) is the basic capacity, defined as \( \text{ton/h} \) of undersize that a particular screen can process per unit of screen area. \( B \) is the amount of oversize in the feed (percentage of material with \( d_i > h \)). Screens that handle great amount of oversize operate more efficiently because that material is directly recovered over the screen. \( C \) is the amount of half-size under in the feed (percentage of material with \( d_i < 0.5 h \)). Feeds containing a large proportion of material considerably smaller than the aperture size, manage to manipulate it more easily. \( D \) is the location of the deck. Lower decks receive material
harder to handle than those decks that take fresh feeds; therefore the capacity decreases with position. $E$ is for wet screening. If dry screening is performed, as in the urea process, its value is set to one. $F_D$ is the bulk density factor; denser materials are separated more easily than lighter materials. $G$ is the amount of near-size material in the feed (percentage with material of $0.75 \ h < d < 1.25 \ h$). Feeds with large quantities of particles close to the aperture size present more difficult separation because the passage of undersize material is inhibited.\(^{24}\)

Karra\(^{11}\) used the standard expressions given in the Nordbeg Process Machinery Reference Manual\(^{25}\) for parameters $A$, $B$, $C$, $D$, $E$ and $F_D$ and developed his own correlation for the near-size material factor $G$. The corresponding equations are presented in Table 1.

### Table 1. Equations for screen correction factors.\(^{11}\)

<table>
<thead>
<tr>
<th>Factor</th>
<th>Equation</th>
<th>Reference</th>
</tr>
</thead>
</table>
| $A$    | $A = 12.1286 h^{0.3162} - 10.2991$ if $h < 50.8 \ mm$  
|        | $A = 0.3388 h + 14.4122$ if $h \geq 50.8 \ mm$ | $h =$ opening aperture (mm) |
| $B$    | $B = -0.012 Q + 1.6$ if $Q \leq 87$  
|        | $B = 0.0425 Q + 4.275$ if $Q > 87$ | $Q =$ % of oversize in feed to deck |
| $C$    | $C = 0.012 R + 0.7$ if $R \leq 30$  
|        | $C = 0.1528 R^{0.564} - 30 < R < 55$  
|        | $C = 0.0061 R^{1.37}$ if $55 \leq R < 80$  
|        | $C = 0.05 R - 1.5$ if $R \geq 80$ | $R =$ % of half-size in feed to deck |
| $D$    | $D = 1.1 - 0.1L$ | $L =$ deck location (top = 1, bottom = 2) |
| $E$    | $E = 1.0$ if $T < 1$ | $T = 1.26 h$ |
| $F_D$  | $F_D = \rho_B / 1602$ | $\rho_B =$ bulk density (kg/m$^3$) |
| $G$    | $G = 0.975(1 - X^{0.511})$ | $X =$ % of near-mesh in the feed |

Once $d_{50}$ is determined, its relationship with the $T_i$ function has to be defined. Several standard functional forms are available to describe it, Karra\(^{11}\) proposed:

$$T_i = 1 - \exp\left[-0.693(d_i / d_{50})^m\right]$$  \hspace{1cm} (13)

This correlation, recognized as the Plitt equation, is expressed in terms of the well known Plitt’s adjustable parameter $m$, which defines the classification sharpness.\(^{26}\)

### Parameter estimation

In this work, the Karra\(^{11}\) model was adapted to describe the performance of two large-scale double-deck screens from an industrial urea granulation plant, once all the experimental data were reconciled.

Each deck was considered as a separate screen; therefore a set of parameters was adjusted for the top deck and another for bottom deck. Following the procedure described in the previous section, the correction factors $A$ to $G$ were determined for both decks.
using the expressions given in Table 1. The screen factor \( E \), corresponding to wet screening, was equaled to 1.

For the special case under consideration, the bulk density corresponding to each industrial deck was not a measurable property because it varied along the screen length with the quality of the material being classified. For this reason, it was set as a fitting parameter for both decks.

Once the theoretical undersize \( U^T \) and all the correction parameters that defined \( K \) were evaluated for each test hour and deck, the \( d_{50} \) were calculated by means of Equation (11) and the corresponding partition coefficients curves were established.

The adjustment of the bulk density \( (\rho_B) \), the Plitt parameter \( (m) \) and the exponent in \( d_{50} \) \((a)\) for each deck, was performed by using the Athena Visual Studio Software\(^\text{23}\) with the aim of reproducing the respective cumulative undersize and oversize streams. The undersize from the top deck was considered as the feed to the bottom deck.

**Results**

Table 2 reports the estimated model parameters. The fitted values are in agreement with the usual operation of both decks. In fact and due to the greater amount of oversize material (i.e. higher bed porosity), the bulk density for the top deck results lower than the one corresponding to the bottom deck. The solid urea density is 1330 kg/m\(^3\), so the bulk densities found for the top and bottom decks indicate bed porosities of about 58% and 47% respectively, which are expected values. The separation sharpness as a function of the feed quality is clearly represented through the adjusted Plitt parameter. The top deck handles material with a greater proportion of oversize than the bottom deck, being more effective in the classification process. Thus, the higher efficiency of the first deck, given by a significant higher \( m \) value, is confirmed.

