Optimal Allocations of Area Margins and Spares to Accommodate HEN Cleaning Schedules*

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Abstract—Due to fouling in one or more heat-transfer unit, the target temperatures of process streams in a heat exchanger network (HEN) may not be achievable after a sufficiently long period of operation. To circumvent this practical problem, a given HEN design should be modified to accommodate online cleaning operations via proper allocation of area margins and spares. Specifically, the candidate units for incorporating margins can be first identified on the basis of sensitivity analysis. The potential locations of additional auxiliary heaters/coolers and bypasses can then be determined according to the heat-load loops and paths embedded in HEN so as to facilitate compensation of the removed duties during defouling. The optimal design refinements and also the spare-supported cleaning schedule can be generated by solving a mathematical programming model formulated on the basis of a superstructure. A simple example is presented in this paper to illustrate the proposed modeling procedure and to demonstrate the feasibility of this scheduling approach.

I. INTRODUCTION

In a chemical process, efficient energy recovery and reuse is usually the key to reducing operating cost, while the heat-exchanger network (HEN) is often adopted to facilitate such a purpose. However, fouling in the heat exchangers inevitably causes significant reduction in their heat duties. In order to solve this problem, a proper cleaning schedule must be implemented over the entire operation horizon.

A programming approach has often been taken in the past to produce the aforementioned HEN cleaning schedules. Smalëi et al. [1] first constructed a mixed integer nonlinear programming (MINLP) model for the thin-juice preheat chain in a sugar refinery, while Lavaja and Bagajewicz [2] formulated a mixed-integer linear programming (MILP) model to relieve computation load. Although a cleaning schedule generated with any of the existing methods could be considered for schedule synthesis is the duration in months between the ending and beginning instances of two consecutive planned plant shutdowns. To relieve computation load, this schedule horizon is partitioned into n separate periods according to Figure 1. Each time period is further divided to two sub-periods: a cleaning sub-period, and an operating sub-period. To relieve computation load, these time periods are fixed, i.e.

\[ \tau_1 = \tau_2 = \ldots = \tau_n = \tau \]  
\[ t_f = \sum_{p=1}^{n} \tau_p = n \tau \]  

where, \( \tau \) is the total duration of schedule horizon; \( \tau_p \) denotes the length of period \( p (p=1,2,\ldots,n) \) and is a given constant. The durations of all cleaning sub-periods are assumed to be \( f_c \), which is an a priori given constant. The beginning and end times of a cleaning sub-period are denoted as \( bcp \) and \( ecp \), respectively, while those of an operating sub-period are denoted as \( bop \) and \( eop \), respectively.

\[ P_k = \{ p \mid p \text{ is the numerical label of a period in year } k \} \]  

Figure 1. Horizon partitioning.

II. PARTITIONING OF SCHEDULE HORIZON

In this work, the maximum length of time period that can be considered for schedule synthesis is the duration in months between the ending and beginning instances of two consecutive planned plant shutdowns. To relieve computation load, this schedule horizon is partitioned into \( n \) separate periods according to Figure 1. Each time period is further divided to two sub-periods: a cleaning sub-period, and an operating sub-period. To relieve computation load, these time periods are fixed, i.e.

III. BINARY VARIABLES TO FACILITATE SELECTIONS FOR EXCHANGER CLEANING AND SPARE SUBSTITUTION

For illustration convenience, let us introduce the following two label sets to collect and classify the process streams in a
given HEN:

\[ I = \{ i | i \text{ is the label of a hot stream in a given HEN} \} \]

\[ J = \{ j | j \text{ is the label of a cold stream in a given HEN} \} \]

In addition, the set of all heat exchangers in this HEN can be defined as

\[ E = \{ (i, j) \mid (i, j) \text{ denotes an exchanger in a given HEN, } i \in I, j \in J \} \]

And the set of spares can be written as

\[ S = \{ s | s \text{ is the label of a spare} \} \]

Therefore, the selections of exchangers to be cleaned can be facilitated with the following binary variables:

\[ Y_{i,j,p} = \begin{cases} 1 & \text{if heat exchanger } (i, j) \text{ is cleaned in period } p \\ 0 & \text{otherwise} \end{cases} \]

