Robust Distributed Control for Plantwide Processes Based on Dissipativity in Quadratic Differential Forms

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Abstract—Based on dissipativity theory, a robust distributed control approach for plantwide chemical processes are developed in this work. The plantwide process is represented as a network of process units and (distributed) controllers. The model of each process unit with uncertainty is represented as a polytopic set. The effects of uncertainties of process units on plantwide stability are analyzed based on the concept of dissipativity. As a result, the plantwide robust stability condition is formulated in terms of dissipativity, which is conveniently represented as the linear combination of the dissipativity conditions of individual process units and controllers, based on the topologies of the process and controller networks. Dissipativity conditions (storage functions and supply rates) in the Quadratic Differential Forms are adopted to reduce the conservativeness of the proposed robust control approach. Furthermore, the dissipativity conditions that individual controllers need to satisfy to ensure plantwide robust stability is developed. A robust distributed control synthesis approach for designing individual controllers is also developed.

I. INTRODUCTION

Complex process plants increasingly appear in the modern process industry, due to considerable economic efficiency that complex and interactive process designs can offer. Such processes often of large scales. Furthermore, they include material recycle loops and heat integration, leading to very strong interactions between process units, such as the example shown in [1], [2]. As a result, the traditional centralized control structure is becoming extremely difficult and often infeasible for plantwide control. Distributed plantwide control approaches are promising, where the plantwide process is viewed as a network of process units and controlled by a network of autonomous controllers which communicate and cooperate with each other to achieve plantwide stability and performance. More details of distributed control can be found in the review papers [3], [4] and the references therein.

In general, distributed control approaches employ the process models to anticipate the behaviour of controlled process units, see the examples in distributed model predictive control (DMPC) [1], [5], [6], [7], [8]. However, model-based design can be sensitive to the model uncertainty. Furthermore, the interactions between process units can significantly amplify the effects of uncertainties of individual process models and adversely impact on the plantwide stability, as illustrated in [9]. There are a number of ways to represent process uncertainties. One is to consider the uncertainty as an unknown dynamical system, which is connected to the nominal model [10], [11], [12]. Another way is to parameterize the process model uncertainty by a class of possible models [13], [14]. In this work, we adopt the second approach and represent the model of a process unit to be within a polytopic set confined by different models on the vertices (which can be developed by using, e.g., linear parameter varying (LPV) identification methods from the plant data [15], [16], [17]. In this formalism, robustness analysis and robust control design can be developed based on the models at the vertices due to the convexity of the polytopic set, e.g., [18], [19], [20], [21], [22]. However, most of these studies are focused on standalone systems.

In this work, a robust distributed control approach is developed using polytopic sets of process models. The effects of uncertainties of process units on plantwide stability are analyzed based on the concept of dissipativity. As an input-output property, dissipativity is very useful in interaction analysis and plantwide control [2], [23], [24], [6]. In this proposed approach, the plantwide robust stability condition is formulated in terms of dissipativity, which is conveniently represented as the linear combination of the dissipativity conditions of individual process units and controllers, based on the topologies of the process and controller networks. The dissipativity conditions that individual controllers need to satisfy to ensure plantwide robust stability is developed. A robust distributed control synthesis approach for designing individual controllers is also developed. Dissipativity conditions (storage functions and supply rates) in the Quadratic Differential Forms are adopted to capture more detailed dynamics of process units and plantwide systems [25], [26] to derive a less conservative robust stability condition.

In Section II, the notations are given. Dissipative dynamical systems and kernel representation with parametric uncertainty are also outlined. In Section III, the main results are presented in two parts: the robustness analysis of plantwide systems based on QDF dissipativity, and robust distributed control design. In Section III-B, the conclusion is given.

