Improved Repetitive Controller for Active Power Filter

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Abstract—In order to eliminate a certain type of harmonics in power grid, this paper presents a novel compound controller to improve the performance of active power filter (APF) based on synchronous frame. The proposed controller can work effectively with the help of a proportional-integral (PI) control and an advanced repetitive controller. By optimizing the internal model of the repetitive controller, the controller can deal with the current from grid directly, which simplifies the APF system dramatically. Because there is no need to add extra sensors and the APF system does not require a harmonic detector. Moreover, the proposed controller which has a shorter convergence time for its smaller repetitive period can eliminate the harmonics faster than the traditional one. And its repetitive cycle is only one-sixth as that of the traditional. In addition, thanks to an advanced voltage phase collector, the voltage phase collected from the grid becomes more accurate, which improves the compensation performance of APF. The controller design algorithm is also given in detail. Theoretical analysis and results obtained by simulations validate the superiority of the proposed controller.

Index Terms—Active power filter; fast repetitive control; harmonic compensation; compound controller.

I. INTRODUCTION

With increasing use of nonlinear loads in the power grid, devices such as power electronic converters have introduced numerous harmonics into the power system. These harmonic currents lead to power loss, voltage fluctuation, temperature rise of transformers, malfunction of electronic equipment, etc. [1]. Harmonic problems have done great harms to distribution systems and power grid.

As a flexible mean of harmonic compensation, the active power filter (APF) can eliminate harmonics generated by various kinds of devices and provide fast response to load variation. Normally, The performance of APF is mainly influenced by its control strategy which depends most on the controller [2-3]. The way of designing a controller with excellent dynamic and static performance is a hot issue in APF research [4]. Conventional PI control, as a simple and effective control method, is widely used in APF current controllers. However, in practical applications of APF, the reference harmonic current contains harmonic currents of different frequencies, and the PI regulator can not meet this requirement because of the limitation of its bandwidth. In order to crack this limitation, the repetitive control strategy which is based on internal model principle is used. Repetitive controller can be targeted to suppress and eliminate stable periodic signal that located in closed-loop [5-6], which can directly improve the accuracy of APF. However, owing to the delay of internal model, a single repetitive controller can not meet the control requirement because of its poor dynamic performance [7]. Thus, a series of compound controllers and improved methods are presented in [8-9]. The repetitive control strategy, as well as an optimized current controller which is based on the synchronous coordinate system, is proposed to improve the efficiency of control scheme in [10-11]. A fast repetitive control strategy is proposed in [12] by optimizing the internal model to shorten the repetitive period.

Based on some optimized structures and methods that mentioned above, a compound controller based on the synchronous coordinate system, which is consisted of a PI controller and an improved repetitive controller is presented.

Firstly, a certain type of harmonic currents in the grid is analyzed in the synchronous coordinate frame, and then, a rapid compound controller which consists of PI and repetitive control is proposed. On this basis, this paper introduces the structure of a new repetitive controller whose repetitive period is only one sixth as that of traditional traditional, which can accelerate the response speed of APF system and deal with the signal from grid directly. By collecting the current signal directly from the grid, the structure of the APF system is further simplified, this means the signal processing time will become shorter and the performance of compensation will be more accurate, because the control performance of APF is not influenced by the harmonic tracking process any more. In practical power system, the grid voltage is often not completely a sinusoidal wave. In order to solve this problem, a new-type grid voltage phase collector is used, which makes the tracking accuracy of current loop higher than that of traditional control strategies. Secondly, comparisons between the proposed control strategy and the traditional control strategy demonstrate the rapid convergence performance of the control strategy. After that, this paper calculates the optimal controller parameters to ensure the reliability. Finally, the results from simulation verify the feasibility and validity of the proposed control strategy.

II. CONTROL STRATEGY ANALYSIS OF APF SYSTEM
A. Topology of System Structure

The topology of a three-phase shunt APF with LCL output filter is shown in Fig. 1. The load is a nonlinear device, and it is usually a six-pulse rectifier which used as the front-ends of industrial AC drives. A shunt APF is used as a source of harmonic compensation, whose main circuit is a three-phase voltage source inverter (VSI) works in parallel with the nonlinear load.