The occasion to consider a spare only arises after making the decision to remove and clean an online unit from HEN. All such options can be represented with another set of binary variables and the corresponding logic constraints, i.e.

\[ X_{i,j,p,s} = \begin{cases} 1 & \text{if heat exchanger } (i, j) \text{ is replaced with spare } s \text{ in period } p \\ 0 & \text{otherwise} \end{cases} \]

that is,

\[ X_{i,j,p,s} = \begin{cases} 0 & \text{if } Y_{i,j,p} = 0 \\ 1 & \text{if } Y_{i,j,p} = 1 \end{cases} \]

To limit the search space, the following selection criteria are also adopted

(i) Spare \( s \) can only be used to replace \( N_s \) different units one-at-a-time in separate periods.

(ii) Every online unit can be substituted only with the same spare.

Finally, spare \( s \) in period \( p \) can only be used for a single heat exchanger. That is,

\[ \sum_{(i,j) \in E} X_{i,j,p,s} \leq 1 \quad \forall p \in P, s \in S \]

IV. FOULING MODEL

Fouling develops in almost heat exchanger during operation, hence the overall heat-transfer coefficient of every exchanger in HEN may decrease with time according to the following formula

\[ U_{ij}(t) = \left[ \frac{1}{U_{ij}^0} + r_{ij}(t) \right]^{-1} \]

where, \((i, j) \in E \); \( U_{ij}(t) \) is the overall heat-transfer coefficient of exchanger \((i, j)\) at time \( t \) and \( U_{ij}^0 \) represents the corresponding constant when the heat-transfer surface is clean. The time function \( r_{ij}(t) \) is the fouling factor of exchanger \((i, j)\) at time \( t \). There are two different fouling models to describe the fouling factor. The linear fouling model is given by

\[ r_{ij}(t) = \dot{r}_{ij} \]

Whereas the exponentially asymptotic model is given by

\[ r_{ij}(t) = r_{ij}^0 \left[ 1 - \exp \left( -K_{ij}t \right) \right] \]

where, \( \dot{r}_{ij} \) is the constant fouling rate; \( r_{ij}^0 \) is the asymptotic maximum fouling resistance; \( K_{ij} \) is the characteristic fouling speed.

V. OVERALL HEAT TRANSFER COEFFICIENTS

There are four scenarios should be considered in modeling the overall heat-transfer coefficient in period \( p \):

(i) Exchanger \((i, j)\) is not cleaned in period \( p \) \((p \geq 2)\), but in at least one of the prior periods unit \((i,j)\) has been cleaned, i.e., \( Y_{i,j,p} = 0 \) and \( \prod_{k=1}^{p-1} (1 - Y_{i,j,k}) = 0 \) for \( p = 2, 3, \ldots, n \).

(ii) Exchanger \((i, j)\) is cleaned in period \( p \) and a spare is used to take its place, i.e., \( Y_{i,j,p} = 1 \) and \( \sum_{s \in S} X_{i,j,p,s} = 1 \) for \( p = 1, 2, \ldots, n \).

(iii) Exchanger \((i, j)\) has never been cleaned since the first period, i.e., \( Y_{i,j,1} = Y_{i,j,2} = \cdots = Y_{i,j,p} = 0 \) for \( p = 1, 2, \ldots, n \). Note that the overall heat-transfer coefficient in this scenario is zero.

The overall heat-transfer coefficient of exchanger \((i, j)\) at time point \( bcp \) and \( ecp \) during period \( p \) can be expressed with four corresponding terms on the right-hand side of the following equation:

\[ U_{i,j,p}^{bcp} = \sum_{k=1}^{p} \left[ a_{i,j,k}^{bcp} \prod_{m=1}^{k-1} (1 - Y_{i,j,m}) \right] + bsp_{i,j,p}^{*} \sum_{s \in S} X_{i,j,p,s} Y_{i,j,p} + c_{i,j,p}^{*} \prod_{s \in S} (1 - Y_{i,j,s}) \]

