Synthesis of Flexible Multi-Period Heat Exchanger Networks for a Changing Utility Cost Scenario*
Adeniyi J. Isafiade

Abstract—Key process parameters in the synthesis of heat exchanger networks, such as process stream supply and target temperatures and process stream flowrates, may vary from time to time due to issues such as changing environmental conditions, plant start-ups/shut-downs, changes in product quality demand, etc. Also some other key design parameters which may also change from time to time include the availability of utilities as well as their costs. These changes may be due to factors such as seasonality issues, e.g. for utilities sourced from renewable energies, or government policies in form of tax, availability of utilities due to shortage of supply, etc. This implies that heat exchanger networks should not only be designed to be flexible in order to satisfy heat demand under changing process parameter scenarios, but should also be flexible in situations where utility costs as well as their availability change from time to time. Hence this paper aims to extend existing stage-wise superstructure (SWS) based multi-period heat exchanger network synthesis methods to be capable of satisfying the heat demand under scenarios where both process stream parameters and utility parameters such costs change from time to time in a pre-defined manner. The approach used entails extending the current multi-period SWS model through the inclusion of additional time index to represent future costs of utilities. The model is applied to one example so as to demonstrate its benefits.

I. INTRODUCTION

The synthesis of heat exchanger networks (HENs) has received significant attention in the last four decades due to issues such as global energy crises and climate change. However, the focus has mostly been on achieving a simultaneous reduction in both energy usage and the associated capital costs in single period scenarios [1]. Single period in this context implies that process stream parameters such as supply and target temperatures and flow rates do not change with time. However, in reality, this is not the case due to the fact that changes in environmental conditions, plant start-ups/shut-downs, changes in feed quality, changes in product quality demand, process upsets, and even deliberate changes in some of these parameters by plant operators, may influence stream parameters which may result in them changing from time to time. Changes of this nature, especially those that can be pre-determined, can be referred to as multi-period changes. This implies that heat exchanger networks need to be designed to be flexible in order to satisfy the multi-period profile of process heat demand in a cost efficient manner. Some of the methods that have been developed for multi-period heat exchanger network synthesis have been based on sequential, simultaneous and stochastic approaches. Under the sequential approach, we have the technique of Floudas and Grossmann [2], which is a multi-period version of the linear program (LP) and mixed integer linear program (MILP) of Papoulias and Grossmann [3] for single period problems. The aim in this method is to determine the minimum utility required for each period of operation in a minimum number of units network. This method was further extended by Floudas and Grossmann [4] to a scenario where the multi-period minimum investment cost network that corresponds to the minimum utility and minimum number of units targets obtained from the LP-MILP model of Floudas and Grossmann [2], is automatically generated based on a non-linear program (NLP). This extension by Floudas and Grossmann [4] is based on insights from the single period case previously presented by Floudas, et al. [5]. Other multi-period sequential based approaches include the works of Mian, et al. [6] and Mian, et al. [7]. The work of Mian, et al. [6] which is also an extension of the multi-period models of Floudas and Grossmann [2] and Floudas and Grossmann [4], also involves the multi-period utility integration and scheduling technique presented by Marechal and Kalitventzeff [8]. The technique aims to select an optimal utility, including its scheduling, among a host of options of utilities. Mian, et al. [7] further extended the work of Mian, et al. [6] through the inclusion of material and electrical storage.

Some of the papers under the category of simultaneous based approaches for the synthesis of multi-period HENs include the works of Aaltola [9], Verheyen and Zhang [10], Isafiade and Fraser [11], Isafiade, et al. [12], Isafiade and Short [13], Short, et al. [14], Sadeli and Chang [15], Jiang and Chang [16]. The technique presented by Aaltola [9], used an average area approach in the multi-period objective function. The average area approach implies that the size of the heat exchanger connecting the same pair of streams in more than one period of operation and in the same interval of the multi-period SWS model is the average of the areas required by the stream pair in the different periods of operations. Verheyen and Zhang [10] on the other hand used the maximum area approach in the SWS multi-period objective function. The maximum area approach ensures that the size of the heat exchanger selected to exchange heat between the same pair of streams in the same interval of the superstructure, but at different periods, is the maximum area required. It is worth stating that according to Isafiade and Fraser [11], these two approaches would fail to give the correct weighting in terms of quantity of utilities used in each period for cases where the period durations are unequal. Hence Isafiade and Fraser [11] modified the objective function of the multi-period SWS based model to address the aforementioned limitation in the

