A Simple PID Controller for a Magnetic Bearing with Four Poles and Interconnected Magnetic Flux*

Christian Tshizubu, José Andrés Santisteban, IEEE Senior Member, Universidade Federal Fluminense

Abstract—The conventional structures of active magnetic bearings to support rotors dispose of eight poles operating in such a way that two adjacent poles share the same magnetic flux. In this work, a different electromagnetic structure is presented which is based on the so named bearingless motor with split windings. While this device is originally supplied by alternate electrical currents, in this case, as the purpose is to generate only radial forces, the windings are supplied by continuous currents forming a magnetic flux with four equivalent poles. To test this approach, a conventional induction motor was used and its rotor was appropriately modified. It will be shown that although the magnetic fluxes are coupled, for low speed rotation, all the controlled plant can be modeled as an unstable second order system, so PID controllers can be appropriately designed. In order to test this approach, a workbench consisting of displacement sensors, signal conditioners and two microcontrollers development boards was implemented. For different radial loads and changes of displacement references, the experimental results were well succeeded.

I. INTRODUCTION

The advance in the control systems of alternate current (AC) electrical motors has helped to rapidly develop the magnetic bearings technologies. Since then, innumerate approaches related to this subject have been tested and proposed in the literature [1]. The benefits of rotating components to work in high speed, high power and without mechanical contact or friction, in certain environments, have placed the magnetic bearings as the main substitute to the well know mechanical bearings.

The Fig. 1 depicts the conventional structure of an active magnetic bearing.

However, despite all these benefits this technology is still in a primary stage of diffusion with few practical implementations in large scale, except in the East Asian countries like China, Japan and South Korea where projects using this technology are implemented in fast speed. Thus, in general, this technology is still considered new and expensive to implement, and requires high skilled manpower.

In order to outline this issue, one of the options could be the use of existing equipment in order to produce a magnetic bearing. This option will eventually spare time of building a magnetic bearing from zero. In this sense, this paper will show this approach.

The stator winding of a conventional three-phase induction motor, easily found in the market, with four revolving magnetic poles can be modified to be supplied with a two-phase electric grid and even maintain the four revolving magnetic poles. These poles have interconnected magnetic fluxes. In order to use this motor as a magnetic bearing, it is sufficient to supply one of the phases, by instance A, while the second phase, called B, is turned off. On the other hand, the rotor cage, in which the current is induced, is substituted by a laminated unit of Silicon steel in order to avoid the induction of any current. In this way, with appropriated control currents, radial forces can be generated to centralize the laminated rotor.

The control currents are imposed from reference currents obtained by the processing of a closed loop control of the displacement errors in each axis until they get null [3]. In this work, these objectives were reached using two microcontrollers that were programmed to implement digital Proportional-Integral-Derivative controllers with Anti-Windup characteristic.

II. DESCRIPTION OF THE SYSTEM

Fig. 2 depicts the experimental structure used in this work. It consists of a rotor in the vertical orientation. At the top of it (d = 500 mm.) there is an auxiliary disc that allows some load tests. Two pairs of eddy current displacement sensors are
located in orthogonal directions at $c = 430 \text{ mm}$. Near to the middle ($b = 280 \text{ mm}$), as explained before, the rotor structure of a two phase inductor motor was utilized but changing the squirrel cage by a laminated unit. Fig. 3 shows the actual distribution of its windings. The airgap between the rotor and the stator is 1 mm. At the bottom, the rotor is supported by a spherical piece. The net mass of the rotor is 3.3 Kg. Additionally, a security bearing was fixed to the rotor to avoid the contact between the stator and the laminated rotor.

![Figure 2: Rotor and components.](image)

The windings with terminals A1-A2 and A5-A6 generate magnetic fluxes aligned with the axis “$y$” while the windings with terminals A3-A4 and A7-A8 generate magnetic flux aligned with the axis “$x$”.

Different authors discuss the dynamics of rotors and, in particular, [1] shows the dynamics of a rotor supported by eight poles active magnetic bearings. Four degrees of freedom are controlled. Meanwhile, in this work only two degrees of freedom are going to be controlled. In this way, the adopted model is described by (1), as discussed in [5].

$$
\ddot{z}_s + G_r \dot{z}_s - K_{xtr} \dot{z}_s = K_{uhr} u
$$

Where, $z_s$ is the states vector, $z_s = [x \ y]^T$; $G_r = I^{-1} G$; $K_{xtr} = k_x b^2 t^{-1}$; $K_{uhr} = k_i b c t^{-1}$ and $I$ is the inertial matrix ($I_x = I_y = 28200 \text{ Kgmm}^2$). $G$ is the gyroscopic matrix, $u$ is the currents vector and finally, $k_x$ and $k_i$ are the coefficients of the linearized model of the electromagnetic vector of forces $F$.

$$
F = k_x z_s + k_i u.
$$

Where, $k_x = (2 \mu_o A N_2 i / h_o^2)$, $k_i = (2 \mu_o A N_2 i / h_o^2)$, $h_o$ is the nominal air gap (1 mm.), $A$ is one quarter of the longitudinal rotor area (4673.12 mm²); $\mu_o$ is the magnetic permeability ($1.256 \times 10^{-6} \text{ N/ A}^2$), $i$ is the total current in A and $N$ is the equivalent number of turns (50) per winding.