II. NOTATIONS AND PRELIMINARIES

A. Notations

Script letters denote dynamical systems, such as \( \mathcal{P} \) for the process unit and \( \mathcal{C} \) for the controller. The blackboard bold letters denote matrices, such as \( A, B, \cdots, \mathbb{R} \). Represents the set of real numbers, and \( \mathbb{Z}^+ \) is reserved for the set of non-negative integers. \( Q \in \mathbb{R}^{m \times n} \) represents the dimension of the matrix \( Q = m \times n \), \( 0 \) and \( I \) represent zero matrix and the identity matrix with appropriate dimensions respectively. The operator \( \text{col}(y_1, y_2, \cdots) \) denotes the column vector consisting of \( y_1, y_2, \cdots \). \( \mathbb{R}[x] \) denotes the set of one-variable polynomial matrices. \( t \) is reserved for time instance, which
belongs to the set \([0, \infty)\). \(\nabla\) is the differential operator for quadratic differential forms, e.g.,
\[
\nabla X = (0 \ X) + (\frac{\partial}{\partial x}).
\]
\(B.\) Dissipative dynamical systems

A system is said to be dissipative with respect to the supply rate \(s(y(t), u(t)) \in \mathbb{R}^{m+n}\), if there exists a non-negative storage function \(V(t)\) and the following inequality is satisfied,
\[
\int_{t=0}^{\infty} s(y(t), u(t)) \geq V(t) \geq 0,
\]
which is called as dissipation inequality. Storage function is a Lyapunov-like function such as \(V(t) = x^T(t)Px(t)\) with the non-negative definite matrix \(P\). In the dissipativity formalism, the system dynamics can be quantitatively formulated as the supply rate in terms of system inputs and system outputs. This is illustrated by the conventional supply rate in a quadratic form
\[
s(y(t), u(t)) = (\begin{pmatrix} y(t) \\ u(t) \end{pmatrix})^T \begin{pmatrix} Q & S \\ \bar{S} & R \end{pmatrix} \begin{pmatrix} y(t) \\ u(t) \end{pmatrix}.
\]

Considering the special case with the coefficients \(Q = -I, S = 0, R = \gamma^2 I\). The dissipativity condition implies an upper bound of the \(L_2\)-gain of the dynamical system:
\[
\|y\|^2 \leq \gamma^2 \|u\|^2.
\]

Another special case of dissipativity is passivity, with the supply rate
\[
s(y(t), u(t)) = y(t)^T u(t),
\]
i.e., \(Q = 0, S = \frac{1}{2} I, R = 0\).

In this paper, we adopt dissipativity in quadratic differential forms (QDF), i.e., QDF-dissipativity. Compared to the conventional quadratic supply rate, this formalism allows more dynamical features of a system to be captured by using trajectories of inputs and outputs. It effectively reduces the conservativeness in dissipativity-based approaches. Define the extended signal spaces of the output and input as \(\hat{y}(t)\) and \(\hat{u}(t)\) respectively:
\[
\hat{y}(t) = \text{col} \left( y(t), \frac{d}{dt} y(t), \cdots, \frac{d^n}{dt^n} y(t) \right)
\]
and
\[
\hat{u}(t) = \text{col} \left( u(t), \frac{d}{dt} u(t), \cdots, \frac{d^n}{dt^n} u(t) \right).
\]

Also denote the collection of both the output and input variables as the manifest variable
\[
w(t) = (y^T(t), u^T(t))^T
\]
and its extended variable
\[
\tilde{w}(t) = (\hat{y}^T(t), \hat{u}^T(t))^T.
\]

The QDF-supply rate can be defined as
\[
s(t) = Qw(t) = \tilde{w}^T(t) \Phi \tilde{w}(t),
\]
where the coefficients matrix
\[
\Phi = \begin{pmatrix} \hat{Q} & \hat{S} \\ \hat{S}^T & \hat{R} \end{pmatrix}.
\]
The supply rate \(Q\Phi(t)\) is said to be induced by the matrix \(\Phi\).