The exponent in the \( d_{50} \) correlation \((a)\) can be considered as an indicator of the cut size evolution during the operation. According to the fitting results, the \( d_{50} \) for the top deck remains almost equal to the screen aperture (for very high \( m \) values \( T_i \) tends to 1, see Equation 13). This result is in agreement with the higher efficiencies found for this first deck. Contrary, the \( d_{50} \) for the bottom deck is a strong function of the operating conditions and differs considerably from the corresponding \( h \) value. Indeed, the highest \( d_{50}/h \) ratio over time obtained for the top deck was 0.4% against 31.9% for the bottom one.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Top deck</th>
<th>Bottom deck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density ((\rho_B)) (kg/m(^3))</td>
<td>560</td>
<td>705</td>
</tr>
<tr>
<td>Power of (d_{50}) equation ((a))</td>
<td>-0.0023</td>
<td>-0.2862</td>
</tr>
<tr>
<td>Plitt’s parameter ((m))</td>
<td>25.097</td>
<td>3.758</td>
</tr>
</tbody>
</table>

Figure 6 shows the experimental and predicted oversize \((O)\) and undersize \((U+P)\) particle size distributions \((PSDs)\) for the top deck. Two samples from screen A (Figures 6 a and c) and two from screen B (Figures 6 b and d) among the 36 acquired, have been included in order to illustrate the goodness of the model. The samples presented in Figure 6 c and d represent the worst \(PSDs\) prediction obtained for screens A
and B, respectively. Considering the relatively high number of experimental points (i.e. 468, 18 samples x 13 classes x 2 screens) to be predicted for each stream (i.e., undersize and oversize) by estimating only 3 parameters, it can be concluded that the fitted model satisfactorily reproduces the available experimental data. In fact, for the oversize stream, 75% of the predicted points have less than 5% of error, 81% less than 10% and 92% less than 20%. For the undersize stream the results are even better, 97.7% of the data are within 5% of error, 98.8% within 10% and 99.6% within 20%.

Figure 6. Some selected experimental and predicted size distributions for the top deck.

Figure 7 presents some selected experimental and predicted PSDs of the oversize \( P \) and undersize streams \( U \) for the bottom decks of screens A and B. The oversize corresponds to the product stream and the undersize to the fraction of fines recycled directly from the double-deck screens to the granulator. Again, two distributions from each screen are shown, being Figures 7 c and d representative of the worst predictions. Regarding these bottom decks, 84% of the predicted points for the oversize stream have less than 30% error and for the undersize stream 81% have less than 30% error. In general, the prediction is not as good as that obtained for the top deck. Nevertheless, and taking into account the errors inherent to sample collection, it can be considered to be in reasonable agreement with the experimental data.

The model previously described, together with the parameters presented in Table 2, can be considered a suitable mathematical representation of the industrial double-deck screens. It is worth to mention that this model is capable of satisfactorily predict changes in the PSD and mass flowrates of the feed stream. Besides, it gives valuable information about the influence of the screening operation on the particle size.
distributions of the fines recycled to the granulation unit, the oversize fed to the double-roll crushers and the product derived to storage facilities.

![Graphs showing size distributions for Screens A and B](image)

**Figure 7.** Selected experimental and predicted size distributions for the bottom deck.

**Conclusions**

The simple empirical model proposed by Karra,\textsuperscript{11} for the mineral processing industry, was satisfactorily adapted for predicting the classification performance of large-scale double-deck screens from an industrial urea granulation plant. The data reconciliation procedure was found as an unavoidable step to estimate the unknown mass flow rates and correct the experimental measured data to satisfy the material balances for every particle size interval. By applying this new technique to the industrial granulation plant under study, it was possible to obtain a set of reliable and consistent experimental data for subsequent modeling, supporting its usefulness for handling data belonging to solid processes.

The presented model allowed predicting successfully the experimental $d_{50}$, a parameter found to be a key model variable. The goodness of the fitted model was widely confirmed over a large number of samples collected from the industrial units, operating with different feed PSDs and mass flowrates.

Market conditions for agricultural commodities and consequently for the urea are exceptionally favorable since the first half of 2007. The demand of urea is forecast to grow about 40% in less than three years. In this context, knowledge improvements to operate more efficiently urea granulation plants will be extremely worthy. In line with the projected urea market conditions, the presented mathematical model is a valuable
contribution for the understanding of the urea classification step and the improvement of the existing knowledge in granulation circuits. It is useful not only to predict the particle size distribution of the product, crusher feed and undersize recycle for specified conditions, but also to determine the effect of different operating variables (such as deck apertures) on the screen performance. In addition, it represents an accurate module for being used in plant simulators of the complete urea granulation circuit.

Literature Cited