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methods of Aaltola [9] and Verheyen and Zhang [10]. Isafiade, et al. [12] further extended the SWS based approach by developing a set of solution procedure using a reduced superstructure approach. In this method, the mixed integer non-linear program (MINLP) multi-period SWS model is solved a number of times so as to generate a set of promising matches which are then used to initialise a reduced multi-period MINLP model. Short, et al. [14] developed a new technique for multi-period heat exchanger network synthesis (HENS) which focused on using a correction factor approach in the objective function of the model so as to ensure that resulting networks are as close as possible to realistic designs. This is necessary since most of the exiting methods have been based on simplified heat exchanger design equations. The papers by Sadeli and Chang [15] and Jiang and Chang [16] are both based on the time sharing mechanism. This approach was developed by the authors due to the fact that period durations may change from time to time in an unpredictable manner, which may then result in some exchangers being overdesigned or redundant for some periods of operations. In the time sharing scheme approach, different stream pairs in different periods of operations can share the same heat exchanger. It is worth stating that according to Isafiade and Short [13], key shortcomings associated with the time sharing approach is that exchangers would have to be thoroughly cleaned from time to time, and the resulting networks, which would be complex in terms of piping, would require complex instrumentations. Hence Isafiade and Short [13] developed a technique, which is also based on the SWS approach, whose focus was to address the aforementioned shortcomings of the time sharing method. The approach of Isafiade and Short [13] entails a set of solution procedure which requires fixing the size of selected heat exchangers in one step, after which the flexibility of the fixed exchangers are tested for a range of combinations of period durations in a further synthesis step.

Other simultaneous based techniques that have been adopted for the synthesis of multi-period HENs include the work of Kang, et al. [17], Kang, et al. [18] and Isafiade and Odejobi [19]. The approach of Kang, et al. [17] involves firstly, the synthesis of a representative single period network which is then optimised for other periods in terms of feasible heat exchange. In the case of Kang, et al. [18], the method developed is for scenarios where the period durations are significantly different. The technique presented by Isafiade and Odejobi [19] was aimed at improving the search space for an optimal solution in the existing multi-period SWS based HEN model. This the authors accomplished by positioning multiple utilities not only at the ends of the superstructure as is currently done in existing methods, but also in all intervals of the superstructure where streams of the opposite kind exist. The authors also presented an initialisation technique which is sequential, so as to improve on the quality of potential solutions.

It is worth stating that stochastic based approaches have also been adopted for the synthesis of multi-period HENs. Examples include the works of Xiangkun, et al. [20] and Ahmad, et al. [21]. The technique of Xiangkun, et al. [20] involves the use of multi-stream heat exchangers in place of single stream exchangers in the multi-period network. The heat exchanger areas are then optimised using genetic/simulated annealing algorithm. The work of Ahmad, et al. [21] used simulated annealing. The authors were of the view that one of the limitations associated with this approach is that higher computational times are required in order to search for solutions within the region of the global optimal solution.

It is worth stating at this point that the existing multi-period synthesis methods that have been reviewed so far have all focused on the multi-period profile of process stream parameters such as supply and target temperatures as well as stream flow rates. No consideration has been given to potential changes in parameters and data associated with the utilities in terms of their cost and availabilities, as these changes could also be multi-period in nature. As an illustration, the cost of utilities may change from time to time due to various factors such as changes in government policies, inflation, inconsistency in utility supply, tax, etc. In the case of availability of utilities, this may also change from time to time, especially for those utilities that are generated from sources that are seasonal in nature, e.g. biomass, wind and solar energy sources. This implies that multi-period heat exchanger networks should be designed not only to be flexible to satisfy heat demand for changing process stream parameters, but also to be robust and flexible to exchange heat in a cost efficient manner even when utility costs or utility availabilities change. This is necessary because heat exchanger areas would have been fixed once design is completed whereas utility costs may only change maybe on a yearly basis. Also, in terms of availability of utilities, back-up plans for alternative sources of utilities, together with the influence of these alternative sources on the network design, needs to be considered during the network synthesis stage. This paper intends to address the case where utility cost may change in a specified manner within a period of time. It is hoped that the other aspect which involves considering changes in availability of utilities will be considered in future studies.