\(C.\) Kernel representation with parametric uncertainty

Suppose a dynamical system described by the kernel representation
\[
R \left( \frac{d}{dt} \right) w(t) = \left( R_0 I + R_1 \frac{d}{dt} + \cdots + R_n \left( \frac{d}{dt} \right)^n \right) w(t) = 0.
\]

This representation can also be re-written in a polynomial form
\[
R(\xi)w(t) = 0,
\]
where
\[
R(\xi) = R_0 I + R_1 \xi + \cdots + R_n \xi^n \in \mathbb{R}^{p \times n}[\xi].
\]

For example, the system
\[
G(s) = \frac{s}{s + 1}
\]
can be written in the above form such that
\[
R \left( \begin{pmatrix} 0 \\ \nu \end{pmatrix} \right) = (\xi + 1, -\xi) \left( \begin{pmatrix} 0 \\ \nu \end{pmatrix} \right) = 0.
\]

Considering the dynamical system
\[
R_\Delta(\xi)w(t) = 0
\]
in the presence of parametric uncertainty, the system matrices \(R_\Delta(\xi)\) can be parameterized by bounded parameter \(\theta\), such that
\[
R_\Delta(\xi) = \sum_{i=0}^{\nu} \theta_i R_i(\xi).
\]

Herein, the system model is assumed to lie in a convex hull
\[
\pi = \text{Co}\{R_1, \cdots, R_\nu, \cdots, R_\nu\},
\]
where \(R_i\) is the model at the \(i\)-th vertex (for \(i = 1, \cdots, \nu\)) and \(\nu\) is the number of the vertices. Furthermore, \(\theta_i\) is the unknown parameter of the model at \(i\)-th vertex with properties:
\[
0 \leq \theta_i \leq 1, \text{ and } \sum_{i=0}^{\nu} \theta_i = 1.
\]

Alternatively, the system can be re-written as:
\[
\sum_{i=1}^{\nu} \theta_i R_i(\xi) \left( \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} \right) = 0.
\]

In particular, if a system described by the kernel representation
\[
\sum_{i=1}^{\nu} \theta_i (R_{\theta_i}(\xi), R_{u_i}(\xi)) \left( \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} \right) = 0,
\]
then the dissipativity of such system can be determined by solving optimization problem subject to the condition given in the following result.
Proposition 1: Suppose a dynamical system described by kernel representation with the unknown parameters \( \theta_i \) which is governed by (22), where \( 0 \leq \theta_i \leq 1 \) and \( \sum_{i=0}^{\nu} \theta_i = 1 \). The system is dissipative with respect to the supply rate \( Q_{\Phi \Delta} \) which is induced by the polynomial matrices

\[
\Phi_{\Delta}(\zeta, \eta) = \sum_{i=0}^{\nu} \theta_i \left( \begin{array}{c} Q_i \nabla Y_i \\ S_i \nabla Y_i \\ R_i \nabla Y_i \end{array} \right),
\]

if there exist a storage function \( Q_{\Phi \Delta} \) induced by

\[
\Psi_{\Delta} = \sum_{i=0}^{\nu} \theta_i \left( \begin{array}{c} X_i \nabla Y_i \\ Y_i \nabla Y_i \\ Z_i \nabla Y_i \end{array} \right) \geq 0,
\]

and matrices \( E_i \) for \( i = 1, \cdots, \nu \), such that the following inequalities are satisfied:

\[
\begin{align}
(Q_i - \nabla X_i, S_i - \nabla Y_i, R_i - \nabla Z_i) + E_i \tilde{R}_i + \tilde{R}_i^T E_i^T & \geq 0,
\end{align}
\]

where

\[
\tilde{R}_i = \left( \tilde{R}_{y_i}, \tilde{R}_{u_i} \right)
\]

\[
\tilde{R}_{y_i} = \left( \begin{array}{ccc}
\bar{A}_0 & \cdots & \bar{A}_n \\
0 & \ddots & \cdots \\
0 & \cdots & \bar{A}_n \end{array} \right) 
\]

\[
\tilde{R}_{u_i} = \left( \begin{array}{ccc}
\bar{B}_0 & \cdots & \bar{B}_m \\
0 & \ddots & \cdots \\
0 & \cdots & \bar{B}_m \end{array} \right),
\]

and \( \bar{A}_l \) and \( \bar{B}_l \) for \( l = 1, \cdots, n \) are the coefficients in \( R_{y_i} \) and \( R_{u_i} \), respectively.

