Prediction of Cone-in-cone Blender Efficiencies and Scale-up Parameters From Knowledge of Basic Material Properties

Author: Kerry Johanson

Engineering Research Center for Particle Science and Technology PO Box 116135, University of Florida, Gainesville, FL, 32611-6135, kjohanson@erc.ufl.edu

Abstract:

Material blending can be done in either a continuous mode or a recycle batch mode. There are many commercial blenders available today, with new versions being developed continually. Most of these blenders require mechanical means to mix material. The size of the blender required to achieve good mixing depends on the residence time distribution function. It also depends on the relative volume of the concentration fluctuations passing through a continuous mode blender or the time available for completion of a batch mixing process. The cone-in-cone blender offers the advantage in that it can be constructed to meet almost any size requirement. It is most useful when utilized in a continuous blending mode. However, it can be adapted to batch mode operation by using external recycle.

Knowledge of the material properties and the blender geometry allows calculation of radial velocities developed in mass flow blenders. These velocity calculations can be employed to compute residence time distribution functions for cone-in-cone blenders and estimate the variance reduction factors for these continuous mode blenders. Ideally, the variance reduction factor should be as low as possible to ensure good blending. The reduction factor is a ratio of the output concentration variance to the input concentration variance. The reduction factor depends on the number of concentration fluctuations per blender volume of throughput and the width of the residence time distribution function. In general, increasing the number of concentration fluctuations per blender volume decreases the variance reduction factor.

This paper discusses the effect of varying material properties on blender operation. The main material property governing blender operation is wall friction angle. The primary focus of this paper is to compute the residence time distribution function knowing the blender geometry and material properties, and use that distribution function to estimate blending effectiveness. In fact, this general approach separates the effect of process and product variable on blending effectiveness making blender scale up possible. When this analysis is applied to cone-in-cone blenders, the results indicate that friction angle has little influence on blending effectiveness when the number of concentration fluctuations per blender volume is large, but will significantly effect blending...
behavior if the input concentration variations comprise a predominate percentage of blender volume. The influence of gas pressure gradients on blending efficiency is also considered in this work.

Key Words: Mass flow, Mixing, Residence Time Distributions, Powders, Solid Mechanics

1. Introduction

Acceptable blending requires three things. First, a distribution of residence times of material within the blender. Second, all material within the blender must be in motion. Third, the blending must result in mixing on a scale similar to the size of the final product sample. Wider residence time distribution functions result in better blending. Blending can therefore be accomplished by imposing a velocity profile across a given piece of process equipment resulting in a distribution of residence times within the blender. In mechanical blenders the velocity flow field is complex resulting in particle flow paths that cross multiple times before exiting the blender. Since all particle flow paths do not travel the same distance before exiting the blender and individual particle velocities are different, the complex flow paths result in a residence time distribution function. Ideally, adjacent particles in a blender would have very different flow paths causing significant inter-particle mixing in addition to large residence time distribution functions. However, real blenders always have shear velocity profiles caused by flow around paddles or screw flights combined with the dynamic particle trajectories caused by airborne material. The airborne particles produce significant axial and radial free surface deposits and free surface flow profiles resulting in inter-particle mixing while the velocity field around the paddles results in smoother velocity profiles and axial blending. Both types of velocity profiles result in blending. Thus, to understand blending as an unit operation, one must be able to estimate the velocity profiles in both the axial shear flows and during free surface flows. Every blender will have a combination of these velocity types. Mechanical blenders will generally produce more surface flow than gravity flow blenders. For example, in simple blenders such as the cone-in-cone, the velocity is a relatively smooth function of radial position in the blender. A particle at the blender wall travels slower than a particle entering the center of the blender. These two particles exit the blender at different times causing mixing as material flows through the blender. However, particle velocity paths rarely cross each other in this type of blender resulting in almost pure axial blending. In fact the only radial velocity that occurs in these blenders is caused by the filling process. Here material entering the blender cascades down an inverted conical pile, resulting in significant inter-particle motion. This suggests that blending models could be enhanced by understanding the role of both shear velocity profiles and free surface flow profiles. In a cone-in-cone blender, the radial velocities may not mix material adequately. Consequently, any radial segregation patterns that exist at the entrance of the blender will exist at the exit. This is not a major problem to continuous blending operation provided the entire cross section of the
outlet is used to fill a sample container. If better blending is required, then radial velocity profiles must be used to increase the inter-particle blending. For this reason, cone-in-cone blenders are usually accompanied by a radial distribution chute at the inlet that splits and directs the flow stream to different radial positions in order to minimize radial segregation. Alternatively, a small radial blender could be placed at the exit of the cone-in-cone to intimately mix the material as it leaves the blender. A screw feeder naturally does this to some extent, while a belt feeder retains the radial segregation pattern.

