Design of a Data-Oriented Cascade Control System*

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Abstract—In process industries, PID control has been applied to controlled objects such as chemical plants. A cascade control system is applied in order to improve control performance by using several feedback loops. However, it is complicated to design a cascade control system because this control system includes plural controllers. In this paper, a design scheme of data-oriented cascade control system without system identification is proposed. According to the proposed scheme, PID gains included in plural controllers can be determined by using only closed-loop data. The effectiveness of the proposed scheme is verified by using a simulation example and an experiment of the water level control system.

I. INTRODUCTION

In process industries, PID controller[1] has been widely used because it is simple and physical the meaning of control parameters is clear. A basic control system has a feedback loop and only system output is fed back to controller. However, the control performance can be improved when other physical quantities are fed back to controllers in several feedback loops[2]. The cascade control system has several feedback loops. A basic way to design a cascade control system is as follows:

1) Design of an inner-loop controller based on a controlled object.
2) Design of an outer-loop controller based on inner-loop controller and a controlled object.

It is complicated to design a cascade control system than single loop control. Furthermore, the control performance depends on quality of system identification.

Incidentally, design of a data-oriented control system without system identification has been proposed[3], [4], [5], [6] for single loop control system in recent years. These schemes are to directly calculate control parameters using closed-loop data. In particular, the fictitious reference iterative tuning (FRIT)[5] and data-driven control scheme[6] are developed for linear systems and nonlinear systems, respectively. Moreover, in these schemes[5], [6], the effectiveness of these schemes has been verified by using an experimental result.

The conventional scheme about data-driven cascade control[7] based on FRIT has been proposed. In this scheme, two controllers are designed based on closed-loop data for two linear systems. However, it is important to consider the data-oriented cascade control for nonlinear systems because there are many nonlinear systems in industries.

In this paper, the data-oriented cascade control for nonlinear systems is proposed. Specifically, the control system which has two feedback loops is introduced. Furthermore, outer loop system is nonlinear such as pH control system[8] and chemical reactor control system[9], and inner loop system is linear. In the proposed scheme, FRIT and data-driven control scheme are applied to inner controller and outer controller, respectively.

This paper is organized as follows. Section II provides an overview of the cascade control and an algorithm of the proposed scheme. The FRIT and data-driven control scheme are introduced in section III. The effectiveness of the proposed scheme is demonstrated by applying to a simulation example in section IV. The paper ends with concluding remarks in section V.

II. OVERVIEW OF CASCADE CONTROL

Fig.1 shows the block diagram of a cascade control system. In Fig.1, outer loop and inner loop are primary control loop and secondary control loop, respectively. C1 and G1 are a controller and a controlled object in primary control loop, respectively. C2 and G2 are a controller and a controlled object in secondary control loop, respectively. r(t) is reference signal for C1, w(t) and v(t) are control input of C1 and C2, respectively. u(t) and y(t) are system output of G1 and G2, respectively. Here, control input w(t) is reference signal for C2.

In this paper, G1 and G2 are given by a nonlinear system and a linear system, respectively. Generally, C1 is designed with considering C2 and G1. Therefore, the design procedure of cascade control system is as follows:

1) Obtain a closed-loop data using initial control parameters of C1 and C2.
2) Design C2 based on an above closed-loop data v(t) and u(t).
3) Obtain a closed-loop data using C2 designed in 2).
4) Design C1 based on above closed-loop data w(t) and y(t).

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5) Apply designed $C_1$ and $C_2$ in 2) and 4) to control system.

III. DESIGN OF A DATA-ORIENTED CASCADE CONTROL SYSTEM

In Fig.1, FRIT[5] and data-driven control scheme[6] are respectively applied to linear system and nonlinear system, and a data-oriented cascade control is proposed. In data-driven control scheme[6], control parameters can be determined using FRIT. Therefore, FRIT is a common scheme in the case where $C_1$ and $C_2$ are designed in proposed scheme.