Next, considering low speed rotations, the gyroscopic effect can be neglected, so two decoupled systems can be obtained from (1), as shown in (3).

$$
\ddot{z}_s - K_{xtr} z_s = K_{uhr} u
$$

III. ELECTROMAGNETIC MODELING

In [1], magnetic bearings are classified in two main groups according to how the magnetic force can be calculated: The Maxwell force and the Lorentz force. This work is related to the former.

The Maxwell force acts perpendicularly to the surface of materials with high magnetic permeability $\mu$, and can be obtained from the magnetic energy through the principle of virtual work [6], as shown in (4).

$$
F = \frac{dW_e}{dh} = \frac{1}{2} [i]^T \frac{\partial [\muL]}{\partial h} = \frac{1}{2} [i]^T \frac{\partial [([h_i])]}{\partial h} [i]\n$$

Where $dW_e$ is the variation of stored magnetic energy, $[i]$ is the flux linkage vector (5), $[i]$ is the column vector of currents, dimension 4x1; $[L(h)]$ is the symmetrical matrix of induction, dimension 4x4; $h$ is any displacement along $x$-$y$ axis ($x$, $y$, $z$, or $\gamma$).

In order to model the electromagnetic structure shown in Fig. 3, a simplified one can be introduced. This is depicted in Fig. 4. It is a four poles stator but with concentrated windings. Its equivalent magnetic circuit is shown in Fig. 5.

As the magnetic flux distribution generated by the stator with distributed windings, shown in Fig. 6, is similar to the one depicted in Fig. 4, this approximation can be used.

$$
\begin{bmatrix}
\lambda_{y1} \\
\lambda_{y2} \\
\lambda_{x1} \\
\lambda_{x2}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{R_{q1}} & -\frac{R_{eq}}{R_{eq} + R_{q1}} & -\frac{R_{eq}}{R_{eq} + R_{q2}} & -\frac{R_{eq}}{R_{eq} + R_{q3}} & -\frac{R_{eq}}{R_{eq} + R_{q4}} \\
-\frac{R_{eq}R_{q1}}{R_{eq} + R_{q1}} & \frac{1}{R_{q1}} & -\frac{R_{eq}R_{q2}}{R_{eq} + R_{q2}} & -\frac{R_{eq}R_{q3}}{R_{eq} + R_{q3}} & -\frac{R_{eq}R_{q4}}{R_{eq} + R_{q4}} \\
-\frac{R_{eq}R_{q1}}{R_{eq} + R_{q1}} & -\frac{R_{eq}R_{q2}}{R_{eq} + R_{q2}} & \frac{1}{R_{q2}} & -\frac{R_{eq}R_{q3}}{R_{eq} + R_{q3}} & -\frac{R_{eq}R_{q4}}{R_{eq} + R_{q4}} \\
-\frac{R_{eq}R_{q1}}{R_{eq} + R_{q1}} & -\frac{R_{eq}R_{q2}}{R_{eq} + R_{q2}} & -\frac{R_{eq}R_{q3}}{R_{eq} + R_{q3}} & \frac{1}{R_{q3}} & -\frac{R_{eq}R_{q4}}{R_{eq} + R_{q4}} \\
-\frac{R_{eq}R_{q1}}{R_{eq} + R_{q1}} & -\frac{R_{eq}R_{q2}}{R_{eq} + R_{q2}} & -\frac{R_{eq}R_{q3}}{R_{eq} + R_{q3}} & -\frac{R_{eq}R_{q4}}{R_{eq} + R_{q4}} & \frac{1}{R_{q4}}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
x_1 \\
x_2
\end{bmatrix} = \lambda
$$
The strategy of control selected for this experiment is the Proportional-Integral-Derivative controller with Anti-Windup characteristic. The transfer function of the PID controller is represented by (14).

\[ G_c = \frac{k_p + \frac{k_i}{s} + k_d s}{s} = k_p \left(1 + \frac{1}{T_i s} + T_d s\right) \]  

(14)

Where \( k_p \), \( k_i \), \( k_d \) are respectively the proportional, integral and derivative gains, and \( T_i \), \( T_d \) are respectively the integral and derivative times.