![Fig.1. Structure of the active power filter system](image)

The grid is connected to the APF with a LCL output filter, where \( u_0 \) is the voltage of grid, \( L_s \) is the inner inductance of the power supply, \( C_o \) is the DC capacitor of VSI and \( L_1, C_1, L_2 \) are known as the parameters of LCL output filter.

The simplified control system of APF is shown in Fig. 2, where the system is composed of a voltage loop and a current loop. PI controller in the outer voltage loop is used to maintain a stable DC voltage of APF, and the inner current loop often uses compound control to keep the grid current and voltage in the same phase[13].

B. Analysis of Improved Compound Control Strategy

In general, nonlinear loads generate numerous odd harmonics which cause more damage than even harmonics. The increasing penetration of these odd harmonics have already deteriorated the power quality.

In industrial applications, most of the harmonics generated by nonlinear load are \( 6m \pm 1 \) harmonic currents with \( m = 1, 2, 3, \ldots \) [10]. Considering the typical three-phase diode rectifier and resistance-inductance load, two phases of the three-phase power source are selected randomly to calculate the load currents. Phase a and phase b are taken as examples in this paper as follows

\[
\begin{align*}
i_a(t) &= \sum_{n=1,2,3,\ldots}^\infty (-1)^n I_n \cos(n \omega t) + I_1 \cos(\omega t) \\
i_b(t) &= \sum_{n=1,2,3,\ldots}^\infty (-1)^n I_n \cos\left(\frac{2\pi}{3} + \omega t\right) + I_1 \cos\left(\frac{2\pi}{3}\right)
\end{align*}
\]

where \( I_1 \) is the fundamental current amplitude, \( I_n \) is the \( n \)th current amplitude, \( n \) is the harmonic order, \( \omega \) is the fundamental angular frequency.

By using the 3/2 transformation, system (1) can be converted to the two-phase stationary coordinate system as

\[
\begin{bmatrix}
i_a \\
i_b
\end{bmatrix} =
\begin{bmatrix}
i_a \\
i_a + 2i_b
\end{bmatrix}
\]

By using the following transformation,

\[
\begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix} =
\begin{bmatrix}
i_a \cos \omega t + i_b \sin \omega t \\
i_a \sin \omega t + \sqrt{3} i_b \cos \omega t
\end{bmatrix}
\]

The expression of current in synchronous coordinate system is given as

\[
\begin{bmatrix}
i_\alpha(t) \\
i_\beta(t)
\end{bmatrix} =
\begin{bmatrix}
I_1 + \sum_{n=1,2,3,\ldots}^\infty (-1)^n (I_{n-1} + I_{n+1}) \cos(n \omega t) \\
\sum_{n=1,2,3,\ldots}^\infty (-1)^n (I_{n-1} + I_{n+1}) \sin(n \omega t)
\end{bmatrix}
\]

As shown in Fig. 3, a phase current spectrum is given. From the load current spectrum in the synchronous coordinate system, it can be seen that in addition to the DC component, there are only \( 6m \) harmonic currents left.

According to the characteristics of the typical load current in the \(dq\) coordinate system, a compound current control scheme using PI and repetitive controller is proposed. The block diagram is given in Fig. 4.

![Fig.4. Block diagram of the control system](image)
As illustrated in Fig. 4, the supply current is first transformed in the synchronous coordinate system. The proposed current controller is then executed to regulate the real current $i_{dc1}$ follow its reference $i'_{dc1}$. After that, the PI controller can produce a rapid response to the reference signal fluctuation, and the repetitive control can track the output wave-form accurately. The transfer function of the repetitive controller is given as

$$G_R(s) = \frac{1 + e^{-sT/6}}{1 - e^{-sT/6}}$$

(5)

with the following transfer function in frequency domain,

$$G_R(j\omega) = \frac{1 + e^{-j\omega T/6}}{1 - e^{-j\omega T/6}}$$

(6)

Where $T = 2\pi f_0$ and $\omega = 2\pi f$ is the fundamental angular frequency. Considering the characteristic of the typical three-phase rectifier, we set $\omega = 6m\omega_0 (m = 1, 2, 3, ...)$.

