Parameter identification method for process control systems based on the Newton iteration

Ling Xu, Feng Ding, Yanjun Liu, Junhong Li and Jing Chen

Abstract—The typical second-order system is a widely used model in control system analysis. The typical second-order system responses contain abundant dynamical information and can be modeled and identified. The modeling for the typical second-order system is significant. This paper studies the identification method for the typical second-order system by means of the impulse response experiment and presents a Newton iterative parameter estimation algorithm for the second-order system with under-damping ratio. An example is provided to test the algorithm performance. Moreover, the proposed method is applied to a mechanical system. The simulation results show that the proposed identification method can work effectively.

I. INTRODUCTION

In industrial processes, some dynamical systems are modeled by the second-order models. Many higher-order systems can be approximated to a second-order system under certain conditions. Because the second-order system can capture the dynamical characteristic of the systems, it is used widely for the system analysis and process control. Bruschetta et al. developed a variational integrators identification approach to the second-order mechanical systems [1], [2]; Maamar et al. designed an internal model proportional integral derivative controller for the typical second-order and the second-order plus delay-time systems [3]; You et al. studied the time disturbance of the second-order systems [4]; Shamsuzzoha et al. discussed the proportional integral derivative controller method of the set-point overshoot to the second-order systems and other process models [5]. Xu proposed a proportional derivative controller for the typical second-order systems [6]. This paper studies the parameter estimation method for the typical second-order systems.

The iterative techniques are used widely for solving the solutions of nonlinear equations or matrix equations [7]–[9]. In system identification, the iterative techniques are used to derive the parameter estimation method [10], [11]. Chen et al. presented an iterative parameter identification method based the data filtering [12]; Zhang et al. studied the iterative algorithm for coupled Sylvester matrix equations [13]; Chidume et al. developed the iterative algorithm for solving the approximating solutions for the Hammerstein nonlinear integral equations [14]; Jin et al. studied the iterative identification method based on the least squares for multivariable integrating and unstable processes [15]; Yun investigated the iterative method for solving the roots of a nonlinear equation [16]; Jafari applied the iterative hierarchical least squares identification algorithm to multivariable nonlinear systems [17]; Cordero et al. derived a derivative-free optimal iterative method for solving nonlinear equations [18]; Li presented a Newton iteration parameter estimation algorithm for Hammerstein CARARMA systems [19]. Butyugin brought forward the iterative solvers for time-harmonic Maxwell equations using the domain decomposition and the algebraic multi-grid [20]. According to the Newton iteration principle, this paper derives the Newton iterative estimation algorithm to estimate the parameters of the typical second-order system. For the typical second-order system, the damping ratio is an important system parameter. In general, the damping ratio has four different condition. In this paper, we study the typical second-order system with the under-damping ratio.

In the system identification, the system parameters can be obtained by defining a cost function [21], [22] and utilizing some optimization algorithms [23]–[25]. The parameter estimation algorithms can be used in linear systems and nonlinear systems. Felis et al. applied the modeling and identification method to the emotional aspects of the locomotion [26]. Yang et al. presented a mathematical modeling and parameter optimization approach to the pulsating heat pipes [27]. Moreover, the identification test is the first step for the parameter estimation. Step response tests [28], [29], impulse response tests, frequency response tests and relay feedback tests are widely used in the system identification [30]–[32]. Wang et al. derived an identification method based on a linear regression equation and the least squares by using the step responses [33]; Li et al. presented a parameter identification method for the two-input two-output process from closed-loop step responses [34]; Hidayat et al. designed an identification method for continuous linear time-delay systems by applying the impulse response data [35]; Panda et al. studied a parameter estimation method for the time-delay process from the relay feedback test [36]; Ryu et al. proposed a discrete-time identification method for the final cyclic steady state process from the frequency response [37]. In the previous work, we presented the damping iterative parameter identification method for dynamical systems based...
on the sine signal measurement \[38\]. Moreover, based on the
response signals, the multi-innovation and the hierarchical
identification methods were proposed to estimate the transfer
function of systems \[39\]. In this paper, the impulse response
test is used to gather the measured data for parameter
identification.

The rests of this paper are organized in the following.
Section II derives the Newton iterative algorithm for the
typical second-order system with a under-damping ratio.
Section III provides an examples to show the effectiveness
of the proposed parameter estimation method. Section IV
gives the application of the proposed method to a mechanical
rotation system. Finally, Section V draws some conclusions.