A. System description

The discrete-time nonlinear system is given as follows:

$$y(t) = f(\phi(t-1)), \quad (1)$$

where $y(t)$ is system output, $f(\cdot)$ is nonlinear function, $\phi(t)$ is following information vector.

$$\phi(t-1) := [y(t-1), \ldots, y(t-n_y), \ w(t-1), \ldots, w(t-n_w)], \quad (2)$$

where $w(t)$ is control input. $n_y$ and $n_w$ denote the order of output and input, respectively. In the data-driven control scheme, a closed-loop data is stored as information vector in Eq. (2) to database. Here, current information vector $\phi(t)$ is called ‘query’.

B. PID control law

In this paper, the following control law of $C_1$ and $C_2$ in Fig.1 is considered.

$$\Delta w(t) = K_P(t)e(t) - K_P(t)\Delta y(t) - K_D(t)\Delta^2 y(t) \quad (3)$$

$$e(t) := r(t) - y(t), \quad (4)$$

where $e(t)$ is control error and $r(t)$ is reference signal. $K_P(t)$, $K_I(t)$ and $K_D(t)$ are proportional gain, integral gain and derivative gain, respectively. $\Delta$ denotes differencing operator ($\Delta := 1 - z^{-1}$).

Note that the control law in Eq. (3) is for $C_1$. Therefore, in the control law of $C_2$, $w(t)$ and $y(t)$ in Eq. (3) are replaced to $v(t)$ and $u(t)$, respectively.

C. Fictitious reference iterative tuning: FRIT

In this subsection, the method to design $C_2$ in Fig.1 using FRIT is described. Fig.2 shows the block diagram of the FRIT for $C_2$. The FRIT is a scheme that control parameters are calculated using a closed-loop data $(v_0(t), u_0(t))$ and the fictitious reference signal $\bar{w}(t)$ which is obtained from a closed-loop data.

The following equation is obtained from $C(z^{-1})/\Delta$ in Fig.2.

$$v_0(t) = \frac{C(z^{-1})}{\Delta} \{\bar{w}(t) - u_0(t)\}. \quad (5)$$

The fictitious reference signal $\bar{w}(t)$ is calculated as follows:

$$\bar{w}(t) = C^{-1}(z^{-1})\Delta v_0(t) + u_0(t). \quad (6)$$

Furthermore, the user gives the reference model parameters $d_m$ and $P(z^{-1})$ in Fig.2. $d_m$ is the minimum estimation of the time-delay and $P(z^{-1})$ is a user-specified polynomial of the reference model. In this paper, the minimum estimation of the time-delay $d_m$ is known. On the other hand, $d_m = 0$ when the time-delay is unknown. $P(z^{-1})$ is expressed as follows[10]:

$$P(z^{-1}) := 1 + p_1 z^{-1} + p_2 z^{-2} \quad (7)$$

$$p_1 = -2 \exp(-\frac{\rho \delta}{\mu}) \cos \left( \frac{\sqrt{4\mu^2 - \rho^2}}{2\mu} \right)$$

$$p_2 = \exp(-\frac{\rho}{\mu}) \quad (8)$$

where $T_s$ is sampling time. $\sigma$ and $\delta$ are a parameter related to the rise-time and the damping oscillation, respectively.

$\bar{u}_m(t)$ in Fig.2 is the fictitious reference output corresponding to the fictitious reference signal $\bar{w}(t)$. In the FRIT, control parameters are calculated by minimizing the error between $\bar{u}_m(t)$ and $u_0(t)$ using optimization technique such as ‘fminsearch.m’ of Matlab/Simulink Ver. 8.3.0.532(R2014a), Optimization Toolbox.

Note that Eq. (5) and (6) are for $C_1$. Therefore, $v_0(t), u_0(t)$ and $\bar{w}(t)$ in Eq. (5) and (6) are replaced to $w_0(t), y_0(t)$ and $\bar{r}(t)$, respectively for $C_2$.

D. Design of a data-oriented PID control system

In this subsection, the method to design $C_1$ in Fig.1 using the data-driven control is described. There are three steps in designing the data-driven PID controller.