II. METHODOLOGY

A. Problem Statement

The problem statement of this paper can be stated as follows: Given a set of hot process streams HP to be cooled and cold process streams CP to be heated. The hot and cold process streams have specified supply and target temperatures as well as flowrates at specified P periods of operations. Also given are a set of hot HU and cold CU utilities alongside their cost data which can change within specified TT time intervals. The goal is to synthesise a flexible heat exchanger network which is capable of meeting the demand for heat for all of the process streams in every period of operations and every TT time interval where costs of utilities change, in a cost effective manner.

B. Multi-Period Stage-Wise Superstructure Model

The multi-period SWS used in this study is adapted from Isafiade and Odejobi [19] and is shown in Fig. 1. In this superstructure, multiple options of utilities can be accommodated and each of these utilities is positioned in every interval and every period of the superstructure where process streams of the opposite kind exists. This implies that each
process stream in each interval where they are present has the opportunity of splitting into the number of streams (both process and utilities) of the opposite kind present in the interval for the purpose of exchanging heat. It should be known that as shown in Fig. 1, the process hot streams run from the last temperature interval to the second interval in every period of operation while the cold process streams run from the first temperature interval to the third interval in every period of operation. Each of the hot utilities are made to participate in intervals 2, 3 and 4, in every period of operation, while each of the cold utilities are made to participate in intervals 1, 2, and 3, in every period of operation. The advantages of this approach is that more options of heat exchange, in every period of operation, are included for every process hot and cold streams, since they also have the opportunity of not only exchanging heat with process streams in the intermediate intervals, but with utilities as well. However, a major shortcoming of this approach is that the number of binary variables will increase significantly. Isafiade and Odejobi [19] overcame this by developing a sequential initialisation procedure.

C. Model Equations

Even though the superstructure adopted in the proposed model of this paper is that of Isafiade and Odejobi [19], the set of model equations used to represent the superstructure mathematically in a solver environment differs. The major difference lies in the fact that since the aim of this paper is to extend the flexibility of multi-period models to scenarios where the cost of utilities may change from time to time, additional index ‘tt’, is included alongside the index ‘p’ which represents periods of operations. The newly added index tt represents each of the time intervals in which each utility takes on a new cost. The set of equations used to model the superstructure includes the following: process streams overall and interval heat balance in every time interval tt and every period of operation p, monotonicity of temperatures along the superstructure for each time interval tt and each period of operation p, exchanger heat load logical constraint, exchanger approach temperatures for each time interval tt and each period of operation p, the maximum area expression and the objective function. All of these model equations, except for the maximum heat exchanger area equation and objective function, are shown in the appendix of this paper. Due to space limitation, the detailed explanations for each set of model equations shown in the appendix are not stated. The reader is referred to Verheyen and Zhang [10] and Isafiade, et al. [12] for the detailed explanations. Shown in (1) is the maximum heat exchanger area expression as used in this paper.

\[
\begin{align*}
A_{i,j,k} & \geq \frac{q_{i,j,k,tt}}{\left( LMTD_{i,j,k,tt} \right) \left( U_{ij} \right)} \\
\hspace{1cm} & \forall i \in HP, j \in CP, k \in K, p \in P, tt \in TT
\end{align*}
\]

where \(q_{i,j,k,tt}\) is the amount of heat transferred from hot process stream \(i\) to cold process stream \(j\) in temperature interval \(k\), period \(p\), and time interval \(tt\), \(LMTD_{i,j,k,tt}\) is the logarithmic mean temperature difference in match \(i,j,k\) in period \(p\), and time interval \(tt\), while \(U_{ij}\) is the overall heat transfer coefficient between hot process stream \(i\) and cold process stream \(j\). It should be known that (1) differs from the maximum area expression in other SWS based multi-period models in that it contains the index \(tt\). Note that the expression for \(LMTD_{i,j,k,tt}\) and how it relates to other parameters and variables of the multi-period model is shown in the appendix. The purpose of (1) as expressed in this paper is to ensure that the representative heat exchanger area \(A_{i,j,k} selected in the synthesis process to exchange heat between the same pair of streams in the same stage of the SWS model, and in more than one period of operation, is the maximum of the areas required for each of the periods of operation. The time interval index \(tt\) in the variables \(q_{i,j,k,tt}\) and \(LMTD_{i,j,k,tt}\), and parameter \(U_{ij}\) on the right hand side of (1), ensures that the maximum heat exchanger area \(A_{i,j,k}\) is not only large enough to operate in every period of operation, but also of the right size to operate when cost of utilities increase, which is represented by change in time intervals.

