Design of Centralized PID Controllers for TITO Processes*

Byeong Eon Park, Su Whan Sung, In-Beum Lee

Abstract—A new method for designing the centralized proportional-integral-derivative (PID) controllers in two-input two-output (TITO) processes is proposed. The proposed method has two diagonal part PID controllers and two off-diagonal part PID controllers. The diagonal part PID controllers are to attenuate the interactions of the TITO processes and the off-diagonal part PID controllers are to track the setpoint. The diagonal part PID controllers are directly tuned by the conventional single-input single-output (SISO) PID tuning methods on the basis of the diagonal part of a process model matrix. And the tuning parameters of the off-diagonal part PID controllers are calculated by minimizing the effects of the off-diagonal components of the open-loop transfer function matrix in frequency domain. The proposed control method shows better decoupling and setpoint tracking performance than previous approaches.

I. INTRODUCTION

TITO processes are frequently encountered in chemical and petroleum industry. Unfortunately, it is not easy to achieve high control performances for the TITO processes if the loop interaction and time delay are significant. Hence, a plenty of control schemes have been introduced to control the TITO processes with high control performance by attenuating the interactions in a systemic way in several previous decades. One of them is a model predictive control (MPC) that calculates a present controller output by minimizing the predicted future control error based on the process model by solving a constrained optimization problem in each sampling time [1]. Even though the MPC theoretically provides an optimal control performance and manages a process constraint, PID controllers are still the most popular process control scheme for industrial field because of their simplicity, robustness and easy maintenance. One of the commonly used PID control schemes for the TITO processes is a multi-loop (decentralized) PID control shown in Figure 1. In order to pair process inputs and outputs, the relative gain array (RGA) analysis or the singular value analysis (SVA) [2] have been used. These methods can provide most effective process input and output pairing to achieve high control performance attenuating the process interactions for the multi-loop PID control scheme. And the tuning methods for the multi-loop PID control scheme have been proposed in several papers [3]–[5]. Nevertheless, if the loop interactions are significant, the multi-loop control PID control schemes are structurally not recommended to attenuate the loop interactions. So, decouplers have been developed to overcome the limitations of the multi-loop control scheme. It is shown in Figure 2. Typically, most decouplers are designed on the basis of the steady-state model or dynamic model of the processes [6]–[9]. The static decouplers are based on the steady-state model so that they cannot guarantee acceptable transient decoupling performance. While the dynamic decouplers can provide good transient decoupling performance theoretically, however, it is...
Another control scheme to control TITO processes is a centralized PID control shown in Figure 3. When the process interactions are highly interactive, the centralized PID control is superior to the multi-loop PID control scheme since the centralized PID control can attenuate the process interactions from the structural point of view. Liesleho proposed a $n \times n$ centralized control design method on the basis of SISO PID design method [11]. Wang et al. proposed an automatic tuning method to design the centralized PID controller using the sequential relay feedback method [12]. Recently, a tuning method for the centralized PI controller based on a synthesis method was introduced [13]. It uses the RGA analysis to approximate the inverse of the process transfer function matrix. The above-mentioned design methods for the centralized PID controller cannot guarantee acceptable decoupling performance in a time delay process. To consider the significant time delay effect of the TITO process more systematically, Morilla et al. used a control structure of the ideal PID control with a time delay for the centralized control [14]. The Morilla’s method shows good control performance for the TITO processes with significant time delay. However, the Morilla’s tuning algorithm is too complex to apply in industry, and it requires user-defined parameter; desired control trajectory.