Proof: According to the definition as given in Section II-B, it implies the following dissipation inequality,

\[
\hat{w}(k)^T (\Phi_{\Delta} - \nabla \Psi_{\Delta}) \hat{w}(k) \geq 0.
\]

Together with system model \( \hat{R}\hat{w} = 0 \), Finsler’s Lemma can then be applied to obtain the dissipativity conditions in the LMI form, Eq. (25a) and (25b).

III. MAIN RESULTS

A. Robustness analysis for plantwide systems in the presence of model uncertainties

Tippett and Bao [6] introduced a framework to study the plantwide system using a network perspective. This framework is extended to develop the analysis of robust plantwide stability for a chemical process plant.

As shown in Figure 1, a plantwide system consists of process units \( \mathcal{P}_{\Delta} \) and distributed controllers \( C_i \), \( i = 1 \cdots n_p \). Suppose process unit \( \mathcal{P}_i \) is controlled by a (local) controller \( C_i \), which is capable of communicating with other controllers via the controller network.

As illustrated in Figure 2, the input of each process unit \( \mathcal{P}_{\Delta} \), can be classified into three groups: the process input from the interconnected process units \( u_{p_i} \), the input determined by the local controller \( u_{c_i} \) and the disturbance \( d_i \). The topologies of the process network and controller network are denoted as \( H_p \) and \( H_c \), respectively, which are constant matrices only containing 1 and 0. Some elements of the process output \( y_i \) (selected by filter \( F_p \)) are fed into the \( j \)-th unit via the process network \( H_{p_j} \), i.e., \( u_{p_j} = H_{p_j} y_i \), where \( H_p \) and \( F_p \) are constant matrices only containing elements 1 and 0. Stack the input and output signals for all process units, \( u_p = \text{col}(u_{p_1}, u_{p_2}, \cdots, u_{p_{n_p}}) \) and \( y = \text{col}(y_1, y_2, \cdots, y_{n_p}) \). Then we have \( u_p = H_p F_p y \).

For \( C_i \), the (input and output) ports are divided into the local and remote ports as depicted in Figure 3. The local input of \( C_i \) is the selected local process output \( y_i \) by filter \( F_1 \), i.e., \( u_{l_i} = F_1 y_i \). The remote input \( u_{r_i} \) from other interconnected controllers via controller network \( H_c \). At the output ports, \( C_i \) sends the control action \( y_i \) via the local port to \( P_i \), and deliver the trajectories of the predicted system response \( u_{r_i} \) to other controllers via the remote port. In a compact form, we have \( u_l = F_1 y \).

The model of \( P_i \) can be described by the following kernel representation:

\[
P_i : R_{\Delta_i} \frac{d}{dt} \begin{pmatrix} y_i \\ u_{c_i} \\ u_{p_i} \\ d_i \end{pmatrix} = 0,
\]

To improve readability, the time variable \( t \) is dropped in the notations for all signals throughout this paper. To take the
model uncertainty into account, the system matrices in $R_{\Delta_i}$ are assumed to be a member in the convex hull as defined in II-C, where the system matrices can be described by

$$R_{\Delta_i} = \theta_1 R_{t_1} + \theta_2 R_{t_2} + \cdots + \theta_{\nu} R_{t_\nu},$$  \hspace{1cm} (28)

where $R_{t_i}$ for $i = 1, \ldots, \nu$ is the system matrix at $i$-th vertex and $\theta_i$ belongs to a convex set and satisfies that $1 \geq \theta_i \geq 0$ and $\sum_{i=1}^{\nu} \theta_i = 1$. For the interconnection among process units, based on the partitioning of process model (27), the supply rate of $\mathcal{P}_{\Delta}$ is induced by the following matrix:

$$\Phi_{\Delta} = \sum_{i=1}^{\nu} \theta_i \begin{pmatrix} \hat{Q}_{i} & \hat{S}_{i} & \hat{R}_{i} \\ \hat{Q}_{i}^T & \hat{S}_{i}^T & \hat{R}_{i}^T \\ \hat{S}_{i}^T & \hat{R}_{i}^T & \hat{R}_{i} \end{pmatrix}.$$  \hspace{1cm} (29)