Understanding blending in a cone-in-cone will require computing the velocity profile in the inner and outer cone, developing the velocity flow pattern for the material flowing down a pile, and creating models for free surface segregation, and applying convection dispersion equations in appropriate blender flow zones to compute blender output concentrations. This information can then be used to compute the residence time distribution function for a given blender. Once the residence time distribution function is known, any output concentration can be predicted by combining the input concentration function and the residence time distribution using a convolution integral. The standard way of evaluating continuous powder blender performance is to place a sudden impulse of markers in the blender then operate the blender while maintaining the level in the blender and observe the discharge profile of markers leaving the blender. This blending profile, when normalize to produce a total integrated concentration of one, is the residence time distribution function. The problem with this analysis is that changing the material, adjusting the rate, or slight modifications to the blender requires a whole new blender test to develop the new residence time distribution function. Thus scale-up of blender performance is nearly impossible for most blenders.

However, residence time distribution functions could be computed provided all the velocity profiles in the blender where known with sufficient accuracy. The bulk material velocities are functions of material properties, blender geometries, and blender operation parameters. In addition, segregation occurring in the blender during filling and continuous operation moves mixture components relative to each other during blending. One would think that this relative motion would enhance blending. However, segregation is never a random process and, as such, does not result in dispersion. Instead, segregation produces a relative particle component flux in the direction of the body force that causes segregation resulting in separation of well mixed products and decreased blender performance. If the segregation velocities were known they could be used along with the computed velocity profiles in the blender to predict the blender performance. This paper discusses how to compute blender velocity profiles from blender geometry and material properties and how to determine the residence time distribution function from these computed velocities. It is the goal of this paper to introduce this methodology as a means of estimating blender performance. Luckily velocity profiles do not need to be extremely accurate to produce reasonable blending estimates. The simple cone-in-cone geometry will be used as a test case.
2. Blender velocity profiles

A cone-in-cone blender is a mass flow hopper that has a significant velocity profile across it. Simply stated, mass flow is a condition that produces significant material movement in the entire process equipment as material passes through or discharges (BMHB 1987). There are no stagnant regions in a mass flow bin. However, depending on the hopper shape and wall friction angle, a significant velocity profile can exist in a mass flow bin creating a residence time distribution. This property has been used successfully to create mass flow blenders (Dau 2003, Johanson 1982). The radial stress theory has been successfully used to compute the velocity in conical hoppers. These theories predict a relationship between the conical hopper half angle and the friction angle that is compatible with radial stress conditions. Figure 1 shows the calculated relationship between conical hopper angle and wall friction angle based on the radial stress theory (Jenike and Johanson 1967, Nedderman 1992). It was shown that wall friction and hopper wall conditions that exceed the limiting line given in Figure 1 resulted either no flow along the walls or funnel flow.

![Figure 1. Typical mass flow funnel flow limit](image)

The radial stress theory asserts that stress states within process equipment are compatible with radial stress fields will produce mass flow in that process equipment. Non-radial stress states produce funnel flow behavior (stagnant region formation). In addition, the more frictional the wall surface, the steeper the velocity profile across the bin. Figure 2 shows the computed velocity profile from the radial stress theory for a conical hopper with a hopper slope angle of 20 degrees measured from the vertical. This figure shows the increase
in the steepness of the velocity profile as the wall friction angle increases. This radial stress theory will be discussed in more detail below.

Figure 2. Conical velocity profiles in a 20 degree hopper with various friction angle along the bin wall.

Computed radial velocities are well established for conical hoppers and could be extended to cone-in-cone geometries. The cone-in-cone consists of an internal cone placed inside a conical hopper. These cones have a common apex. The top diameter of the inner cone is determined by extending the inner cone up to the arc that has a center at the apex of the outer cone and inscribes the top diameter of the outer conical hopper as indicated in Figure 3.