II. THE NEWTON ITERATIVE PARAMETER ESTIMATION

In process control, the typical second-order systems are
used widely for describing and analyzing the dynamical
characteristic of systems. The transfer function is a useful
mathematical model for describing control systems. Generally,
the transfer function of the typical second-order system is
represented by

\[
G(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2},
\]

(1)

where \(\xi\) is the damping ratio and \(\omega_n\) is the undamped
natural angular frequency. These parameters denote the system
performance. Once these parameters are estimated, we can
achieve the model of the typical second-order system. In
this study, the under-damping typical second-order system is
considered to present the identification method.

When the damping ratio \(0 < \xi < 1\), it is called under-
damping. For convenience, let \(a := \xi \omega_n, b := \omega_n^2\). Then
Equation (1) can be rewritten as

\[
G(s) = \frac{b}{s^2 + 2as + b}.
\]

In order to collect the measured data, a unit impulse signal
is taken as the input of the system. Because the Laplace
transform of the unit impulse signal is \(R(s) = 1\), the Laplace
transform of the output is given by

\[
Y(s) = G(s)R(s) = \frac{b}{s^2 + 2as + b}.
\]

The output response is the inverse Laplace transform of \(Y(s)\).
Taking the inverse Laplace transform gives

\[
y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left[\frac{b}{s^2 + 2as + b}\right]
= \mathcal{L}^{-1}\left[\frac{b}{(s + a)^2 + \sqrt{b-a^2}}\right]
= \mathcal{L}^{-1}\left[\frac{b}{\sqrt{b-a^2}}\frac{1}{(s + a)^2 + (\sqrt{b-a^2})^2}\right]
= \frac{b}{\sqrt{b-a^2}} e^{-at} \sin \sqrt{b-a^2} t.
\]

(2)

Define the difference between the observation output and
the model output as

\[
e_i := y(t_i) - \frac{b}{\sqrt{b-a^2}} e^{-at_i} \sin \sqrt{b-a^2} t_i,
\]

where \(y(t_i)\) \((i = 1, 2, \cdots, L)\) are the measured data and \(L\) is
the data length.

Define the cost function

\[
J(\theta) = \frac{1}{2} \sum_{i=1}^{L} e_i^2 = \frac{1}{2} \sum_{i=1}^{L} \left[y(t_i) - \frac{b}{\sqrt{b-a^2}} e^{-at_i} \sin \sqrt{b-a^2} t_i \right]^2,
\]

(3)

where \(\theta\) is the parameter vector \(\theta := [a, b]^T \in \mathbb{R}^2\).

Taking the first-order partial derivative of \(J(\theta)\) with
respect to \(\theta\) gives

\[
f_a(\theta) := \frac{\partial J(\theta)}{\partial a} = \frac{1}{2} \sum_{i=1}^{L} e_i e^{-at_i} \cos \sqrt{b-a^2} t_i + \frac{b}{\sqrt{b-a^2}} e^{-at_i} \sin \sqrt{b-a^2} t_i.
\]

\[
f_b(\theta) := \frac{\partial J(\theta)}{\partial b} = -\frac{1}{2} \sum_{i=1}^{L} e_i e^{-at_i} \left(\frac{b-2a^2}{\sqrt{b-a^2}} \sin \sqrt{b-a^2} t_i + b t_i e^{-at_i} \cos \sqrt{b-a^2} t_i\right).
\]

(4)

(5)

Equations (4)–(5) form the gradient of the cost function
\(J(\theta)\). Define the gradient of the cost function as

\[
F(\theta) := \text{grad}(J(\theta)) = \left[\frac{\partial J(\theta)}{\partial a}, \frac{\partial J(\theta)}{\partial b}\right]^T \in \mathbb{R}^2.
\]