The objective function shown in (2) comprises a simultaneous minimisation of a sum of the annual operating cost (AOC) and annualised capital costs (ACC). In (2), the AOC is summed over the number of periods and time intervals in which the process operates. In this equation, TAC represents the total annual cost, \(DOP_{i,j,p}\) represents the duration of period \(p\) in time interval \(tt\), NOP is the number of periods, HUC represents cost per unit of hot utility, CUC represents cost per unit of cold utility, AF represents annualisation factor (0.2 used in this paper), CF represents exchanger installation cost (8333.3 $ used in this paper), \(y_{i,j,k}\) represents binary variable which indicates whether an exchanger is selected or not, AC represents cost per area of heat exchanger (641.7 $ used in this paper) while \(ACi\) represents area cost index for exchanger (1 used in this paper). The approach adopted in this paper to cater for possible changes in utility prices is novel in that previous HENS methods have mostly involved single periods of operations, while those which considered multiple periods of operations have assumed that utility prices will be fixed for the duration of the plant life. The form that the additional time interval, which represents time periods where prices of utilities may change despite changing periods of operations, has been used is novel in that the maximum area expression in (1) will ensure that heat exchanger areas are not significantly oversized irrespective of the period of operation or yearly time interval being served in terms of heat exchange. Furthermore, the form in which (2) has been presented ensures that the correct quantity of utilities used in each period of operation and each yearly time interval is adequately accounted for.

The MINLP model of this paper is solved using General Algebraic modelling Systems (GAMS), version 24.0.2, as the solver environment [22], Microsoft Windows 7 Enterprise™ 64 bit, Intel® Core™ i5-3210M processor running at 2.50 GHz with 4 GB of installed memory is used as the platform for running the solver. The solver, which is DICOPT uses CPLEX for the mixed integer linear program master problems and CONOPT for the non-linear program sub-problems.

III. Example

The example considered in this paper comprises 3 process hot process streams, 4 process cold streams, 3 hot utilities (high, medium and low pressure steam levels), and 1
cold utility. Each of these streams, including the utilities are available in 3 periods of operations as illustrated in Table I. 

\[
\min TAC
= \left\{ \sum_{i \in I} \sum_{j \in J} \left( \sum_{k \in K} \left( \frac{DOP_{tt,p}}{\sum_{p=1}^{\text{NOF}} DOP_{tt,p}} \right) \sum_{i \in \text{HP}} \sum_{j \in \text{CU}} \sum_{k \in \text{K}} \text{UC}_j \cdot q_{i,j,k,p,tt} \right) \right\}
\]

\[
+ \left( \left( \frac{DOP_{t1,p}}{\sum_{p=1}^{\text{NOF}} DOP_{t1,p}} \right) \sum_{i \in \text{HU}} \sum_{j \in \text{CP}} \sum_{k \in \text{K}} \text{HU}_i \cdot q_{i,j,k,p,tt} \right) \right\}
\]

\[
+ AF \left\{ \sum_{i \in \text{HP}} \sum_{j \in \text{HU}} \sum_{k \in \text{K}} \sum_{\text{CU}_k} \text{CU}_j \cdot A_{i,j,k} \right\}
\]

\[
\sum_{i \in \text{HP}} \sum_{j \in \text{HU}} \sum_{k \in \text{K}} \sum_{\text{CU}_k} \text{CU}_j \cdot A_{i,j,k}
\]

\[
i \in H \quad j \in C \quad k \in K \quad s \in S \quad p \in P \quad tt \in TT
\]