where, the label “**” means time points \( bcp \) and \( ecp \); \( p = 1, 2, \ldots, n \); \( a_{i,j,k,p}^{bcp} \), \( bsp_{i,j,p}^{*} \), \( c_{i,j,p}^{*} \), and \( ecp_{i,j}^{bcp} \) are the corresponding overall heat-transfer coefficient in scenarios (i), (ii) and (iii), respectively. More specifically, the three coefficients mentioned above in the time point \( bcp \) can be expressed explicitly as

\[ a_{i,j,k,p}^{bcp} = \frac{1}{\eta_{U_{ij}^{0}}} + r_{ij}^{bcp} \left[ 1 - \exp \left( -K_{ij}(p-k) \right) \right] \]

\[ bsp_{i,j,p}^{*} = \frac{1}{U_{ij}^{0}} + r_{ij}^{*} \left[ 1 - \exp \left( -K_{ij}(p-1) \right) \right] \]
After a time interval of length $\eta_i$, the three coefficients mentioned above in the time point $eop$ can be expressed as

$$a_{ij,p}^{eop} = \frac{1}{\eta_i U_{ij}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} (p - k) \right) \right]$$

$$b_{ij,p}^{eop} = \frac{1}{\eta_i U_{ij}^{mov}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} f_c \right) \right]$$

$$c_{ij,p}^{eop} = \frac{1}{U_{ij}^{\ddagger}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} \tau^* \right) \right]$$

Note that, the overall heat-transfer coefficient of exchanger $(i,j)$ at time point $bop$ and $eop$ during period $p$ can be expressed as

$$U_{ij}^{\ddagger} = \sum_{p=1}^{p_{max}} a_{ij,p}^{bop} + b_{ij,p}^{bop} \sum_{p=1}^{p_{max}} X_{ij,p}^{bop} Y_{ij,p}^{bop} + c_{ij,p}^{bop} \sum_{p=1}^{p_{max}} (1 - Y_{ij,p}^{bop})$$

$$\forall \, p \in P, (i,j) \in E, s \in S$$

where, the label $s$ can be replaced with time points $bop$ and $eop$; $p = 1,2,\cdots, n$; note that, since exchanger $(i,j)$ is not cleaned during period $p$ in scenario (i) and (iii), the equations of $a_{ij,p}^{eop}$ and $c_{ij,p}^{eop}$ should also be applicable for time point $bop$ as well. That is, $a_{ij,p} = a_{ij,k,p}$ and $c_{ij,p} = c_{ij,k,p}$; $b_{ij,p}$ denotes the overall heat-transfer coefficient at time point $bop$ during period $p$ in scenario (ii), i.e.

$$b_{ij,p}^{eop} = \eta_i U_{ij}^{\ddagger}$$

Finally, the overall heat-transfer coefficients of exchanger $(i,j)$ at time point $eop$ during period $p$ can be presented as

$$a_{ij,k,p}^{eop} = \frac{1}{\eta_i U_{ij}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} \left( (p - k + 1) \tau + f_c \right) \right) \right]$$

$$b_{ij,p}^{eop} = \frac{1}{\eta_i U_{ij}^{mov}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} \tau \right) \right]$$

$$c_{ij,p}^{eop} = \frac{1}{U_{ij}^{\ddagger}} + r_{ei} \left[ 1 - \exp \left( -K_{ij} \tau^* \right) \right]$$

VI. OVERALL HEAT-TRANSFER COEFFICIENTS

By assuming counter-current flow in every heat exchanger, the corresponding energy balance can be written as

$$Q_{ij}(t) = F^{\text{in}}_{ij} C^C_{ij} \left( T^{\text{in}}_{ij}(t) - T^{\text{out}}_{ij}(t) \right)$$

$$= F^{C}_{ij} C^C_{ij} \left( T^{C}_{ij}(t) - T^{C}_{out,ij}(t) \right)$$

$$= U_{ij}^{\ddagger} A_{ij} \frac{\Delta T_i(t) - \Delta T_j(t)}{\ln \left( \Delta T_i(t)/\Delta T_j(t) \right)}$$