Taking the second-order partial derivative of \(J(\theta)\) and form-
ing the Hessian matrix, we have

\[
H(\theta) := \frac{\partial^2 J(\theta)}{\partial \theta \partial \theta^T} = \frac{\partial^2 J(\theta)}{\partial \theta_a \partial \theta_a} \left(\begin{array}{cc} h_{11}(\theta) & h_{12}(\theta) \\ h_{21}(\theta) & h_{22}(\theta) \end{array}\right) \in \mathbb{R}^{2 \times 2},
\]

where

\[
h_{11}(\theta) := \frac{\partial^2 J(\theta)}{\partial \theta_a^2} = \sum_{i=1}^{L} \left(\frac{b e^{-at_i}}{\sqrt{b-a^2}} \left(t_i - \frac{a}{b-a^2}\right) \sin \sqrt{b-a^2} t_i + \left(\frac{b t_i e^{-at_i}}{\sqrt{b-a^2}} + \frac{3a^2}{(b-a^2)^2} \right) \cos \sqrt{b-a^2} t_i \right)
\]

\[
h_{12}(\theta) := \frac{\partial^2 J(\theta)}{\partial \theta_a \partial \theta_b} = \sum_{i=1}^{L} \left(\frac{b e^{-at_i}}{\sqrt{b-a^2}} \left(\frac{b}{2b-2a^2-1} \right) \cos \sqrt{b-a^2} t_i \right)
\]

\[
\times \sin^2 \sqrt{b-a^2} t_i + \left(\frac{b t_i e^{-at_i}}{(b-a^2)^2} \left(\frac{ab}{2b-2a^2} \right) \left(\frac{b}{2b-2a^2-1} \right) \cos \sqrt{b-a^2} t_i \right).
\]
Let $k = 1, 2, 3, \ldots$ be the iterative variable and $\hat{\theta}_k := [\hat{a}_k, \hat{b}_k]^T$ denote the parameter estimates of $\theta$. Minimizing the cost function $J(\theta)$, we can derive the Newton iterative parameter estimation algorithm

$$\hat{\theta}_{k+1} = \hat{\theta}_k - H^{-1}(\hat{\theta}_k)F(\hat{\theta}_k),$$

(6)

$$F(\hat{\theta}_k) = [f_a(\hat{\theta}_k), f_b(\hat{\theta}_k)]^T,$$

(7)

$$H(\hat{\theta}_k) = \begin{bmatrix} h_{11}(\hat{\theta}_k) & h_{12}(\hat{\theta}_k) \\ h_{21}(\hat{\theta}_k) & h_{22}(\hat{\theta}_k) \end{bmatrix},$$

(8)

$$f_a(\hat{\theta}_k) = \sum_{i=1}^{L} \left\{ \frac{\epsilon_i e^{-\hat{a}_i t_i}}{\sqrt{b_k - \hat{a}_k^2}} \left( b_k t_i - \frac{\hat{a}_k}{b_k - \hat{a}_k^2} \right) \right\} \times \sin \sqrt{b_k - \hat{a}_k^2} t_i,$$

(9)

$$f_b(\hat{\theta}_k) = \sum_{i=1}^{L} \frac{\epsilon_i e^{-\hat{a}_i t_i}}{2(b_k - \hat{a}_k^2)} \left( \frac{b_k - 2\hat{a}_k}{b_k - \hat{a}_k^2} \sin \sqrt{b_k - \hat{a}_k^2} t_i + \hat{b}_k t_i \cos \sqrt{b_k - \hat{a}_k^2} t_i \right),$$

(10)

$$h_{11}(\hat{\theta}_k) = \sum_{i=1}^{L} \left\{ \frac{\epsilon_i e^{-\hat{a}_i t_i}}{\sqrt{b_k - \hat{a}_k^2}} \left( t_i - \frac{\hat{a}_k}{b_k - \hat{a}_k^2} \right) \sin \sqrt{b_k - \hat{a}_k^2} t_i \right\} \times \sin \sqrt{b_k - \hat{a}_k^2} t_i,$$

$$+ \frac{\epsilon_i b_k e^{-\hat{a}_i t_i}}{b_k - \hat{a}_k^2} \cos \sqrt{b_k - \hat{a}_k^2} t_i - \frac{\epsilon_i}{b_k - \hat{a}_k^2} \frac{b_k e^{-\hat{a}_i t_i}}{b_k - \hat{a}_k^2} \sin \sqrt{b_k - \hat{a}_k^2} t_i \times \sin \sqrt{b_k - \hat{a}_k^2} t_i,$$