1) Design procedure:

[STEP1] Create the initial database

The historical data is required to use the data-driven control scheme. The historical data denotes the following equation as an information vector $\phi(t)$:

$$\phi(j) := [\phi(j), K(j)], \quad (j = 1, 2, \ldots, N) \quad (9)$$

where $N$ is the number of initial data. $\phi(t)$ and $K(t)$ are defined as follows:

$$\phi(t) := [r(t+1), r(t), y(t), \ldots, y(t-n_y+1), w(t-1), \ldots, w(t-n_w+1)] \quad (10)$$

$$K(t) := [K_P(t), K_I(t), K_D(t)]. \quad (11)$$

Here, $K(1) = K(2) = \ldots = K(N)$ in initial database.
[STEP2] Calculate distance and select neighbors’ data
The distance between query \( \bar{\phi}(t) \) and information vectors \( \bar{\phi}(j) \) in database is calculated by \( L_1 \) norm with some weights as follows:

\[
d(t(\bar{\phi}(t), \bar{\phi}(j)) = \sum_{l=1}^{m} \frac{\phi_i(t) - \phi_i(j)}{\max(\phi_i(m)) - \min(\phi_i(m))},
\]

(12)

where \( \phi_i(j) \) denotes \( l \)th element of \( j \)th information vector \( \bar{\phi}(j) \). Furthermore, \( \max(\phi_i(m)) \) and \( \min(\phi_i(m)) \) are maximum and minimum value of \( l \)th element in database, respectively.

The neighbors’ data are defined as information vector which are based on smallest distance \( d \). In this paper, the number of neighbors’ data \( k \) are selected for calculating control parameters following [STEP3].

[STEP3] Calculate PID gains
Based on neighbors’ data in [STEP2], control parameters are calculated by the following equation:

\[
K(t) = \sum_{i=1}^{k} w_i K_i(t), \quad \sum_{i=1}^{k} w_i = 1,
\]

(13)

where \( w_i \) is weighting parameter corresponding to \( K_i(t) \) and it is given by the following equation:

\[
w_i = \frac{1/(1+d_i)}{\sum_{i=1}^{k} 1/(1+d_i)}
\]

(14)

A learning method is required to calculate effective control parameters of initial database. Therefore, an off-line learning method using FRIT after obtaining initial database is described in next section.

2) Off-line learning scheme of data-driven PID controller based on FRIT: In this section, the off-line learning scheme based on FRIT is described. First, the distance between the query of closed-loop data \( \bar{\phi}(t) \) and information vector in database is calculated using Eq.(12). Second, neighbors’ data are selected based on smallest \( d(\bar{\phi}(t), \bar{\phi}(j)) \). Finally, PID gains \( K^{old}(t) \) are calculated using Eq.(13) based on above neighbors’ data. In the following off-line learning equation, \( K^{new}(t) \) is the learned PID gains from \( K^{old}(t) \) using steepest descent method.

\[
K^{new}(t) = K^{old}(t) - \eta \frac{\partial J(t+1)}{\partial K(t)}
\]

(15)

where \( \eta \) is learning rate, and \( J(t+1) \) is a criterion based on FRIT as follows:

\[
J(t) := \frac{1}{2} \varepsilon(t)^2
\]

(17)

\[
\varepsilon(t) := y_0(t) - \bar{y}_m(t)
\]

(18)

where \( y_0(t) \) is output of closed-loop data, and the fictitious reference output \( \bar{y}_m(t) \) is given by following equation:

\[
\bar{y}_m(t) = \frac{z^{-d_m(t)} P(1)}{P(z^{-1})} \bar{r}(t)
\]

(19)

Here, \( \partial J(t+1)/\partial K(t) \) of Eq. (15) is expanded as follows:

\[
\begin{bmatrix}
\frac{\partial J(t+1)}{\partial K_P(t)} = & \frac{\partial J(t+1)}{\partial \bar{y}_m(t+1)} \frac{\partial \bar{y}_m(t+1)}{\partial \bar{r}(t)} \frac{\partial \bar{r}(t)}{\partial K_P(t)} = & -\frac{K^{old}_P(t)}{\varepsilon(t+1) P(1) \Delta \bar{y}_0(t)}
\end{bmatrix}
\]

(20)

where \( \Gamma(t) \) is given by following equation.