The inner cone outlet diameter is chosen to specify the fraction of global flow rate directed through the inner cone. This fraction is based on the ratio of flow areas at the top and bottom diameters of the cones. A typical cone-in-cone can achieve a maximum velocity ratio between inner and outer cones of about five to one. Equation 1 can be used to estimate the inner cone diameter \(d_B\) required for a given velocity ratio \(R=V_{\text{inner}}/V_{\text{outer}}\).

\[
d_B = \frac{D_B}{\sqrt{\left(\frac{\sin(\theta_o)}{\sin(\theta_i)}\right)^2 + 1}} - 1
\]  

\(R=V_{\text{inner}}/V_{\text{outer}}\)
In addition to this global velocity pattern, the nature of mass flow will cause a velocity profile across the inner cone and in the annular region between the cones. Figure 2 contains typical radial velocities for the inner conical hopper. However, the radial stress and velocity equations must be derived for the case of flow in an annular mass flow channel. The derivation of these equations is shown below. The starting point is the equation of motion (Equation 2) with acceleration terms neglected. As a result, the radial stress theory applies to slow moving conditions in process equipment.

\[ \nabla \cdot \tau = \gamma g_z - \nabla P \quad (2) \]

This equation can be reduced to the following with appropriate assumptions outlined in axial symmetric flow and the radial stress theory.

\[ \frac{1}{r^2} \frac{\partial (r^2 \sigma_r)}{\partial r} + \frac{1}{r \sin(\theta)} \frac{\partial (r \tau_{r\theta} \sin(\theta))}{\partial \theta} \sigma_\theta + \sigma_\phi = \gamma g_r - \frac{\partial P}{\partial r} \quad (3) \]

\[ \frac{1}{r^2} \frac{\partial (r^2 \tau_{r\theta})}{\partial r} + \frac{1}{r \sin(\theta)} \frac{\partial (r \sigma_\theta \sin(\theta))}{\partial \theta} + \frac{\tau_{r\theta}}{r} \cot(\theta) \sigma_\phi = \gamma g_\theta - \frac{1}{r} \frac{\partial P}{\partial \theta} \quad (4) \]
It is important to point out that there are four unknown stresses in the two above equations. Additional relationships between stress components are required to solve these two equations. The required relationship comes from the effective yield locus of the bulk solid (see Figure 4).

Figure 4. Limiting stress state definitions

The effective yield locus describes the stress state that exists when bulk materials deform without volume change and provides the constitutive relationship required to create closure with this set of equations. This results in the follow equations describing stress condition in flowing bulk materials (see Equations 5 through 10).

\[
\sigma_r = \sigma \cdot (1 - \sin(\delta) \cdot \cos(2 \cdot \omega)) \\
\sigma_\theta = \sigma \cdot (1 + \sin(\delta) \cdot \cos(2 \cdot \omega)) \\
\tau_{r\theta} = \sigma \cdot \sin(\delta) \cdot \sin(2 \cdot \omega) \\
\sigma_1 = \sigma \cdot (1 + \sin(\delta)) \\
\sigma_3 = \sigma \cdot (1 - \sin(\delta)) \\
\sigma_\phi = \sigma \cdot (1 + \sin(\delta))
\]
The last equation required for the radial stress model comes from work done by (Sokolovski, 1965). He states that the mean solid stress near the apex of a wedge shaped section of a bulk material is proportional to the radial distance from the wedge apex to the point of interest. This yields Equation 11 for the mean stress.

\[
\sigma = \gamma \cdot g \cdot r \cdot s(\theta)  \tag{12}
\]

Substitution of Equations 5 through 12 into Equations 3 and 4 results in two equations in terms of the mean solids stress (\(\sigma\)) and the principal stress direction angle (\(\omega\)). These derived equations assume that the effective angle of internal friction is constant. This is brief a review of the standard radial stress theory for the benefit of those readers not familiar with the process. It should be pointed out that Equations 13 and 14 include a pressure gradient term not normally included in the traditional radial stress theory proposed by earlier authors. The full derivation of this will not be presented here. However, the A-terms in Equation 13 and Equation 14 are the ratio of radial and polar pressure gradients in the process equipment. These new equations can be used to study the relationships between pressure gradient and solids velocity patterns.