(11)

$$h_{12}(\hat{\theta}_k) = \sum_{i=1}^{L} \left\{ \frac{b_k t_i e^{-\hat{a}_i t_i}}{b_k - \hat{a}_k^2} \left( \frac{b_k}{2b_k - 2\hat{a}_k^2} - 1 \right) \cos \sqrt{b_k - \hat{a}_k^2} t_i \right\},$$

(12)

$$h_{21}(\hat{\theta}_k) = \sum_{i=1}^{L} \left\{ \frac{b_k t_i e^{-\hat{a}_i t_i}}{b_k - \hat{a}_k^2} \left( \frac{b_k}{2b_k - 2\hat{a}_k^2} - 1 \right) \sin \sqrt{b_k - \hat{a}_k^2} t_i \right\},$$

(13)

$$h_{22}(\hat{\theta}_k) = \sum_{i=1}^{L} \left\{ \frac{b_k t_i e^{-\hat{a}_i t_i}}{b_k - \hat{a}_k^2} \left( \frac{b_k}{2b_k - 2\hat{a}_k^2} - 1 \right) \times \sin^2 \sqrt{b_k - \hat{a}_k^2} t_i \cos \sqrt{b_k - \hat{a}_k^2} t_i \right\},$$

(14)

$$\hat{a}_k = \sqrt{\hat{b}_k - \hat{a}_k^2},$$

(15)

The steps of fulfilling the Newton iterative algorithm are listed in the following.

1) To initialize: let $k = 0$ and let $\hat{\theta}_0 = [\hat{a}_0, \hat{b}_0]^T$ be a random vector. Pre-set a small positive number $\varepsilon > 0$.

2) Collect the measured data $\{t_i, y(t_i), i = 1, 2, 3, \ldots, L\}$.

3) Compute $e_k$ using (15); compute $f_a(\hat{\theta}_k), f_b(\hat{\theta}_k)$ using (9)–(10) and form $F(\hat{\theta}_k)$ using (7).

4) Compute $h_{ij}(\hat{\theta}_k) (i = 1, 2, j = 1, 2)$ using (11)–(14); Form $H(\hat{\theta}_k)$ using (8).

5) Update the parameter estimation vector $\hat{\theta}_{k+1}$ through (6), if $\|\hat{\theta}_{k+1} - \hat{\theta}_k\| > \varepsilon$ increase $k$ by 1 and go to
Step 3; otherwise terminate the iterative procedure and
obtain the parameter estimation vector \( \hat{\theta}_k \).

6) Compute the parameter estimates \( \hat{\theta}_{hk} \) and \( \hat{\xi}_k \) using (16).

III. SIMULATION EXAMPLE

In this section, an example is provided to illustrate the
effectiveness of the proposed Newton iterative estimation
method.

Consider a typical second-order system with the under-
damping ratio

\[
G(s) = \frac{4}{s^2 + 2s + 4},
\]

where \( \xi = 0.5, \omega_n = 2 \).

In simulation experiment, we use the unit impulse re-
gponse to generate the measured data. Meanwhile, the distur-
bance signal \( \nu(t) \) of an uncorrelated noise sequence with
zero mean and variance \( \sigma^2 = 0.10^2 \) and \( \sigma^2 = 0.50^2 \) is added
to the system responses, respectively.

Using the proposed Newton iterative algorithm to estimate
the parameters, the parameter estimates and their estimation
errors are shown in Tables I–II and the estimation errors
\( \delta := \| \hat{\theta}_k - \theta \| /\| \theta \| \) versus \( k \) are shown in Figure 1.

![Fig. 1. The estimation errors \( \delta \) versus \( k \) for Example 1](image1)

![Fig. 2. A mechanical rotation system](image2)

Figure 2 shows the mechanical model of the mechanical
rotation system.

According to the principle of mechanics and selecting the
spring as the separation body, we obtain the torque balance
equation

\[
T_M(t) = M[\theta_k(t) - \theta_o(t)],
\]

where \( M \) is the torsional stiffness coefficient, \( \theta_k(t) \) is the
input rotation angle, \( \theta_o(t) \) is the output rotation angle and
\( T_M(t) \) is the spring torque.

Selecting the viscous damper as the separation body, we
obtain the movement equation

\[
T_C(t) = C \frac{d\theta_o(t)}{dt},
\]

where \( C \) is the viscous damping coefficient, \( T_C(t) \) is the
viscous damper torque.