\[
\Gamma(t) = -\Delta \varepsilon(t) - \{K^{old}_P(t) + K^{old}_D(t)\} \bar{y}_0(t) + \left\{K^{old}_P(t) + 2K^{old}_D(t)\right\} y_0(t-1) - K^{old}_D(t) \bar{y}_D(t) - 2.
\]

(21)

Eq. (20) shows the off-line learning scheme because it includes the fictitious reference signal \( \bar{r}(t) \) of FRIT. Therefore, \( K^{new}(t) \) can be updated off-line by repeated calculation using the initial database so that minimizing the criterion of Eq. (17). The updated PID gains \( K^{new}(t) \) is applied to control system by [STEP1]-[STEP3] in subsection III-D after above off-line learning.

IV. NUMERICAL EXAMPLE

A. Polystyrene polymerization reactor
In this section, the effectiveness of the proposed scheme is numerically evaluated using the polystyrene reactor model [9]. Fig.3 shows the schematic figure of the polystyrene reactor. The polystyrene reactor is to generate polystyrene by polymerization reaction of styrene. At that time, it is important to control temperature of a reactor and a jacket for polymerization reaction of styrene.

The controlled object \( G_1 \) and the system output \( y(t) \) of the inner loop in Fig.1 are a reactor and the reactor temperature in Fig.3, respectively. Moreover, \( G_2 \) and \( u(t) \) of outer loop in Fig.1 are a jacket and jacket temperature in Fig.3, respectively. Furthermore, \( G_1 \) and \( G_2 \) are given as a linear system and a nonlinear system in the next subsection.
C. The scheme to obtain initial PID gains

In this paper, the control tuning scheme without system identification based on the generalized minimum variance control law [13] is utilized to get a closed-loop data of design procedure 2) in section II. In particular, $C_1$ and $C_2$ are designed based on reference [13] as fixed PID controllers.

In this subsection, reference [13] scheme is described. First, controlled object is expressed as follows:

$$A(z^{-1})y(t) = z^{-d_{a}+1} B(z^{-1}) w(t) + \xi(t)/\Delta$$ (25)

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \ldots + a_m z^{-m}$$ (26)

where $\xi(t)$ is a white Gaussian noise whose mean and variance are 0 and $\sigma_{\xi}^2$, respectively. $m$ denotes the number of order of $B(z^{-1})$ and $m = 3$ in this paper. Furthermore, $\sigma$, $\delta$ included $P(z^{-1})$ and $d_m$ are set as same parameters in TABLE I.

PID gains of $C_1$ and $C_2$ are calculated so that the following criterion are minimized:

$$J = E[\phi^2(t+1)]$$ (27)

$$\phi(t+1) := P(z^{-1})y(t+1) + \lambda \Delta w(t) - P(1)r(t),$$ (28)

where $\lambda$ is a weighting factor of control input $w(t)$. In this paper, $\lambda$ of $C_1$ and $C_2$ are set as $\lambda = 0.5$. $\lambda = 3$ respectively.

Note that Eq. (25) and Eq. (27) are for $C_1$. Therefore, $w(t)$ and $y(t)$ in Eq. (25) and Eq. (27) are replaced to $v(t)$ and $u(t)$ when $C_2$ is considered.

First, the inner-loop controller $C_2$ is designed using the step response data and the following PID gains are calculated.

$$K_p = 0.57, K_i = 0.05, K_d = 0.07.$$ (29)

Next, the outer-loop controller $C_1$ is designed using the step response data included $C_2$ in which PID gains of $C_2$ are Eq. (29). At that time, PID gains of $C_1$ are calculated as follows:

$$K_p = 0.15, K_i = 0.01, K_d = 0.80.$$ (30)

In next section, the effectiveness of the proposed scheme is shown using the initial PID gains of Eq. (29) and (30).

D. Control results

In order to show the effectiveness of the proposed scheme, three types of the control results are compared. The three types of the control results are (i) fixed PID gains of $C_1$ and $C_2$ are previous section, (ii) only $C_2$ is tuned by FRIT and (iii) $C_1$ is tuned by the data-driven control scheme.