\[
\frac{ds}{d\theta} = \frac{\left[ \sin(2 \cdot \omega) + \sin(\delta) \cdot \cot(\theta) \cdot \cos(2 \cdot \omega) - \sin(\delta) \cdot \cot(\theta) - \sin(\delta) \cdot \sin(2 \cdot \omega) \right] \cdot s + \left[ -A_1 \cdot \cos(2 \cdot \omega) + \cos(2 \cdot \omega) \cdot \sin(\theta) + \right]}{A_2 \cdot \sin(2 \cdot \omega) + \sin(2 \cdot \omega) \cdot \cos(\theta)} \tag{13}
\]

\[
\frac{d\omega}{d\theta} = \frac{1}{2} \cdot \frac{\left[ \cos(\delta) \cdot \sin(\delta) \cdot \cot(\theta) \cdot (\sin(2 \cdot \omega) + 1) \cdot (\sin(\delta) + 1) \right] \cdot s + \left[ A_1 \cdot (\sin(\delta) \cdot \cos(2 \cdot \omega) + 1) + \right]}{A_2 \cdot \sin(2 \cdot \omega) \cdot \sin(\delta) + \cos(2 \cdot \omega) \cdot \cos(\theta) \cdot \sin(\delta) + \cos(\theta) \cdot \sin(\delta) \cdot (\cos(2 \cdot \omega) + \sin(\delta))} - 1 \tag{14}
\]

Where:
With the exception of the pressure gradient terms, the velocity profile derivation up to this point parallels the radial stress equations for a cone. However, the boundary conditions are different for the annular flow channel. The conical hopper uses the assumption of axisymmetric flow to deduce that the principle stress direction was aligned with the hopper centerline. This boundary condition does not apply to the annular region where material flows on two wall surfaces. Instead, wall friction boundary conditions must exist at both the inner and outer walls. Figure 5 shows a Mohr circle compatible with the critical state of stress.

![Figure 5. Definition of wall friction angle relationship in terms of limiting state model.](image)

The line in Figure 5 indicates a typical cumbic friction condition. There are two wall friction operation points that satisfy both the wall friction condition and the limiting state of stress conditions. If the hopper converges, then the intersection at point A, representing the passive state of stress, applies. If the hopper diverges, then the active stress at intersection point B applies.

For the purposes of this paper, only converging cone-in-cone geometries will be considered and the passive stress state (point A) applies. The stress state represented by point A implies that the direction between the coordinate axis and the major principal stress is given by Equation 17.

$$\omega = \frac{1}{2} \left[ \phi_w + a \sin \left( \frac{\sin(\phi_w)}{\sin(\delta)} \right) \right] \quad \text{at} \quad \theta = \theta_o$$  \hspace{1cm} (17)
This boundary condition applies at the outer cone hopper wall. However, the boundary condition along the inner wall is the negative of this since the direction of major principle stress is rotated counter clockwise from the coordinate axis (see Equation 18).

\[
\omega = \frac{-1}{2} \left( \phi_w + a \sin \left( \frac{\sin(\phi_w)}{\sin(\delta)} \right) \right) \quad \text{at} \quad \theta = \theta_i \quad (18)
\]

Equations 13 and 14 can be integrated using boundary conditions given in Equations 15 and 16 to produce radial stress solutions for annular geometries. Figure 5 shows some of the solutions for the case of a 10 degree inner cone. The line given in this figure is the limit of radial stress in an annulus. In a cone, the radial stress limit is also the boundary between mass flow and funnel flow behavior. The mass flow limit for a conical hopper is also shown on Figure 6. Both the conical and annular radial stress solutions have the same type of behavior, but there is a shift of the limiting radial stress line towards flatter hoppers. Thus, the cone-in-cone geometry extends the mass flow limit to flatter outer cone angles.

![Figure 6. Mass flow limit for cone-in-cone hopper with inner cone of 10 degrees.](image)

Figure 7 shows the predicted mass flow limit lines for several inner conical hopper configurations. In general, if the inner hopper is designed for mass flow with a hopper angle of \( \theta_c \), then the outer hopper angle can be designed with a
slope angle given by the balanced friction angle limit line found in Figure 7. This represents the case where the inner and outer cones have the same frictional properties conditions. Conversely, if an existing conical hopper has a frictional condition that puts it in the funnel flow region, then Figure 6 could be used to determine the size and shape of the inner cone that would be required to cause flow along the walls. For example, consider the case of a 35° conical hopper with a wall friction angle of 21°. This cone would be in the funnel flow regime as indicated by the plus sign in Figure 7. However, if an inner cone with a 10° slope was placed within the existing funnel flow hopper, then mass flow could be achieved.