Selecting the gear as the separation body, we obtain the
torque balance equation:

\[
J \frac{d^2 \theta_o(t)}{dt^2} = T_M(t) - T_C(t),
\]

where \( J \) is the rotary inertia of the rotation body. Substituting
(17) and (18) into (19) gets

\[
J \frac{d^2 \theta_o(t)}{dt^2} + C \frac{d\theta_o(t)}{dt} + M\theta_o(t) = M\theta_k(t).
\]

IV. APPLICATION TO A MECHANICAL ROTATION SYSTEM

In order to show the effectiveness of the proposed Newton
iterative method, a practical application to the mechanical
rotation system is provided in this section. In mechanical
equipments, the mechanical rotation systems are used widely.

![Table I](image3)

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \sigma_0 )</th>
<th>( \xi )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.974099</td>
<td>0.55035</td>
<td>0.03489</td>
</tr>
<tr>
<td>2</td>
<td>1.974747</td>
<td>0.53674</td>
<td>0.02856</td>
</tr>
<tr>
<td>3</td>
<td>1.978355</td>
<td>0.52758</td>
<td>0.02343</td>
</tr>
<tr>
<td>4</td>
<td>1.981977</td>
<td>0.52107</td>
<td>0.01914</td>
</tr>
<tr>
<td>5</td>
<td>1.985164</td>
<td>0.51629</td>
<td>0.01584</td>
</tr>
<tr>
<td>10</td>
<td>1.994806</td>
<td>0.50490</td>
<td>0.00533</td>
</tr>
<tr>
<td>15</td>
<td>1.999944</td>
<td>0.50006</td>
<td>0.00006</td>
</tr>
<tr>
<td>20</td>
<td>1.999437</td>
<td>0.50051</td>
<td>0.00058</td>
</tr>
<tr>
<td>25</td>
<td>1.999881</td>
<td>0.50017</td>
<td>0.00019</td>
</tr>
<tr>
<td>30</td>
<td>1.999994</td>
<td>0.50006</td>
<td>0.00006</td>
</tr>
<tr>
<td>True value</td>
<td>2.000000</td>
<td>0.50000</td>
<td></td>
</tr>
</tbody>
</table>

![Table II](image4)

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \sigma_0 )</th>
<th>( \xi )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.974804</td>
<td>0.51990</td>
<td>0.03155</td>
</tr>
<tr>
<td>2</td>
<td>1.974137</td>
<td>0.51424</td>
<td>0.02778</td>
</tr>
<tr>
<td>3</td>
<td>1.974501</td>
<td>0.50003</td>
<td>0.02464</td>
</tr>
<tr>
<td>5</td>
<td>1.983784</td>
<td>0.49989</td>
<td>0.01580</td>
</tr>
<tr>
<td>10</td>
<td>1.987079</td>
<td>0.49689</td>
<td>0.01287</td>
</tr>
<tr>
<td>15</td>
<td>1.998817</td>
<td>0.49593</td>
<td>0.01193</td>
</tr>
<tr>
<td>20</td>
<td>1.998853</td>
<td>0.49563</td>
<td>0.01163</td>
</tr>
<tr>
<td>25</td>
<td>1.998865</td>
<td>0.49553</td>
<td>0.01153</td>
</tr>
<tr>
<td>30</td>
<td>1.998865</td>
<td>0.49553</td>
<td>0.01153</td>
</tr>
<tr>
<td>True value</td>
<td>2.000000</td>
<td>0.50000</td>
<td></td>
</tr>
</tbody>
</table>
According to the definition of the transfer function, we obtain the transfer function of the mechanical rotation system:

\[ G(s) = \Theta_o(s) \Theta_i(s) = \frac{M}{s^2 + Cs + M} = \frac{M/J}{s^2 + C/s + M/J} \]  

(20)

Comparing (20) with the typical second-order system model, we have

\[ \omega_n = \sqrt{M/J}, \xi = C/2\sqrt{MJ}. \]

For the mechanical rotation system, the unknown parameters are \( M, J \) and \( C \). In fact, the mathematical model which is deduced in (20) has two unknown parameters \( \omega_n \) and \( \xi \). But for the system modeling, the aim is to obtain the relationship between the input and the output. Therefore, once the parameters \( \omega_n \) and \( \xi \) are estimated, the system model is obtained. Here, we use the proposed Newton iterative estimation method to estimate these parameters for the mechanical rotation system.