In repetitive control, the period delay caused by the internal model controller lags the system response directly, which affects the dynamic response performance of repetitive controller and will finally lead to an unsatisfactory tracking performance of the APF. As shown in (5), the new internal model controller whose repetitive period is only one-sixth of the conventional can solve this problem well. By optimizing the internal model, the repetitive control can make a more rapid response to the signal fluctuation. And according to Euler formula, the transfer function of the repetitive controller can be modified as

$$G_R(i) = \frac{1 + \cos 2m\pi i}{1 - \cos 2m\pi i}$$

(7)

From the above equation, it can be found that this controller has infinite gains with $m = 1, 2, 3, ...$. That is to say, harmonic currents in $6m$ order in coordinate system, which act as $6m \pm 1$ in traditional coordinate, will all be eliminated through using the proposed control strategy. Consequently, the compensation performance of APF will be improved obviously. In addition, this controller is easy to be implemented digitally because of the optimized control scheme and simplified system model. For this reason, this controller is easy to be applied in practical engineering.

In practical power grid, the supply voltage is not completely a sine wave, because it usually contains a lot of harmonic components [11]. Since the conventional phase locked loop cannot guarantee the accuracy of the phase signal of the grid. In order to resolve this problem, a high-precision voltage phase controller (VPC) is adopted here to improve the accuracy of the collected voltage phase. As shown in Fig. 5(a), by using a band-pass filter (BPF) tuned at the basic frequency, this collector can reject all harmonic components in the supply voltage. After collecting the fundamental supply voltage signal, a PI controller is used to improve the tracking accuracy of the dynamic voltage phase. It can be seen from Fig. 5(b) that compared to the grid voltage phase, the phase collected by the conventional phase lock loop (PLL) is seriously distorted between 0.02s and 0.06s, while the voltage collector phase keeps almost the same with the grid phase. This means that the proposed collector can avoid most of the influences makes by the voltage harmonics, and then the compensation of APF will be more accurate.

III. PERFORMANCE ANALYSIS AND PARAMETER DESIGN OF COMPOUND CONTROLLER

A. System Rapidity Analysis

For the purpose of analyzing the convergence performance of APF system, a closed-loop control block diagram is given in Fig. 6. where $I_{L_k}(s)$ is the load harmonic currents, $e_{k-1}(s)$ is the grid side harmonic current, $K(s)$ is the expression of PI controller, $G_R(s)$ is the compensation part of the repetitive controller, $G_d(s)$ is the function of controlled object.

The analysis on the performance of the compound control is based on discrete-time domain. And $k$ is the repetitive cycle. The dynamic response time is set to be $T/6$ in which $T$ is the period of the traditional repetitive controller.
From Fig. 6, (8), (9) and (10) can be obtained as

\[ U_{k+1}(s) = U_k(s) + e_{k+1}(s) \]  
\[ U_{p,k+1}(s) = K(s)e_{k+1}(s) \]  
\[ e_{k+1}(s) = I_{L,k+1}(s) + U_{p,k+1}(s)G_F(s) - G_c(s)G_F(s)[U_{k+1}(s) + U_{k}(s)] \]

From (10), the error current in the last period in the supply side is given as

\[ e_k(s) = I_{L,k}(s) + U_{p,k}(s)G_F(s) - G_c(s)G_F(s)[U_{k}(s) + U_{k-1}(s)] \]

When the control system works in the steady state, the harmonic current of the load side remains unchanged during the two adjacent periods,

\[ I_{L,k}(s) = I_{L,k+1}(s) \]