In this research, a new design method to tune the centralized PID controller is proposed in a conceptually straightforward way. It goes through the following steps. First, the effective process input and output are paired using the RGA analysis. Second, when the RGA analysis indicates the process input 1 paired with the process output 1 and the process input 2 paired with the process output 2 are the best pairing, the diagonal-part PID controllers are tuned by the conventional SISO PID tuning methods [14-16]. Third, the off-diagonal part PID controllers are tuned to remove the interactions of the process. The off-diagonal part PID controllers have a time delay term to compensate the time delay in a structural point of view. The proposed method calculates the tuning parameters in an analytic way without solving an iterative optimization problem and requiring additional user-defined parameters such as gain margin or phase margin.
\[ G_c(s) = \begin{bmatrix} 0.825 + 0.0389 \frac{1}{s} + 0.310s & (-0.224 - 0.0148 \frac{1}{s} - 0.233s)e^{-2s} \\ 0.358 + 0.0132 \frac{1}{s} + 0.167s)e^{-4s} & -0.189 - 0.0100 \frac{1}{s} - 0.195s \end{bmatrix} \]  
(18)

\[ G_c(s) = \begin{bmatrix} 66.667 + 1.0582 \frac{1}{s} + 190.468s & (-2.525 - 0.668 \frac{1}{s} - 0.356s)e^{-6s} \\ 48.668 + 0.829 \frac{1}{s} + 135.187s & -32.500 - 0.833 \frac{1}{s} - 116.675s \end{bmatrix} \]  
(20)

\[ G_c(s) = \begin{bmatrix} -166.779 - 41.221 \frac{1}{s} - 262.677s & (30.133 + 1.059 \frac{1}{s} - 181.3609s)e^{-1.9s} \\ -58.505 - 14.825 \frac{1}{s} - 90.934s)e^{-12.43s} & 456.170 + 6.936 \frac{1}{s} + 5562.5s \end{bmatrix} \]  
(22)

\[ G_c(s) = \begin{bmatrix} -2.293 - 0.254 \frac{1}{s} - 0.812s & (2.135 + 0.183 \frac{1}{s} + 0.290) \\ -1.447 - 0.165 \frac{1}{s} - 0.513s)e^{-1.45s} & 3.614 + 0.311 \frac{1}{s} + 0.492s \end{bmatrix} \]  
(24)

II. PROPOSED DESIGN METHOD FOR CENTRALIZED PID CONTROLLER

Consider the following transfer function matrix of TITO process.

\[ \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \]  
(1)

where \( G_{ij}(s) \), \( i=1,2; j=1,2 \) represents the transfer function of each sub-process. \( Y_1(s) \) and \( Y_2(s) \) are the process outputs. \( U_1(s) \) and \( U_2(s) \) are the process inputs. The \( 2 \times 2 \) centralized PID controllers can be represented by (2), where \( G_{ij}(s) \), \( i=1,2; j=1,2 \) are the PID controllers. \( Y_{1i}(s) \) and \( Y_{2i}(s) \) are the controller setpoints.

\[ \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} Y_{11}(s) - Y_1(s) \\ Y_{21}(s) - Y_2(s) \end{bmatrix} \]  
(2)

The closed-loop response equations in (3) and (4) are obtained by combining (1) and (2).

Here, without loss of generality, it is assumed that the RGA analysis provides the best paring of \( Y_1 \) paired with \( U_1 \) and \( Y_2 \) paired with \( U_2 \). Then, the transfer function of (4) should be diagonal to remove the loop interactions by satisfying (5) and (6).

Letting \( G_{ij}(s) = A_{ij}(s)e^{-\theta_{ij}s} \), \( i=1,2; j=1,2 \), (5) and (6) can be rewritten as (7) and (8). Note that the controller transfer function of (7) and (8) have the time delay terms \( e^{-\theta_{21}-\theta_{11}s} \) and \( e^{-\theta_{22}-\theta_{12}s} \). It is not easy to approximate the frequency response of the time delay term for the ideal PID structure because they show totally different frequency response, while it is relatively easy to approximate the time delay free term for the ideal PID structure. So, the ideal PID structure plus time delay term for a positive time delay and the ideal PID structure for a negative time delay are chosen to remove process interactions as (9) - (12).