In a similar fashion, the storage function can be defined in the above parameter-dependent form.

Denote $\mathcal{C} = \text{diag}(C_1, \ldots, C_1, \ldots, C_{\nu})$. The supply rate of $\mathcal{C}$ is induced by the partitioned matrix

$$\Phi_{C} = \begin{pmatrix} Q_{C} & Q_{C} & \hat{S}_{C} & \hat{S}_{C} \\ Q_{C}^T & \hat{S}_{C}^T & \hat{S}_{C} & \hat{S}_{C}^T \\ \hat{S}_{C}^T & \hat{S}_{C} & \hat{R}_{C} & \hat{R}_{C} \\ \hat{S}_{C} & \hat{R}_{C} & \hat{R}_{C} & \hat{R}_{C} \end{pmatrix}.$$  \hspace{1cm} (30)

Based on the configuration of the plantwide system and the above defined dissipativity of the subsystems therein, the plantwide dissipativity is presented in the following result.

**Proposition 2:** Consider a plantwide system as shown in Figure 1. It consists of a collection of the process units $\mathcal{P}_{\Delta}$ and a collection of the distributed controllers $\mathcal{C}$. Denote $Q_{\Phi_{\Delta}}$ and $Q_{\Phi_{C}}$ as the supply rates of $\mathcal{P}_{\Delta}$ and $\mathcal{C}$ respectively, which are induced by the matrix pencil $\Phi_{\Delta}$ and the matrix $\Phi_{C}$. $Q_{\Phi_{\Delta}}$ and $Q_{\Phi_{C}}$ denote the storage function of $\mathcal{P}_{\Delta}$ and $\mathcal{C}$, induced by $\Psi_{\Delta}$ and $\Psi_{C}$ respectively. The plantwide system is dissipative with respect to the supply rate $Q_{\mu}$ if there exist non-negative storage functions $Q_{\Psi_{\Delta}}$ and $Q_{\Psi_{C}}$ that satisfy the following dissipation inequality, for $i = 1, \ldots, \nu$,

$$\int_{0}^{\infty} Q_{\Phi_{\Delta}} dt + \int_{0}^{\infty} Q_{\Phi_{C}} dt \geq Q_{\Psi_{\Delta}} + Q_{\Psi_{C}} \geq 0.$$  \hspace{1cm} (31)

with $Q_{\mu}$ induced by the matrix

$$\mu = \sum_{i=1}^{\nu} \theta_i \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix},$$  \hspace{1cm} (32)

where

$$X_{11} = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}.$$  \hspace{1cm} (33a)

Together with considering the following relations illustrated in the plantwide configuration as shown in Figure 1,

$$u_p = H_p f_p y, \hspace{0.5cm} u_c = y_1, \hspace{0.5cm} y_r = H_r y_r, \hspace{0.5cm} u_i = F_i y.$$  \hspace{1cm} (35)

the summation of $Q_{\Phi_{\Delta}}$ and $Q_{\Phi_{C}}$ reduces to the supply rate induced by the matrix $\mu$ as (32). From the viewpoint of the plantwide system, the entire plant can be viewed as shown in Figure 4, where $y_{pw} = \text{col}(y, y_1, y_r)$ is the plantwide output and $d$ is the disturbance. To combine with Theorem 1 in [6], the condition for robust plantwide stability can be derived from the plantwide dissipativity in Proposition 2. This result is presented as below.