where, $(i,j) \in E$; $Q_{ij}$ is the heat duty (kJ/s) of exchanger $(i,j)$; $F^{\text{in}}_{ij}$ and $F^{C}_{ij}$ denote respectively the mass flow rates (kg/s) of hot stream $i$ and cold stream $j$; $C^H_{ij}$ and $C^C_{ij}$ denote respectively the heat capacities (kJ/kg-K) of hot stream $i$ and cold stream $j$; $A_{ij}$ is the heat transfer area (m$^2$) of exchanger $(i,j)$; $T^{\text{in}}_{ij}$ and $T^{\text{out}}_{ij}$ denote respectively the inlet temperatures (K) of hot stream $i$ and cold stream $j$; $T^{\text{out}}_{out,ij}$ and $T^{C}_{out,ij}$ denote respectively the outlet temperatures (K) of hot stream $i$ and cold stream $j$; $\Delta T_i(t)$ and $\Delta T_j(t)$ denote respectively the temperature difference $T_i(t)$ and $T_j(t)$; $\Delta T_2(t)$ denotes the temperature difference between $T^{\text{out}}_{out,ij}(t)$ and $T^{C}_{out,ij}(t)$. Equation (18) can then be rearranged to produce an expression for the outlet temperature of the hot stream, i.e.

$$T^{\text{out}}_{ij}(t) = \frac{R_{ij}(t)}{R_{ij}(t - 1) - 1} \frac{U_{ij}(t) A_{ij}}{F^{C}_{ij} C^C_{ij}} \left( R_{ij}(t - 1) - 1 \right)$$

where, the constant $R_{ij}$ is defined as

$$R_{ij} = \frac{F^{C}_{ij} C^C_{ij}}{F^{\text{in}}_{ij} C^C_{ij}} \frac{T^{\text{out}}_{out,ij}(t) - T^{\text{out}}_{in,ij}(t)}{T^{C}_{out,ij}(t) - T^{C}_{out,ij}(t)}$$

To simplify model formulation, let us introduce two additional model parameters $d_{ij}$ and $d_{ij}^{eop}$:

$$d_{ij} = \frac{A_{ij}}{F^{C}_{ij} C^C_{ij}} \left( R_{ij} - 1 \right)$$

$$d_{ij}^{eop} = \frac{A_{ij}}{F^{C}_{ij} C^C_{ij}} \left( R_{ij} - 1 \right)$$

where, $A_{ij}$ which is a variable is the heat-transfer area of the spare exchanger. After substituting (16) and (17) into (19), one can then obtain the expression of the outlet hot stream temperatures at the aforementioned four time points.

Finally, the outlet temperatures of cold stream at different time instances can be determined according to (19), i.e.

$$T^{C}_{out,ij}(t) = T^{C}_{out,ij} + \frac{T^{H}_{out,ij} - T^{H}_{out,ij}(p - 1)}{R_{ij}}$$

where, $(i,j) \in E$; $p = 1,2,\cdots, n$; $tp \in \{bop, eop, bop, eop\}$; $T^{C}_{out,ij}(tp)$ denotes the outlet temperature of cold stream $j$ at time point $tp$ in period $p$

VII. OBJECTIVE FUNCTION-TOTAL ANNUAL COST

The total annual cost (TAC) associated with operating and cleaning a given HEN can be approximated by summing the total annualized capital cost (TACC) for purchasing the spares and the average value of the total annual operating cost (TAOC), i.e.

$$\text{TAC} = \text{TACC} + \overline{\text{TAOC}}$$
The former cost is computed with the following formula

\[
\text{TACC} = C_c \sum_{i \in S} A_i^0 + C_u \sum_{(i,p) \in E} \left( (A_i^0)^{0.6} - (A_i^0)^{0.6} \right) + C_{\text{heat}} \sum_{j \in I_{\text{AO}}} \left( A_j^{\text{AHT}} \right)^{0.6} + C_{\text{cooler}} \sum_{j \in I_{\text{AO}}} \left( A_j^{\text{ACL}} \right)^{0.6}
\]

where, \( I_{\text{AO}} \) represents the set of cold streams needing auxiliary heaters; \( I_{\text{AH}} \) represents the set of hot streams needing auxiliary coolers; \( C_c \), \( C_u \), \( C_{\text{heater}} \), and \( C_{\text{cooler}} \) represent respectively the capital cost coefficients of spare heat exchanger, auxiliary heater and cooler; \( A_i^0 \), \( A_j^{\text{AHT}} \) and \( A_j^{\text{ACL}} \) represent respectively the area of overdesigned heat-exchanger, \( A_j^{\text{AHT}} \) and \( A_j^{\text{ACL}} \) can be calculated by the following two equations.