Time delay-free terms of (9), (10) and (11), (12) are linear combination of the tuning parameter and the Laplace variable \( s \). Therefore, we can apply linear least square method to obtain the tuning parameters of the off-diagonal part PID controllers by approximating the frequency response of \((-A_{12}(s)/A_{11}(s))G_{22}(s)\) and \((-A_{21}(s)/A_{22}(s))G_{11}(s)\). For convenience, let \( Q_{12}(s) = -(A_{21}(s)/A_{11}(s))G_{22}(s) \) and \( Q_{21}(s) = -(A_{12}(s)/A_{22}(s))G_{11}(s) \). And, the Laplace variable \( s \) is replaced to \( j\omega_k \), \( k=1,2,...,n \) to obtain the frequency response data where \( \omega \) denotes frequency. Then, the following linear equation can be straightforwardly derived by following:

\[ B = \Phi P . \]  
(13)

Thus, the matrix \( B, \Phi \) and \( P \) are

\[ B = \begin{bmatrix} Q(j\omega_0) \\ \vdots \\ Q(j\omega_n) \end{bmatrix}, \quad \Phi = \begin{bmatrix} 1 & 1/j\omega_0 & j\omega_0 \\ \vdots & \vdots & \vdots \\ 1 & 1/j\omega_n & j\omega_n \end{bmatrix}, \quad P = \begin{bmatrix} k_{c12} \\ k_{d12}/\tau_{d12} \\ k_{c12}\tau_{d12} \end{bmatrix} . \]  
(14)

Since (13) and (14) have imaginary term, the solution of the linear least square method should be calculated by

\[ P = [\Phi^T \Phi]^{-1}[\Phi^T B^r] \]  
(15)

where,

\[ B^r = \begin{bmatrix} \text{Re}(B) \\ \text{Im}(B) \end{bmatrix}, \quad \Phi^r = \begin{bmatrix} \text{Re}(\Phi) \\ \text{Im}(\Phi) \end{bmatrix} . \]  
(16)

525
TABLE I. CONTROL PERFORMANCE EVALUATION USING RMSE

<table>
<thead>
<tr>
<th>Tuning method</th>
<th>Wood and Berry column</th>
<th>Wardle and Wood column</th>
<th>Tyreus stabilizer</th>
<th>Vinante and Luyben plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_1$</td>
<td>$Y_2$</td>
<td>$Y_1$</td>
<td>$Y_2$</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.100</td>
<td>0.184</td>
<td>0.087</td>
<td>0.105</td>
</tr>
<tr>
<td>BLT</td>
<td>0.117</td>
<td>0.289</td>
<td>0.110</td>
<td>0.193</td>
</tr>
<tr>
<td>Wang</td>
<td>0.135</td>
<td>0.184</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

It means that the PID tuning parameters can be analytically obtained from (15) without solving iterative nonlinear optimization problem. The PID tuning parameters of $G_{e12}(s)$ can be determined in the same way with the case of $G_{e12}(s)$.

III. CASE STUDY

Four kinds of typical TITO processes are controlled by the proposed method and Luyben’s biggest log-modulus (BLT) method [3] and Wang’s method [12] are simulated to demonstrate that the proposed method shows almost equivalent or better control performances compared with previous approaches as well as it does not require the user-defined parameters. As shown in Table 1, the root-mean-square error (RMSE) of the proposed method and the other tuning methods are presented to evaluate the control performance of the proposed method. The BLT tuning method is applied to the four kinds of typical TITO process, and the Wang’s method is only applied to Wood and Berry column and Vinante and Luyben plant since the tuning parameters for the other processes are not included in Wang’s paper.

A. Wood and Berry Column

$$G(s) = \begin{bmatrix} 12.8e^{-2s} & -18.9e^{-3s} \\ 16.7s + 1 & 21s + 1 \\ 6.6e^{-7s} & -19.4e^{-3s} \\ 10.9s + 1 & 14.4s + 1 \end{bmatrix}$$ (17)

Wood and Berry column is presented in (17) [15]. Before designing the centralized PID controller using the proposed method, the RGA analysis should be carried out to find the best pairing. In this case, the RGA analysis shows that $Y_1$ paired with $U_1$ and $Y_2$ paired with $U_2$ are the best. So, the diagonal part PID controllers are simply tuned by the ITAE-1 tuning method [16]. And the parameters of the off-diagonal part PID controllers are straightforwardly calculated by solving the least squares method. The time delays of the off-diagonal part PID controllers are 2 and 4, respectively. Then, the centralized PID controller of (18) is obtained and the control performance is shown in Figure 4, confirming that the proposed method provides almost complete decoupling performance and similar setpoint tracking performance compared with the BLT method and Wang’s method.