**Theorem 1:** Consider a plantwide system as described in Proposition 2. This plantwide system is input-output stable, if the entire plant is dissipative with respect to the supply rate $Q_{\mu}$ as given in Proposition 2 and the following linear matrix inequality is satisfied:

$$X_{11} < 0$$  \hspace{1cm} (36)

for $i = 1, \ldots, \nu$.

**Proof:** For vanishing disturbances, the supply rate of the entire plant reduces to the quadratic function induced by
\[ \sum_{i=1}^{\nu} \theta_i X_{11i} \]. Applying the stability condition in Theorem 1 in [6], the condition for plantwide (input-output) stability becomes
\[ \sum_{i=1}^{\nu} \theta_i X_{11i} < 0. \] (37)
As \( \theta > 0 \), the sufficient condition for (37) is
\[ X_{11i} < 0 \quad \forall i. \] (38)

B. Robust distributed control design

In this section, we present the robust distributed control design to realize a \( \Phi \)-dissipative controller for satisfying the plantwide dissipativity conditions as developed in III-A. The proposed design procedure inherits the framework as presented in our previous work [27], where the controller to be realized is built upon a “seed” system by using polynomial J-factorization (see the detailed algorithm in [28]). Technically, it starts from the “seed” system with image representation
\[
\begin{pmatrix}
y_e' \\
u'_c
\end{pmatrix} = M \left( \frac{d}{dt} \right) \ell,
\] (39)
which is dissipative with respect to the supply rate
\[ J = \left( \begin{array}{cc} -I & 0 \\ 0 & I \end{array} \right) \] (40)
e.g., passive systems and the feedback law \( u = -y \). Then, polynomial J-factorization is employed to find out the controller with image representation
\[
\begin{pmatrix}
y_e' \\
u'_c
\end{pmatrix} = L \left( \frac{d}{dt} \right) M \left( \frac{d}{dt} \right) \ell,
\] (41)
such that it is \( \Phi \)-dissipative.

Combined with the dissipativity conditions developed in Section III-A, the distributed control design is divided into two steps: 1. Determine the required dissipativity for the controller to ensure the robust plantwide stability. 2. Design a controller satisfying the required dissipativity determined from the previous step.

In first step, we cast the problem of determination of plantwide dissipativity into the following linear matrix inequality (LMI) problem, which can be effectively solved by any semi-definite programming tools[29].

**Problem 1: (Offline)** Consider a plantwide system as shown in Figure 1 that consists of the process units \( P_\Delta \) and the distributed controllers \( C \). \( Q_{\Delta \Phi} \) and \( Q_{\Phi} \) denote the supply rate of \( P_\Delta \) and \( C \), respectively. Also, given the storage function of the \( i \)-th process and the distributed controllers \( \Psi_i \).

\[
\begin{pmatrix}
Q_{x_i} - \nabla X_{x_i} \\
S_{x_i} - \nabla Y_{x_i} \\
S_{\ell_i} - \nabla Y_{\ell_i} \\
R_{x_i} - \nabla Z_{x_i} + \tilde{R}_{x_i} \Psi_{x_i} + \tilde{R}_{\ell_i} \Psi_{\ell_i} \\
& \geq 0
\end{pmatrix}
\] (42a)

(Dissipativity of \( i \)-th Process)

\[
\begin{pmatrix}
Q_{c_i} - \nabla X_{c_i} \\
S_{c_i} - \nabla Y_{c_i} \\
S_{\ell_i} - \nabla Y_{\ell_i} \\
R_{c_i} - \nabla Z_{c_i}
\end{pmatrix} \geq 0
\] (42b)

(Dissipativity of \( i \)-th Controller)

\( \tilde{X}_{11} < 0 \) (42c)

**Problem 2: (Stability of Plantwide Outputs)**

\[
\begin{pmatrix}
D_{q_i} \\
D_{y_i} \\
D_{z_i}
\end{pmatrix} \geq 0 \quad \forall i
\] (42d)

and the coefficients of \( Q_{x_i} - D_{q_i} \) and \( R_{c_i} - D_{y_i} \) satisfy the conditions given in Lemma 3 in [27].