The periods of operations are assumed to have equal durations, which is 4 months each in an operational year. The data for the utilities are shown in Table II. In this table, the cost of each of the utilities increases from Year 1 to Year 3, where each year constitutes time interval tt. So the goal of the model developed in this paper is to ensure that the resulting network of exchangers would be robust and flexible enough to exchange heat in a cost effective manner as changes from one period to another occurs as well as when utility costs increase from one time interval to another. This implies that the exchangers which would have been fixed based on the results of the multi-period SWS model, would not have to be retrofitted in order to continually have a cost efficient operation when utility costs change. In solving this example, two solution approaches are developed in this paper. The first approach (called the proposed model) is the goal of the technique being developed in this paper and is favoured over the second approach due to the fact that it is simultaneous in nature, unlike the second approach that is sequential in nature. The second approach is developed in this paper so as to have a basis for illustrating the benefits of the proposed model. The first approach entails solving the set of multi-period model equations which include the newly added time interval index tt, i.e. the set of equations in the appendix and (1) and (2), simultaneously. The second approach, called the benchmark method, on the other hand entails first solving the set of multi-period model equations shown in the appendix and in (1) and (2) in the conventional way, i.e. without the index tt, hence it is solved for a fixed time interval. The purpose of this is to generate a multi-period network based on any of the utility costs, i.e. one time interval, highlighted in Table II. In this example, the set of costs listed for Year 1 was used in this first step. The next step then entails fixing the heat exchanger area sizes of the multi-period network to the matches, including the sizes obtained, in the first step. Once this is done, the multi-period network is then tested in terms of feasibility of satisfying heat demand for all periods in each of the other 2 years, one after the other. The detailed solutions for both approaches are shown in Table III. In this table, the AOC increases from Year 1 to Year 3. This is the case since utility costs increase from Year 1 to Year 3. However the determination of the heat exchanger area sizes required to satisfy these heat duties in each of the periods, as well as in each of the yearly time interval in which utility costs increase, were obtained simultaneously. Table IV shows a comparison of the heat exchanger areas obtained when the synthesis is done simultaneously, which is the proposed approach of this paper, unlike when the synthesis is done sequentially using conventional multi-period synthesis approach (referred to as the benchmark approach in the table). Fig. 2 illustrates the network for the proposed approach. In Table IV, the ACC of the proposed approach is higher than that of the sequential approach by 26%. However in Table V, it can be seen that the higher ACC obtained for the proposed approach makes provision for a better usage of utilities for each of the time

![Figure 1. Multi-period stage-wise superstructure as used by Isafiade and Odejobi [19]](image-url)
TABLE II. UTILITY DATA FOR EXAMPLE CONSIDERED

<table>
<thead>
<tr>
<th>Utility</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>High pressure steam, HU1 (400°C)</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Medium pressure steam, HU2 (300°C)</td>
<td>100</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>Low pressure steam, HU3 (250°C)</td>
<td>70</td>
<td>120</td>
<td>170</td>
</tr>
<tr>
<td>Cooling water, CU1 (0°C~10°C)</td>
<td>1.3</td>
<td>1.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>

TABLE III. UTILITY USAGE FOR BOTH APPROACHES

<table>
<thead>
<tr>
<th>Utilities</th>
<th>Costs for proposed approach</th>
<th>Costs for benchmark approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>HU1</td>
<td>Periods</td>
<td>Year 1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1,412,583</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1,344,945</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1,265,040</td>
</tr>
<tr>
<td>HU2</td>
<td>1</td>
<td>13,354</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>14,323</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>13,484</td>
</tr>
<tr>
<td>CU1</td>
<td>1</td>
<td>16,793</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>17,546</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>17,546</td>
</tr>
<tr>
<td>Total ($/yr)</td>
<td>4,064,136</td>
<td>5,435,792</td>
</tr>
</tbody>
</table>

TABLE IV. COMPARISON OF HEAT EXCHANGER AREAS BETWEEN PROPOSED MODEL AND BENCHMARK APPROACH

<table>
<thead>
<tr>
<th>Units</th>
<th>Heat Exchanger Area (m²)</th>
<th>Proposed model</th>
<th>Benchmark approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,813</td>
<td>5,240</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2,036</td>
<td>2,817</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>263</td>
<td>5,119</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>17,822</td>
<td>9,636</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1,804</td>
<td>1,428</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2,516</td>
<td>2,864</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2,126</td>
<td>4,571</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7,599</td>
<td>2,900</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7,054</td>
<td>864</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>516</td>
<td>4,261,892</td>
<td></td>
</tr>
</tbody>
</table>

ACC ($/yr) 5,785,414

IV. CONCLUSION

In the proposed model of this paper the multi-period network is synthesized simultaneously taking into consideration all periods of operations and yearly time intervals in which utility costs are envisaged to change, by restricting the candidate heat exchangers to be greater than or equal to the areas required in all of the periods and time intervals. This approach of using the maximum area was first developed for HENS problems involving multi-period operations where only process parameters change from time to time [10]. However, in this paper, the maximum area approach has been extended to include other HENS problem data that could change in a multi-period fashion like changes in utility costs. It is hoped that future studies will extend the newly developed synthesis approach to include other problem parameters that may change from time to time like availability of utility energy sources, especially the renewable ones, due to issues like weather conditions and seasonality. Limitations associated with the approach developed in this paper include the fact that one or more of the representative exchangers of the final multi-period network may be overdesigned due to the fact that these exchangers were selected to be the maximum area required considering not only periods of operations, but time intervals where utility costs may change as well. This limitation may be more significant for cases where the period durations differ significantly or even cases where the period durations change in an unpredictable manner. Again, it is hoped that these issues will be addressed in future studies.