![Figure 7. Balanced friction mass flow limiting line for cone-in-cone geometry (internal friction angle $\delta=50$ deg)](image)

None of the assumptions regarding radial velocities are violated in annular conical flow, so the standard radial velocity equations used by (Jenike and Johanson 1967, Nedderman 1992) can be used to predict the velocities in annular conical geometries.

$$V(\theta) = V(0) \cdot \exp \left[ -3 \cdot \frac{\theta}{\theta_l} \cdot \tan(2 \cdot \omega) \cdot d\theta \right]$$  \hspace{1cm} (19)
These velocities can be combined with the inner cone velocities to give the radial velocity pattern in a cone-in-cone hopper. Scaling factors can be applied to the inner and outer velocity profiles to adjust the flow ratio in the inner and outer geometries. Figure 8 shows two expected velocity patterns in a cone-in-cone hopper with a two-to-one velocity flow ratio from the inner to outer cones. These velocities are a function of the wall friction angle. Figure 8a predicts very flat velocity profiles with low friction angle materials. Figure 8b shows steeper velocity profiles with more frictional wall materials.

Figure 8. Velocity profiles in typical cone-in-cone geometries at two different friction conditions.

One of the assumptions of the radial stress theory is that the flow stream lines do not cross. This yields the result that mixing in a cone-in-cone hopper does not give rise to radial blending. If the material enters the bin with a radial segregation pattern, it will exit the bin with the same pattern across the outlet. Consequently, it is recommended that a small mixing device be placed below the outlet of the cone-in-cone hopper to achieve radial blending.

Even with this shortcoming, cone-in-cone blenders can produce reasonable blending with a variety of materials. In fact, since an approximation of the blender velocity profile can be computed from radial stress and velocity calculations, these velocity profiles can be used to predict the residence time distribution in typical cone-in-cone blenders. This gives the ability of separating the effects of blending operation parameters, blender geometrical parameters, and the material properties. If advances are to be made in the area of blender modeling, then an understanding of the influence of these three effects are required. This simple blender allows for the easy separation of variables to
predict blender uniformity thru axil-symmetric stress and velocity assumptions. The process is outlined below. However, it is important to note that this type of process could be implemented in any powder blender. In more complex blenders it may be required to solve fully 3D partial differential equations with free boundary conditions. Advances have been made in finite volume calculations that may help make these calculations possible. It is this author’s view that major advances in blender modeling will not be possible without using the basic approach outlined in this paper. Significant energy should be directed to extend this basic approach to more complex blenders.

In the simple cone-in-cone geometry calculated velocity profiles can be integrated along flow stream lines to determine the time that a particle placed at the top of the blender will exit the blender outlet. Computing the exit times for a layer of particles placed at the top of the blender gives rise to a method of computing the residence time distribution function of the blender. The procedure for computing residence time in the blender from material properties is as follows.

The time required for one complete blender volume to pass through the blender is divided into small equal increments. This incremental time unit is then used to compute the thickness of a mathematical marker layer placed at the top of the blender. The thickness of this layer represents the average distance traveled for material at the top of the bin in one time increment. The cross sectional area at the top of the blender is divided into rings of equal area, creating regions of constant volume. These regions will be subject to radial flow streamline velocities. The position of any of the regions over time can be computed by multiplying the radial velocity by the incremental time and vectorially adding this value to the last marker volume position. The position of these individual marker volumes can be continually monitored to determine the time when they exit the blender. The number of volume elements exiting the blender divided by the total number of initial volume elements in the blender is an approximation of the blender residence time distribution. Perfect plug flow will result in all the marker volumes exiting the blender at the same time (after one blender volume discharge). Any deviation from this will produce a distribution in residence times and will produce blending. Figure 9 shows a calculated residence time distribution function for a cone-in-cone hopper with a 10 degree inner cone and a 20 degree outer cone. The relative flow rate variation between the inner and outer cone was two-to-one and the wall friction angle was 30 degrees measured from the horizontal. This figure also contains the velocity profile computed from this blender geometry. The initial marker layer comes from the center of the blender and exits the blender after only 0.5 blender volumes have passed through the system. The large peak is from the sudden change in the slope of the blender profile and exits the blender after about 1.25 blender volumes have passed through the system. It is important to point out that this figure represents the residence time distribution in terms of the total amount of material passing through the blender instead of the elapsed time in the blender itself. In other words, the number of blender volumes (BV) passing through the system would be equal to the residence time (t\text{residence}) multiplied by
the average mass flow rate \( Q_{s\text{avg}} \) of the material in the blender and divided by the blender mass capacity \( C_{ap\text{blender}} \).

\[
BV = t_{\text{residence}} \cdot \frac{Q_{s\text{avg}}}{C_{ap\text{blender}}}
\]  

(20)

![Residence time distribution function for cone-in-cone geometry shown in Figure 8 with wall friction angle of 30 degrees.](image)

Figure 9. Residence time distribution function for cone-in-cone geometry shown in Figure 8 with wall friction angle of 30 degrees.