\[
Q_j^{\text{AHT}} = U_{\text{heater}} A_j^{\text{AHT}} \left( \frac{T_{\text{in}}^{\text{H}} - T_{\text{in}}^{\text{AHT},p}}{\ln \left( \frac{T_{\text{out}}^{\text{AHT},p}}{T_{\text{in}}^{\text{AHT},p}} \right)} \right) + F c p \left( T_{\text{in}}^{\text{AHT},p} - T_{\text{in}}^* \right)
\]

\[
Q_j^{\text{ACL}} = U_{\text{cooler}} A_j^{\text{ACL}} \left( \frac{\left( \max \left( T_{\text{out}}^{\text{H},p} - T_{\text{in}}^{\text{H}} \right) - T_{\text{out}}^{\text{H}} \right)}{\ln \left( \frac{T_{\text{in}}^*}{T_{\text{out}}^{\text{H}} - T_{\text{in}}^{\text{H}}} \right)} \right) + F c p \left( \max \left( T_{\text{out}}^{\text{H},p} - T_{\text{in}}^{\text{H}} \right) - T_{\text{out}}^{\text{H}} \right)
\]

The average operating cost can be approximated simply by taking the arithmetic average of total operating costs needed in all years in the schedule horizon, i.e.,

\[
\text{TOC}_k = \frac{1}{\gamma} \sum_{k=1}^{\gamma} \text{TOC}_k
\]

where, \( \text{TOC}_k \) denotes the total operating cost in year \( k \), which can be further divided into the total utility cost (\( \text{TUC}_k \)) and the total cleaning cost (\( \text{TCLC}_k \)). Specifically,

\[
\text{TUC}_k = \sum_{p \in P} \left[ C_{\text{H}} \frac{u_c}{\eta_c} \sum_{j \in J} E_{j,p}^c + C_{\text{p}} \frac{u_H}{\eta_H} \sum_{j \in J} E_{j,p}^H \right]
\]

\[
\text{TCLC}_k = C_{\text{C}} \sum_{j \in J} Y_{j,k} + C_{\text{H}} \sum_{j \in J} \sum_{p \in P} X_{j,i,p}
\]

where, \( C_{\text{H}}^c \) and \( C_{\text{H}}^H \) respectively denote the unit costs of cooling and heating utilities in period \( p \); \( \eta_c \) and \( \eta_H \) denote the heat-transfer efficiencies in cooler and heater respectively; \( E_{j,p}^c \) and \( E_{j,p}^H \) denote total amounts of utilities consumed respectively by cold stream \( j \) and hot stream \( i \) in period \( p \); \( C_{\text{C}} \) and \( C_{\text{H}} \) represent the cleaning costs of exchanger and spare respectively. With all aforementioned constraints and the objective function (22), a comprehensive MINLP model can be formulated to facilitate synthesis of optimal cleaning schedule. However, additional constraints are still needed to enhance computation efficiency.

VIII. SENSITIVITY ANALYSIS

To facilitate clear explanation of the proposed analysis, let us consider the HEN design presented in Figure 2, the corresponding stream data in Table 1 and the design specifications in Table 2. In this example, the exponential fouling model is assumed and its asymptotic maximum fouling resistance \( f c \) and the characteristic fouling speed \( f c \) are set at 0.8 (m²K/kW) and 0.3 (month⁻¹) respectively for each unit in HEN. The unit costs of hot and cold utilities are chosen to be \( 7.72 \times 10^{-3} \text{($/KJ)} \) and \( 7.72 \times 10^{-3} \text{($/KJ)} \) respectively, while the cleaning costs of exchanger and spare are assumed to be 4000 ($/cleaning) and 1000 ($/cleaning) respectively. The other model parameters are selected as follows: \( \tau = 1 \text{ month} \); \( f_c = 0.25 \text{ month}^{-1} \); \( \eta_c = 0.9 \); \( C_{\text{exp}} = 550 \text{ (yr m}^{-1.5} \).
On the other hand, it can be determined that a bypass should also be installed on HE3 to facilitate the required load shift on another path. Finally, although obviously not cost effective, the most straightforward approach to bring the final temperatures of process streams to their targets is to install auxiliary heaters and/or coolers at the HEN outlets which are originally without utility users.