First, Fig.5 and Fig.6 show the control results in the case of (i). Specifically, Fig.5 indicates the control result of input and output signal of jacket temperature $y(t)$ has oscillation in 1000-1500[step]. Note that the control error between jacket temperature $u(t)$ and reference signal of jacket temperature $w(t)$ in Fig.5 is small when PID gains of $C_2$ are tuned properly. The above error (integral of squared error: ISE [2]) is calculated as $1.02 \times 10^4$ in order to verify the effectiveness of FRIT in (ii).
Next, Fig.7 and Fig.8 show the control results of (ii) in which PID gains of $C_1$ are Eq. (29) and PID gains of $C_2$ are the following equation calculated by FRIT.

$$K_P = 4.43, K_I = 0.34, K_D = 8.36$$

(31)

The reactor temperature $y(t)$ of Fig.7 has less oscillation in 1000-1500[step] comparing to the control results of Fig.5. Here, $u(t)$ and $w(t)$ in Fig.8 is calculated as $3.15 \times 10^3$ which is smaller than previous ISE: $1.02 \times 10^4$ of Fig.6. Therefore, the control parameters are tuned properly using FRIT. However, the speed of response of the reactor temperature $y(t)$ is not good because PID gains of $C_1$ in Fig.7 is not tuned.

Next, Fig.9 and Fig.10 show the control results of (iii) in which $C_2$ is applied to the data-driven control scheme. Here, PID gains of $C_2$ are Eq. (31). The speed of response of $y(t)$ in Fig.9 is better than Fig.7 because PID gains of $C_1$ in Fig.10 are tuned when reference signal $r(t)$ is changed.

The setting time and ISE in the case of (i), (ii) and (iii) are shown in TABLE II to show the effectiveness of the proposed scheme of (iii). In this paper, the setting time denotes the time that $y(t)$ reaches to $\pm 1\%$ of $r(t)$. In particular, the setting time and ISE for each reference signal about $y$-1st and $y$-2nd in Fig.9 are evaluated. From TABLE II, the effectiveness of the proposed scheme is shown because the setting time and ISE about $y$-2nd are better than $y$-1st.

Finally, Fig.10 shows the control result by using fixed PID gains for $C_2$ in Eq. (29) and data-driven controller for $C_1$. Learning ratio in Eq. (16) is obtained as $\eta = [1.4 \times 10^{-4}], 3.5 \times 10^{-7}, 5.5 \times 10^{-4}$ by using ‘fminsearch.m’ of Matlab/Simulink Ver. 8.3.0.532(R2014a), Optimization Toolbox. The output $y(t)$ in Fig.10 is oscillatory because PID gains of $C_2$ is not tuned. Thus, it is difficult to improve the control result by tuning only $C_1$. The above mentioned control result shows the effectiveness of tuning $C_2$ in the proposed scheme.

V. CONCLUSION

In this paper, the data-oriented cascade control system using closed-loop data has been proposed. The features of the proposed scheme are as follows:

1) Control parameters are calculated without system identification.
learning rates are respectively $[\eta_p, \eta_i, \eta_d] = [2.0 \times 10^{-3}, 7.0 \times 10^{-4}, 2.0 \times 10^{-3}]$, PID gains of $C_2$ are respectively $K_p = 4.43, K_i = 0.34$ and $K_d = 8.36$.

Fig. 9. Control result of primary loop tuning primary controller $C_1$ where learning rates are respectively $[\eta_p, \eta_i, \eta_d] = [1.4 \times 10^{-4}, 3.5 \times 10^{-2}, 5.5 \times 10^{-4}]$, and PID gains of $C_2$ are respectively $K_p = 0.57, K_i = 0.05$ and $K_d = 0.07$.

In numerical example, the setting time and the integral of squared error: ISE are improved in the proposed scheme.

2) Nonlinear system can be controlled using the data-driven control.

3) Several controllers are designed using closed-loop data directly.

In numerical example, the setting time and the integral of squared error: ISE are improved in the proposed scheme. Future work is to verify the effectiveness of the proposed scheme using experiment. This work was supported by JSPS KAKENHI Grant Number 16K14285.

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