Applying an external force to the mechanical rotation system and then rescinding the force immediately, the measured data are listed in Table III. Using the Newton iterative algorithm to estimate the parameters of the mechanical rotation system, the parameter estimates are shown in Table IV.

### Table III

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \theta_o(t) )</th>
<th>( \theta_i(t) )</th>
<th>( \theta_o/t )</th>
<th>( \theta_i/t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.72396</td>
<td>1.10</td>
<td>0.29741</td>
<td>2.10</td>
</tr>
<tr>
<td>0.20</td>
<td>1.14682</td>
<td>1.20</td>
<td>0.18259</td>
<td>2.20</td>
</tr>
<tr>
<td>0.30</td>
<td>1.34102</td>
<td>1.30</td>
<td>0.09560</td>
<td>2.30</td>
</tr>
<tr>
<td>0.40</td>
<td>1.37079</td>
<td>1.40</td>
<td>0.03147</td>
<td>2.40</td>
</tr>
<tr>
<td>0.50</td>
<td>1.29037</td>
<td>1.50</td>
<td>-0.01296</td>
<td>2.50</td>
</tr>
<tr>
<td>0.60</td>
<td>1.14340</td>
<td>1.60</td>
<td>-0.04121</td>
<td>2.60</td>
</tr>
<tr>
<td>0.70</td>
<td>0.96344</td>
<td>1.70</td>
<td>-0.05676</td>
<td>2.70</td>
</tr>
<tr>
<td>0.80</td>
<td>0.77492</td>
<td>1.80</td>
<td>-0.06284</td>
<td>2.80</td>
</tr>
<tr>
<td>0.90</td>
<td>0.59453</td>
<td>1.90</td>
<td>-0.06225</td>
<td>2.90</td>
</tr>
<tr>
<td>1.00</td>
<td>0.43264</td>
<td>2.00</td>
<td>-0.05732</td>
<td>3.00</td>
</tr>
</tbody>
</table>

The collected measured data

In the simulation, the step response experiment are used in order to test the dynamical performance of the mechanical rotation system. The external forces with different values are applied to the system, the step responses are shown in Figure 3.

![Fig. 3. The step response](image)

From the simulation results, we can draw the following conclusions.

1) From the last column in Tables I–II, it can be seen that the parameter estimation errors obtained by the proposed Newton iterative algorithm become smaller gradually with the decreasing of the noise variances.

2) The parameter estimation errors obtained by the Newton iterative algorithm decrease gradually with the increasing of iteration \( k \), see Figure 1. With the decreasing of the noise variances, the parameter estimation accuracy becomes higher.

3) The application results to the mechanical rotation system show that the proposed Newton iterative parameter estimation algorithm is effective. The step responses show that the estimated system model can capture the dynamical characteristic of systems.

### Table IV

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \omega_n )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.11979</td>
<td>0.73110</td>
</tr>
<tr>
<td>2</td>
<td>3.09084</td>
<td>0.72321</td>
</tr>
<tr>
<td>3</td>
<td>3.06997</td>
<td>0.71766</td>
</tr>
<tr>
<td>4</td>
<td>3.05443</td>
<td>0.71362</td>
</tr>
<tr>
<td>5</td>
<td>3.04264</td>
<td>0.70599</td>
</tr>
<tr>
<td>10</td>
<td>3.01326</td>
<td>0.70324</td>
</tr>
<tr>
<td>15</td>
<td>3.00428</td>
<td>0.70104</td>
</tr>
<tr>
<td>20</td>
<td>3.00139</td>
<td>0.70034</td>
</tr>
<tr>
<td>25</td>
<td>3.00045</td>
<td>0.70011</td>
</tr>
<tr>
<td>30</td>
<td>3.00015</td>
<td>0.70003</td>
</tr>
</tbody>
</table>

The parameter estimates and error of the mechanical rotation system

V. CONCLUSIONS

This paper studies parameter estimation algorithms by using the impulse response test for the typical second-order system. For the typical second-order system with underdamping ratio, the Newton iterative parameter estimation algorithms can be derived. The simulation results show that the proposed Newton iterative algorithm works well. Other methods [40]–[46] can be used for studying parameter identification problems of transfer function models.

### REFERENCES