The average residence time \( T_{\text{avg}} \) can be found by noting the time at which half of the markers exit the blender. This number can be used to characterize blenders or other process equipment. However, two blenders may have the same average residence time but cause very different blending behaviors. In fact, it is possible to construct a bin where the material flows in nearly perfect uniform velocity mass flow. The residence time for this case would correspond to the time required for one blender volume to pass through the system. The distribution function in perfect mass flow would be a sharp concentration spike centered around one blender volume. This type of blender profile would not lead to blending and a uniform velocity mass flow bin would be a poor choice for a blender. The width of the residence time distribution is more important in quantifying blending operation. It is the range of residence times in any blender that causes blending. The residence time distribution function could be considered a blender fingerprint. With this distribution function, engineers can
compute the expected concentration profiles for any input condition. Normally this distribution function is a measured quantity. The strength of this paper is the ability to compute the distribution function solely from measured material properties for a given blender geometry. The convolution integral given in Equation 21 provides the means for that calculation.

\[ C_{out}(\tau) = \int_{0}^{\infty} C_{in}(\tau - t) \cdot E(t) \cdot dt \quad (21) \]

Compare the input and output concentration profiles shown in Figure 10. The continuous blender operation smooths the input concentration profile and produces a concentration profile with the same frequency but smaller variation exiting the blender.

![Figure 10. Input (a) and output (b) concentrations for blender with computed residence time distribution given in Figure 8.](image)
The number of fluctuations in a blender volume affects the degree of smoothing observed in the output concentration. One could compute the variance of the output concentration and divide that by the variance of the input concentration to provide a measure of blending effectiveness. This variance reduction factor can provide a means of ranking blender performance. Lower values of the variance reduction factor will result in better blending

\[ VR = \left( \frac{Sdev_{out}}{Sdev_{in}} \right)^2 \]  

(22)

The above example would result in a reduction factor of 0.3. It is important to point out that the greater the frequency of the input concentration, the smaller the output concentration fluctuation. Since the above method allows calculation of the blender residence time distribution as a function of material properties and specific blender design constraints, it can be used for a parametric study of blend efficiencies based on flow properties and design constraints without having to measure blender distribution functions experimentally using expensive scale blender models. For example, consider the case of a cone-in-cone blender with a 20 degree outer cone and a 10 degree inner cone. Consider also the effect of changing the input fluctuation frequency in two nearly identical blenders where the difference in these two blenders is the wall friction angle. Figure 11 shows the effect of blender variance reduction factors (VR) for two blenders where the number of input concentration fluctuation in one blender volume was varied. The number on the x-axis of this graph is the number of fluctuation periods of a sinusoidal varying concentration that enters the blender during the time it would take for one blender volume to pass through the system. The resulting variance reduction factor depends strongly on the number of fluctuations per blender volume. This figure clearly shows that the larger the number of fluctuations per blender volume, the lower the variance reduction factor will be.