TABLE V. TOTAL COST COMPARISON FOR THE TWO SYNTHESIS APPROACHES

<table>
<thead>
<tr>
<th>Costs for proposed approach</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOC</td>
<td>4,064,136</td>
<td>5,435,792</td>
<td>6,807,448</td>
</tr>
<tr>
<td>ACC</td>
<td>5,785,414</td>
<td>5,785,414</td>
<td>5,785,414</td>
</tr>
<tr>
<td>TAC</td>
<td>9,849,094</td>
<td>11,220,595</td>
<td>12,580,973</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Costs for benchmark approach</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOC</td>
<td>6,343,229</td>
<td>8,462,685</td>
<td>9,806,302</td>
</tr>
<tr>
<td>ACC</td>
<td>4,261,892</td>
<td>4,261,892</td>
<td>4,261,892</td>
</tr>
<tr>
<td>TAC</td>
<td>10,605,121</td>
<td>12,724,577</td>
<td>14,068,194</td>
</tr>
</tbody>
</table>

Difference % 7 12 11

Figure 2. Final multi-period network of the proposed approach for the example of this paper.

APPENDIX

Shown below are the set of equations used to model the superstructure.

Overall hot and cold process stream enthalpy balance in each period of operation and each time interval:

\[
(T_{i,p,t}^\text{in} - T_{i,p,t}^\text{out})F_{i,p,t} = \sum_{l \in \text{HP}, p \in P, t \in \text{TT}} q_{l,i,p,t} \]

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\[(T_{f,pt}^i - T_{f,pt}^j)F_{f,pt} = \sum_{k \in K} \sum_{l \in CP} q_{i,j,k,pt} \quad j \in CP, p \in P, tt \in TT\]

Stage enthalpy balance in each period of operation and each time interval for hot and cold streams respectively:

\[ (t_{i,k,pt} - t_{i,k+1,pt})F_{i,pt} = \sum_{k \in K} q_{i,k,pt} \quad k \in K, i \in HP, p \in P, tt \in TT \]

\[ (t_{j,k,pt} - t_{j,k+1,pt})F_{j,pt} = \sum_{k \in K} q_{j,k,pt} \quad k \in K, i \in HP, p \in P, tt \in TT \]

Temperature feasibility along superstructure interval for each period of operation and each time interval:

\[ t_{i,k,pt} \geq t_{i,k+1,pt} \quad i \in HP, p \in P \]

\[ t_{j,k,pt} \geq t_{j,k+1,pt} \quad j \in CP, p \in P \]

Logical constraint:

\[ q_{i,j,k,pt} - \Delta y_{i,k} \leq 0 \]

Temperature difference at hot end and cold end of exchangers for calculation of heat exchanger area:

\[ dt_{i,j,k,pt} \leq t_{i,k,pt} - t_{c,i,j,pt} + \phi (1 - y_{i,j,k}) \quad k \in K \]

\[ i \in HP, \quad j \in CP, \quad p \in P, \quad tt \in TT \]

\[ dt_{i,j,k+1,p} \leq t_{i,k+1,p} - t_{c,i,j,pt} + \phi (1 - y_{i,j,k}) \quad k \in K \]

\[ i \in HP, \quad j \in CP, \quad p \in P, \quad tt \in TT \]

Exchanger minimum approach temperature (EMAT):

\[ dt_{i,j,k,pt} \geq EMAT \quad k \in K, i \in HP, j \in CP, p \in P, tt \in TT \]

Paterson (1984) approximation of the logarithmic mean temperature difference:

\[ \text{LMTD}_{i,j,k,pt} = \frac{2}{3} \left( \frac{(dt_{i,j,k,pt}) (dt_{i,j,k+1,pt})}{2} \right)^{3/2} \]

\[ + \frac{1}{3} \left( \frac{(dt_{i,j,k,pt}) + (dt_{i,j,k+1,pt})}{2} \right) \]

ACKNOWLEDGMENT

This study is supported by the National Research Foundation of South Africa (Grant numbers: 85536 and 87744), Sasol, and the Research Office of the University of Cape Town. These supports are gratefully acknowledged.

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