B. Wardle and Wood Column

$$G(s) = \begin{bmatrix} 0.126e^{-6s} & -0.101e^{-12s} \\ 60s + 1 & (48s + 1)(45s + 1) \\ 0.094e^{-8s} & -0.12e^{-8s} \\ 38s + 1 & 35s + 1 \end{bmatrix}$$ (19)

Wardle and Wood Column can be represented by (19) [17]. The RGA analysis indicates that the $Y_1$ paired with $U_1$ and $Y_2$ paired with $U_2$ are the best. The diagonal part PID controllers are straightforwardly tuned by the SISO IMC tuning method [18]. The tuning parameters of the off-diagonal part PID controllers are calculated by the linear least squares method of (15) as shown in (20). Figure 5 shows that the proposed method provides acceptable tracking performance of the setpoint change and excellent decoupling performance.

C. Tyreus Stabilizer

$$G(s) = \begin{bmatrix} -0.1153(10s + 1)e^{-0.1s} & 0.2429e^{-2s} \\ (4s + 1)^3 & (33s + 1)^2 \\ -0.0887e^{-12s} & 0.2429e^{-0.17s} \\ (43s + 1)(22s + 1) & (44s + 1)(20s + 1) \end{bmatrix}$$ (21)

Equation (21) is the Tyreus stabilizer [19]. The RGA analysis recommends the $Y_1$ paired with $U_1$ and $Y_2$ paired with $U_2$. The diagonal part PID controllers are tuned by the SISO ITAE-2 tuning method [20]. The ITAE-2 combined with model reduction is used to tune the $G_{e13}(s)$ because the $G_{e13}(s)$ is 3rd order process [20], [21]. The calculated centralized PID controllers are presented in (22). Figure 6 shows that the proposed method can control Tyreus stabilizer process in an efficient way. The result shows the proposed method gives remarkable improvement in tracking setpoint change and decoupling performance.

D. Vinante and Luyben Plant

$$G(s) = \begin{bmatrix} -2.2e^{-s} & 1.3e^{-0.3s} \\ 1 + 7s & 1 + 7s \\ -2.8e^{-1.8s} & 4.3e^{-0.35s} \\ 1 + 9.5s & 1 + 9.2s \end{bmatrix}$$ (23)

Vinante and Luyben plant is given in (23) [3]. The RGA analysis shows the $Y_2$ paired with $U_1$ and $Y_3$ paired with $U_3$ are the best. The diagonal part PID controllers are tuned by SISO ITAE-1 tuning rule [16]. The time delay of $G_{e11}(s)$ is larger than the time delay of $G_{e12}(s)$. Therefore, the structure of the off-diagonal part PID controller $G_{e12}(s)$ should be (11). The
off-diagonal part PID parameters can be obtained by solving (13) with linear least squares method, resulting in the centralized PID controller in (24). The result shows that the proposed method guarantees similar tracking performance and better decoupling performance for $Y_1$ when $Y_2$ setpoint is changed without requiring user-defined parameters. It is shown in Figure 7.

IV. CONCLUSION

In this research, a new designing method for the centralized PID controllers is proposed to improve the closed-loop control performance by removing off-diagonal part component of the open-loop transfer function matrix in frequency domain. The proposed method provides the centralized PID controller tuning parameters in a straightforward way without user-defined parameter such as phase margin, gain margin, overshoot and so on. The case studies show that the control performance of the proposed method is almost equivalent or better than previous approaches even it is conceptually straightforward and simple. The proposed method can be implemented by adding just two off-diagonal PID controllers to the already existing multi-loop PID control system without breaking conventional control loops because it uses conventional tuning parameters to tune the diagonal-part PID controllers. Therefore, the proposed method can be applied to industrial field more easily.

REFERENCES