The results of this parametric study also suggest that the wall friction angle does not make a significant difference in blender effectiveness if the number of concentration input cycles is greater than about seven. However, there is a significant difference in blender efficiency in this blender when the number of input concentration cycles per blender volume is small. Thus the cone-in-cone blender operation becomes sensitive to material properties (i.e. wall friction) when attempting to blend fluctuations that occur over large blender volumes.
Blender effectiveness in the cone-in-cone blender depends on the velocity distribution across the blender. Therefore, anything that changes the velocity distribution will also change the blender effectiveness. Differences in wall friction angle change the velocity distribution and will result in changes to blender performance depending on the blender operation. However, Equations 13 and 14 also contain the effect of the local pressure gradient. These pressure gradient values can therefore result in differences in velocity profiles in the cone-in-cone blender. Significant gas pressure gradients can result in cone-in-cone blenders when discharging fine powder materials. These gas pressure gradients arise from the fact that bulk materials are compressible and change porosity as solids contact stresses within the equipment change. Consider, for example, a bulk power material flowing at a slow rate through a cone-in-cone blender. The solid’s stresses increase as the material flows through the bin. This causes some consolidation of the bulk material and squeezes some of the air out of the solid’s pores. This gas leaves the system predominately through the top material free surface. However, the consolidation pressure in the bin or hopper decreases as the material approaches the outlet. Decreasing the consolidation pressure results in expansion of the bulk material. However, some of the gas within the solid’s pores has already left the system through the top of the bin. This results in a net gas deficit near the hopper outlet, creating negative gas pressures within the material and producing significant gas pressure gradients near the outlet. These gas pressure gradients could be responsible for changing the local
velocity profiles in the bin. The cone-in-cone hopper situation is complicated by the fact that the inner cone wall is impervious to gas flow. Thus, there exists a difference in radial gas pressure gradients between the inner cone and the annular region in the cone-in-cone blender. This could significantly change the blender effectiveness of cone-in-cone blenders. Figures 12 and 13 show the expected gas pressure and gas pressure gradient profiles in both the inner cone and annular region for a typical cone-in-cone blender.

![Graph showing gas pressure profiles](image)

**Figure 12.** Gas pressure profile in cone-in-cone blender when operating with fine plastic powder at a flow rate of 1360.8 kg/hr (about 30% of the expected limiting flow rate for this powder). Velocity ratio between inner and outer cone is 2:1.

\[ Q_s = 1360.8 \text{ kg/hr} \]
Once the local gas pressure gradients are known, the radial velocity analysis outlined above can be implemented to compute the expected velocity profiles for the case of the local radial pressure gradients given in Figure 13. This analysis can be applied to both the inner cone and the annular region to determine the velocity profiles at various local gas pressure gradients. Figure 14 shows the expected velocity profiles in the 10 degree inner cone at different radial gas pressure gradients. Note that the negative gas pressure gradient at the bottom of the inner cone is large enough to result in a funnel-flow velocity pattern. If this high gas pressure gradient persisted throughout the inner cone section, it would induce a funnel flow velocity pattern in the blender and result in a preferred flow channel. Luckily the gradients in other hopper elevations are smaller than this value. Likewise, Figure 15 indicates that if gas pressure gradients in the annular section have large enough negative values, then preferred flow channels would develop along the outer wall of the inner cone.
Each of the velocity profiles in these figures has been normalized so the velocity at the centerline equals one.

Figure 14. Normalized velocity profiles for inner cone at various local gas pressure gradients $A1 = (dP/dr / \gamma g)$. 
Figure 15. Normalized velocity profiles in annular section of cone-in-cone hopper as a function of local gas pressure gradient $A_1 = \frac{(dP/dr)}{\gamma g}$.

However, local velocity profiles at any elevation in the blender can be computed by assuming a two-to-one velocity difference between the inner and outer cone and computing the velocity profile from the local gas pressure gradient given in Figure 13. Figure 16 shows several of these velocity profiles in the hopper section of the cone-in-cone blender. This figure indicates that the steep funnel flow velocity pattern persists only near the outlet of the hopper. Velocity profiles near the top of the cone section are actually flatter than those expected for normal mass flow without gas pressure effects. It is not likely that the small zone of steep velocity profiles near the hopper will promote funnel-flow behavior in the entire hopper section. However, increasing the solids flow rate will increase the magnitude of these negative gas pressure gradients in the lower hopper and increase the positive gas pressure gradients in the upper hopper section. This suggests that one should expect to see a flow rate condition that may induce a preferred flow channel formation in the center of the blender during operation at high flow rates. Conversely, the higher gas pressure gradient at the top of the hopper would likely result in overall average velocity profiles flatter than typical velocities in mass flow blenders without gas pressure gradient effects. This suggests that the blending action of the blender may diminish as the flow rate of fine powder increases until the very steep profile in the lower hopper section occupies a significant portion of the bin, at which point the blender velocity profile produces funnel flow behavior. The whole matter of the influence of local gas pressure gradient in cone-in-cone blender is an interesting area for
further study. However, this theoretical analysis suggests that fine powder may be susceptible to the formation of preferred flow channels in cone-in-cone blenders if conditions are right.

![Diagram](image)

**Figure 16.** Velocity profiles in cone-in-cone blender at various locations in the blender. The mass flow rate in this blender is 1360.8 kg/hr with a two to one velocity ratio between the inner cone and the annular section.