X. SUPERSTRUCTURE

Based on the aforementioned decisions on the overdesigned heat exchanger(s) and the locations of bypasses and auxiliary units, one can build a superstructure (see Figure 5) to accommodate the optimal cleaning schedule. For the present example, HE1 is chosen to be margined and the bypasses are placed on HE1, HE2 and HE3. The only auxiliary heater is installed on cold stream C2. A MINLP model can then be constructed to produce the optimal cleaning schedule for minimizing TAC.

![Figure 5. Superstructure.](image)

XI. OPTIMIZATION RESULTS AND DISCUSSIONS

On the basis of sections VIII to X, the proposed MINLP model can be solved with solver SBB in GAMS (version 23.9.5) on a personal computer (Intel Core i7-4790 3.6GHz). As shown in Table III, it is only necessary to buy one spare, which can be used while either HE1, HE2 or HE4 is being cleaned. The heat-transfer area of this spare is 323.38m² and the area of HE1 after adding margin is 530.38m². Since the heat duty of HE1 is biggest in this HEN, cleaning of HE1 is the most cost-effective operation. It may appear that cleaning HE1 and HE2 respectively in the 3rd and 4th month are too early. This decision can be attributed to the use of exponential fouling model with a large characteristic fouling speed ($K_{ij}$), i.e., such a model predicts quick initial fouling. In addition, notice that the conclusions of sensitivity analysis are applicable not only to variations of heat-transfer areas but also overall heat-transfer coefficients. Thus, it can be deduced from Figure 3 that a change in the overall heat-transfer coefficient of HE1 or HE2 should be more sensitive to those of the other units.

Figure 6 and Figure 7 show the opening percentages of bypasses on HE1 and HE2 respectively. It can be observed that the bypass opening on HE1 is relatively small throughout the entire horizon. It varies moderately in response to the development of fouling resistance before and after cleaning. On the other hand, the bypass opening on HE2 are markedly larger than that on HE1 in 3rd, 13th, 16th, 19th and 20th month. The purpose of decreasing the heat duty of HE2 is to increase...
the driving force in HE4 so as to produce a larger heat-transfer rate. The reason for this practice is to cause a heat load shift on the same path (see Figure 4). On the other hand, the opening of bypass on HE2 in 4th and 18th month is due to the fact that HE2 uses a spare with a larger heat-transfer area.

In addition, a cleaning schedule without considering the area margins and bypasses was also generated in this example. The corresponding optimum solution shows that 2 spares are needed. One is used for HE1 and its heat-transfer area is 451.93 m², while the other is for HE2 and the corresponding area is 210 m². Finally, the cost estimates in different scenarios are compared in Table IV. The TAC achieved without cleaning is 2.92 (million $/year). With only the spare-supported cleaning schedule, one can obtain a lower TAC, i.e., 2.87 (million $/year), which represents a 1.7% cost saving. With the additional oversized heat exchanger, the resulting minimum TAC is 2.85 (million $/year), which further improves the TAC saving to 2.4%.

Due to space limitation, only the optimization results of a small HEN are presented above. In fact, more cost-effective cleaning schedules can be synthesized for larger systems. In another example with four hot streams, three cold streams, six heat exchangers, four coolers and one heater, when compared with the base case of no cleaning, a saving of 24.4% was realized with proposed mathematical programming model [6].

XII. CONCLUSIONS

In the present study, the optimal HEN cleaning schedules have been generated by introducing the design options for area margins and spares into the existing model. Such an improved MINLP model can be constructed according to a superstructure in which the additional bypasses and auxiliary units are placed systematically. From the optimization results obtained so far, it can be observed that the sensitivity analysis can indeed be used to effectively identify the candidate heat exchangers needing extra areas in the given HEN and, also, the required heat-load shifts can be realized with the added bypasses and auxiliary units. In terms of the total annual cost, one can see that a much improved cleaning schedule can be generated with the proposed optimization approach.

REFERENCES