By combining (10), (11), (12), the following relationship holds,

\[ e_k(s) - e_{k+1}(s) = [U_{p,k+1}(s) - U_{p,k}(s)]G_F(s) + G_c(s)G_F(s)[U_{k+1}(s) - U_{k-1}(s)] \]

Substituting (8), (9) into (13), we can obtain

\[ \frac{e_k}{e_{k+1}} = \frac{1 + G_F(s)[K(s) - G_c(s)]}{1 + G_F(s)[K(s) + G_c(s)]} = 1 - \frac{2G_F(s)G_c(s)}{1 + G_F(s)[K(s) + G_c(s)]} \]

where the remainder of ratio is the error convergence \( G_{Rem} \). It follows from (14) that the error will be zero finally when \( G_{Rem} \| < 1 \). And smaller the remainder is, the shorter convergence time will be [12].

Similarly, the error ratio of conventional compound controller is given as

\[ \frac{e^*_k}{e^*_{k+1}} = \frac{1 + G_F(s)[K(s) - G_c(s)]}{1 + G_F(s)K(s)} = 1 - \frac{G_F(s)G_c(s)}{1 + G_F(s)K(s)} \]

The ratio of error functions between advanced compound controller and conventional controller can be obtained as

\[ \left| \frac{1 - G_{Rem}}{1 - G_{Rem}^C} \right| = \left| \frac{1 + G_F(s)K(s)}{1 + G_F(s)K(s) + G_F(s)G_c(s)} \right| < 1 \]

It can be concluded from the above equations that the proposed control scheme has a faster convergence speed compared with the conventional ones, and then the performance of the compound controller has been significantly improved.

B. Design of PI Controller Parameters

Although the PI controller cannot achieve zero-error tracking for the bandwidth limitation, but its fast response performance to load fluctuation can make up for the inadequacy of repetitive control [13]. When designing the proportion parameter of the PI controller, it is necessary to ensure that the open-loop bandwidth is greater than the requirement in order to compensate the highest harmonic frequency. Additionally, in digital control system, output of the repetitive controller usually delays about a switching period owing to the influence of the system sampling time and computing time, which leads to the phase delay of the system. So, in order to ensure that the PI controller has a large enough phase margin to keep the closed-loop system stable, the open-loop gain \( k_p \) cannot be too large. At the same time, due to the discreteness of the digital system, there still be a certain amount of errors in the digital system, so an overlarge \( k_p \) will also lead to the loss of the control accuracy and make the system unstable. Besides that, a \( k_p \) which is too small for the PI controller to regulate the current, may reduce the adjustment accuracy and make the response slow, which further prolongs the adjustment time and affects the performance of APF system.

For integral coefficient \( k_i \) whose ability is to eliminate errors while the system is working in steady state, so that the accuracy of APF can be significantly improved. The controller will easily overshoot, which is likely to make the system unstable, if it is designed inappropriately. Based on the above considerations, we set \( k_p = 15 \) and \( k_i = 4 \).

C. Design of Repetitive Controller Parameters

The performance of repetitive controller is influenced by its compensator \( G_c(z) \) dramatically. In the digital design of repetitive controller, \( G_c(z) \) is composed of three main parts and is usually written as \( k_C \Delta C(z) \), where \( k_C \) is the gain coefficient regulating the gain of the repetitive loop. \( C(z) \) provides high frequency attenuation. To avoid over-large high frequency gain to increase the system stability, a low-pass second-order filter is used here [14]. According to Fig. 1, the transfer function of the controlled object can be given as

\[ G_F(s) = \frac{L_1 L_2 C s^2 + L_1}{L_1 L_2 C s^3 + (L_1^2 + L_1 L_2) s} \]

It is a typical second-order oscillating plant, but the phase shifting in low and middle frequency is not obvious, and is nearly close to zero. Therefore the common form of low-pass second-order filter can be given when the filter cutoff frequency is chosen,

\[ C(z) = \frac{s^2}{s^2 + 2s \mu s + s^2} \]

At the same time, in order to prevent the resonance in the controller, and meet the requirement of \( \mu > 0.707 \), we take \( \mu = 0.8 \).