Fig. 8 depicts the block diagram of a PID controller with Anti-Windup resource. The error signal \( e_s \) is the difference
between the controller output \( v \) and the actuator output \( u \). It is then fed to the input of the integrator block after being multiplied by \( \frac{1}{T_t} \).

\[ I = \frac{1}{T_t} e_s + \frac{K}{T_i} e \]  

(15)

Where \( e \) is the control error.

The feedback path around the integrator causes the integrator output to drive towards a value such that the integrator input becomes zero. Hence,

\[ e_s = -\frac{K T_t}{T_i} e. \]  

(16)

Since

\[ e_s = u - v, \]  

(17)

substituting (16) in (17), it follows that

\[ v = u_{lim} + \frac{K T_t}{T_i} e \]  

(18)

Where \( u_{lim} \) is the saturating value of the control variable. Thus, (18) means that the integral block of the system will not wind up.

V. SIMULATION RESULTS

Knowing the parameters of the prototype, the transfer function of (3) is given by (19). The correspondent root locus is plotted in Fig. 9.

\[ G_p = \frac{3.534}{0.0282 s^2 - 2301} \]  

(19)

As known, the Windup effect is caused by the presence of integral block. Without the Anti-Windup, its output can overcome the actuator limits. So the system would run as an open loop because the actuator output would remain at its limit independently of the process output [8].

In a first attempt to stabilize the system using the Dominant Poles Allocation (DPA) method, two zeros and one pole in the origin were arbitrary chosen and added to Fig. 8, resulting in the root locus in Fig. 10. This method helped to determine the preliminary PID gain parameters of the controller.

The zeros and pole parameters chosen were located respectively at \( z_1 = -50, z_2 = -10 \) and \( p = 0 \). With these, the calculated values of the PID parameters are as follow:

\[ kp = 917.280; \; ki = 7644 \; \text{and} \; kd = 15.288 \]

Using the Matlab software tuning tool of PID controller, it is possible to adjust the values of those parameters. The adjusted parameters for a slow response are:

\[ kp = 0.618; \; ki = 2.583 \; \text{and} \; kd = 0.002. \]

Fig. 11 depicts the slow response of the rotor using the calculated values (gray color) and adjusted values of PID parameters (blue color).
The PID controller with the calculated values causes a lower settling time (131ms.) but the output oscillations are quite high.

Figure 11. Simulation with PID gain parameters calculated and tuned.

VI. EXPERIMENTAL RESULTS

In order to test the effectiveness of the designed controller, for each axis, it was used a microcontroller and auxiliary electronic devices as depicted in Fig. 12. Following traditional methods of digital controllers [8], a difference equation was obtained and translated in commands, in C language, loaded to the microcontroller. The sampling frequency was 2kHz.

Figure 12. Experimental workbench – electronic components: a) Currents Supply Unit b) Auxiliary electronics & c) Microcontroller

Fig. 13 shows a photograph of the experimental workbench.

During the experiments, it was noted the necessity of some fine adjusts on the PID parameters. These were made offline. In Table 1, all the tested parameters are shown.

### TABLE I. ADJUST OF PID GAIN PARAMETERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Both axes</th>
<th>Both axes</th>
<th>Both axes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_p$</td>
<td>917.28</td>
<td>0.618</td>
<td>2.13</td>
</tr>
<tr>
<td>$k_i$</td>
<td>7644</td>
<td>2.584</td>
<td>12.31</td>
</tr>
<tr>
<td>$k_d$</td>
<td>15.288</td>
<td>0.002</td>
<td>0.03</td>
</tr>
</tbody>
</table>

In order to test the robustness of the designed PID controller, a step change of reference is applied in both axes. The responses are shown in Fig. 15 and Fig. 16.

Due to imperfections in the alignment of the displacement sensors, it was observed along the tests that the reference value to centralize only the “x” axis was different to the reference value for the “y” axis.

Figure 13. Experimental workbench overview: d) 2-Phase Induction Motor, e) Power Supply Unit and f) Sensors Unit

In Figs. 14, 15 and 16, the displacement range of both axes is from -1 millimeter to 1 millimeter that corresponds to 0 and 4 Volts respectively. The “x”-axis and “y”-axis displacements are respectively represented by the yellow and blue traces.

In Fig. 14, the displacement signals, when both rotor axis are centralized, are shown.

Figure 14. PID controller with Anti-Windup applied on both axes, “x”-axis in yellow and “y”-axis in blue
A different approach of magnetic bearing with four poles and interconnected magnetic flux has been presented. Different to other approaches, in this case it was utilized a distributed windings construction, the same used by conventional rotating AC electrical motors. It was proved that it is possible to adapt the structure of commercial electrical motor to work as AMB.

For this structure, two PID controllers were simulated and experimentally tested using low cost hardware. The presented results confirm the effectiveness of the displacement controller design and of the alternative device.

Finally, the relationships between the radial load and the total current along the “y” axis are shown in Fig. 17 and Fig. 18. As noted, the current is proportional to the radial load.

**REFERENCES**