### 4. Conclusions

The strength of this paper is the ability to compute the residence time distribution function just from knowledge of material properties. The properties induce a unique velocity pattern in a given cone-in-cone geometry that can be approximated using limiting states of stress in the blender. The radial velocity assumption allows radial velocity fields to be extended to annular flow channels. The analysis in this paper suggests that the blender design is somewhat independent of wall friction angle for conditions where there are several input concentration fluctuations in a single blender volume. However, friction angle does play a role in blender performance for conditions where the blender must mix large-period input fluctuations. Radial velocity profiles in cone-in-cone blenders can be extended to include the effect of local gas pressure gradients. There is some evidence that these gas pressure gradients can induce funnel-flow behavior in the blender, depending on the operation flow rate, and may result in preferred flow channels during operation. This is an area for further research. It is the contention of the author that the method of blender evaluation outlined
above could be used with other blender configurations to develop a bridge between blender effectiveness and the material properties which control the velocity profiles in the blender. This general methodology, if successfully implemented, will allow scale-up of blenders. This will likely require solving complex 3D differential equations with free boundaries to obtain an approximation to the local velocity patterns. Even though this is a formidable task, the author suggests this road as the way forward. Advances in volume of fluid finite element methods may provide the necessary computational power to accomplish this task.

5. Nomenclature

A₁ Dimensionless radial pressure gradient body force ratio.
A₂ Dimensionless θ–direction pressure gradient body force ratio.
BV Bin Volume
\( \text{Cap}_{\text{blender}} \) Blender capacity
Cᵢn Input concentration.
Cₒut Output concentration.
E Residence time distribution function.
P Gas pressure. (KPa)
Qₗ Solids flow rate (kg/hr)
r Radial position. (m)
\( r_b \) Distance from the hopper centerline to hopper radial coordinate. (m)
\( R_b \) Distance from the hopper centerline to the hopper wall. (m)
s θ-dependence function for radial stress.
Sdevᵢn Standard deviation of input stream.
Sdevₒut Standard deviation of output stream.
V(θ) Radial velocity. (m/s)
V(0) Velocity at hopper centerline. (m/s)
V(rₗ) Radial Velocity in terms of distance from centerline. (m/s)
V_center Velocity at hopper centerline. (m/s)
VR Variance reduction factor

γ Powder bulk density. (kg/m³)
β Angle between the effective wall body force and the gravitational vector direction. (deg)
\( \delta \) Effective Internal friction angle. (deg)
\( \phi_w \) Wall friction angle. (deg)
\( \phi_{we} \) Effective wall friction angle including gas pressure gradient terms. (deg)
\( \sigma_r \) Normal stress on the plane perpendicular to the radial direction in a spherical coordinate system. (KPa)
\( \sigma_\theta \) Normal stress on the plane perpendicular to the θ–direction in a spherical coordinate system. (KPa)
\( \sigma_\phi \) Normal stress on the plane perpendicular to the \( \phi \)-direction in a spherical coordinate system. (KPa)

\( \sigma \) Mean stress. (KPa)

\( \sigma_1 \) Major principal stress. (KPa)

\( \sigma_3 \) Minor principal stress.

\( \theta_w \) Half angle of conical hopper measured from the vertical. (deg)

\( \theta_i \) Half angle of inner conical hopper measured from the vertical. (deg)

\( \theta_o \) Half angle of outer conical hopper measured from the vertical. (deg)

\( \theta \) \( \theta \)-direction coordinate. (deg).

\( \phi \) \( \phi \)-direction coordinate. (deg)

\( \phi_w \) Wall friction angle. (deg)

\( \phi_{we} \) Effective Wall friction angle. (deg)

\( \tau_{r\phi} \) Shear stress on the plane perpendicular to the radial direction acting in the \( \phi \)-direction in a cylindrical coordinate system. (KPa)

\( \tau_{r\theta} \) Shear stress on the plane perpendicular to the radial direction acting in the \( \theta \)-direction in a cylindrical coordinate system. (KPa)

\( \tau_{\theta\phi} \) Shear stress on the plane perpendicular to the \( \theta \)-direction acting in the \( \phi \)-direction in a cylindrical coordinate system. (KPa)

\( \omega \) Angle between the major principal stress and the \( r \)-coordinate direction. (deg)

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7. References