Under normal circumstances, the zero phase shift filter \( C(z) \) is usually added into the internal model to give the converter negative gain to eliminate the resonant peak at the cut-off frequency. The zero phase shift filter makes the gain remain almost constant when the converter works in the frequency lower than the cut-off frequency, but the gain will decay rapidly when the converter works in the frequency higher than the cut-off frequency. In the form of literature [15], the filter is taken as follows,
\[ C_r(z) = \frac{\sum_{r=1}^{a_0} \alpha_r z^{-r} + \sum_{r=1}^{a_0} \alpha_r z^{-r}}{\alpha_0 + 2} \]  

where \( \alpha_0 \) and \( \alpha_r \) are the weight coefficients. Considering that the controller will become more complicate when the order of \( C_r(z) \) is higher than two, and at the same time give \( k \) a more wide stable range, \( C_r(z) \) is given as

\[ C_r(z) = \frac{z + 1 + z^{-1}}{3} \]  \hspace{1cm} (20)

To compensate the phase delay caused by controlled plant and compensator, an element \( k \) is introduced in the repetitive controller. In addition to the phase delay impacting controlled plant and \( G_r(z) \), the sampling time and PWM control delay time are also considered. Here, takes \( k = 2 \).

IV. SYSTEM SIMULATION

To validate the effectiveness of this proposed control scheme, simulations based on Matlab/simulink is developed. An APF system is established according to Fig. 1, where the six-pulse rectifier bridge with load of resistance and inductance is used as the source of harmonic. Detailed parameters of the system are given in TABLE 1.

<table>
<thead>
<tr>
<th>TABLE 1. SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>parameter</td>
</tr>
<tr>
<td>( L_1/mH )</td>
</tr>
<tr>
<td>( L_2/mH )</td>
</tr>
<tr>
<td>( L_3/mH )</td>
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<tr>
<td>( C/\mu F )</td>
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Fig. 7 shows the waveform of load current. Compared with ideal sine wave, we find that the load current is seriously distorted, which includes a large number of harmonics. According to Fig. 3(a) mentioned in part 2.2, it can be seen that most of these harmonics are \( 6n \pm 1 \). As shown in Fig. 8(a), the controller does not work until 0.04s, so there is not too much change before 0.04s while using neither the traditional control nor the advanced one. After that, it can be seen that there appear some burrs in the supply current especially at 0.04s and 0.06s, when the traditional compound controller is used. However, as shown in Fig. 8 (b), most of harmonics are suppressed while using the proposed controller. It can also be seen that during 0.05s to 0.07s the current distortion has been eliminated more quick than the traditional when the proposed controller is used.

As can be seen in Fig. 9, the error percentage of \( 6k \pm 1 \) harmonic current in traditional and proposed control scheme is all above 50%. That means the ratio of error harmonic current to traditional harmonic current is greater than 1/2, and the harmonic currents have decreased by half. To verify the dynamic response performance of proposed control scheme, the system load is mutated in the circuit at 0.06s. As shown in Fig. 10, it can be seen that the current is not dramatically affected when the load changes. This verifies the stability of the APF system after the introduction of proposed controller. Fourier spectrum analysis shown in Fig. 11, the THD (Total Harmonic Distortion) of supply side reduces from 2.91% to 1.89%, which indicates the good inhibiting effect on harmonics.
V. CONCLUSION

In this paper, an advanced current controller based on synchronous reference frame was proposed. According to the control system block and the convergence function, the static and dynamic performance of APF system were analyzed respectively. The convergence analysis proved the stability of the proposed compound control strategy. A high accuracy supply voltage phase collector was further presented to improve the performance of phase tracking process. The working principle of the compound controller was particularly analyzed and the design of parameters was given in detail. Theoretical analysis and simulation show that the proposed control strategy can improve the stability of APF system and eliminate the $6m\pm1$ harmonics. Consequently, the APF can work with smaller steady-state errors and faster convergence speed compared to the traditional control strategy.

REFERENCES