**Feasibility**

By solving the above LMI problem, the required dissipativity condition (the storage function and supply rate) for each controller can be obtained. The induced matrix of the controller supply rate is re-written as

\[
\Phi_c = \begin{pmatrix}
\tilde{\theta}_{00} \\
\tilde{\theta}_{01} \\
\vdots \\
\tilde{\theta}_{0K}
\end{pmatrix} \begin{pmatrix}
\tilde{\theta}^{(0K)} \\
\tilde{\theta}^{(1K)} \\
\vdots \\
\tilde{\theta}^{(KK)}
\end{pmatrix},
\] (43)

where

\[
\tilde{\theta}_{ij} = \begin{pmatrix}
Q_{x_i} & S_{x_i} \\
S_{\ell_i} & R_{x_i}
\end{pmatrix} \begin{pmatrix}
S_{y_i} & R_{y_i} \\
R_{z_i}
\end{pmatrix}
\] for \( i, j = 1, \cdots, K \) (44a)

\[
\tilde{\theta}^{(0K)} = \begin{pmatrix}
\tilde{\theta}_{01} & \cdots & \tilde{\theta}_{0K}
\end{pmatrix}
\] (44b)

\[
\tilde{\theta}^{(KK)} = \begin{pmatrix}
\tilde{\theta}_{11} & \cdots & \tilde{\theta}_{1K} \\
\vdots & \ddots & \vdots \\
\tilde{\theta}_{K1} & \cdots & \tilde{\theta}_{KK}
\end{pmatrix}
\] (44c)

Then, for the \( i \)-th controller, it can be found out by following the below algorithm.

**Algorithm 1:** [27]

1) Set \( K \) equal to the order of the supply rate
2) Solve the ARE equation

\[
A^T P + PA + \tilde{\theta}^{(KK)} - \begin{pmatrix}
\tilde{\theta}^{(0K)} + B^T P
\end{pmatrix} \tilde{\theta}_{(00)}^{-1} \begin{pmatrix}
\tilde{\theta}^{(0K)} + B^T P
\end{pmatrix} = 0,
\] (45)

where

\[
A = \begin{pmatrix}
0_{q \times q} & 0_{q \times q} & \cdots & 0_{q \times q} \\
I_{q \times q} & 0_{q \times q} & \cdots & 0_{q \times q} \\
\vdots & \vdots & \ddots & \vdots \\
0_{q \times q} & 0_{q \times q} & \cdots & I_{q \times q}
\end{pmatrix},
B = \begin{pmatrix}
0_{q \times q} \\
0_{q \times q} \\
\vdots \\
0_{q \times q}
\end{pmatrix}
\] (46)

3) If there exists a solution in Step 2, then the controller can be obtained from

\[
\begin{pmatrix}
y_{ci} \\
u_{ci}
\end{pmatrix} = L_i \left( \frac{d}{dt} \right) M_i \left( \frac{d}{dt} \right) \ell,
\] (47)

where \( L_i \left( \frac{d}{dt} \right) \) is the adjugate of the \( F_i \left( \frac{d}{dt} \right) \), and \( M_i \left( \frac{d}{dt} \right) \ell \) is a J-dissipative system. Otherwise, set the new value of \( K \) equal to \( K + 1 \), and repeat Step 2.
IV. CONCLUSIONS

In this work, a robust distributed control framework for plantwide systems is developed. Parameter-dependent QDF-dissipativity is employed to capture dynamical features of process units in presence of model uncertainties, which are parametrized by polytopic models. The effects of uncertainties on plantwide stability due to interactions between process units are determined based on the plantwide dissipativity. The dissipativity condition of individual controllers to ensure robust plantwide stability is derived, followed by the control design approach using J-factorization.

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REFERENCES