PID Control Loop Performance Assessment and Diagnosis Based on DEA-Related MCDA *

Zun Wang, Yongming Han, Zhiqiang Geng, Qunxiong Zhu, Yuan Xu, and Yanlin He

Abstract— Control loop performance assessment and diagnosis have been attracting more and more attention in the academia and industry. Both traditional performance assessment method and minimum variance method often require the process model and provide limited information, which is not particularly convenient for practical applications. Therefore, the method based on data envelopment analysis (DEA)-related multiple criteria decision analysis (MCDA) is developed for assessing and diagnosing PID control loop performance, which relies solely upon the collected process data during routine plant operation. The control loop performance is assessed and sorted by utilizing the self-evaluation DEA-related MCDA model. The operation priority of the control loop is ranked and determined by utilizing the cross-evaluation DEA-related MCDA model. The improving direction and quantitative space of control loop performance can be diagnosed by DEA-related MCDA model with slack variables and non-Archimedean infinitesimal ε. The correctness and effectiveness of the proposed method are confirmed and validated by simulation examples.

I. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are practically the most widely adopted controllers in industry, and they can achieve satisfactory performance for extensive processes in a relatively simple way [1]. It has been reported that as many as 60% of all industrial controllers have performance problems [2]. In most cases, poor working controllers can be improved by retuning, i.e., adjusting their parameter settings [3]. Studies have shown that the industrial controllers did not always provide an acceptable level of performance as well as everyone had assumed [4].

Control performance monitoring/assessment (CPM/CPA) is an important asset-management technology to maintain highly efficient operational performance of automation systems. Malfunctions in process-control loops, including sensors and actuators, are very common in industry. Their influence is to introduce excess variation throughout the process thereby reducing machine operability, increasing costs and disrupting the control of final product quality [3], [4].

The traditional control performance assessment mainly involves the indexes of time domain and frequency domain which are related to dynamic quality systems. The settling time, rise time and maximum peak overshoot following a setting value change are traditional time domain characteristics for assessing the transient response characteristics of a closed loop system. Other major indexes such as the integral absolute error (IAE), the integral square error (ISE) and the integral time weighted absolute error (ITAE) are often used to assess the control loop performance [5]. Most of the traditional control loop performance assessment methods require a process model and priori parameters, and therefore the actual implementation is not very convenient.

The minimum variance control (MVC) benchmark introduced by Harris [6] is widely used to assess the control loop performance. The MVC based performance assessment techniques are most useful in determining the improvement potential in variance reduction, or in analyzing the variance components by decomposing the process output variance into the components arising from feedback and feed forward elements. However, the minimum variance benchmark cannot be used to determine the improvement potential in practice because of its theoretical limit (unless the process model and the disturbance model are both perfectly known). In addition, the MVC benchmark requires aggressive manipulated variable action, which often does not represent a user-desired, practically acceptable operating condition. A benchmark representing practically acceptable control performance would be desirable [7], [8].

The MVC based performance assessment methods and most of the other performance monitoring techniques usually just adopt a trigger value of a single performance index to judge the process performance. To solve this problem, the multiple criteria decision analysis (MCDA) method can be chosen for assessment.

Since the work by Charnes et al. [9], data envelopment analysis (DEA) has grown into an exciting and fruitful field, in which operations research and management science researchers, economists, and experts from various application areas have played their respective roles. The DEA has been accepted as a major frontier technique for benchmarking, and has been widely utilized as an assessment methodology for the environmental performance of energy firms [10].

Meanwhile, DEA can be treated as a particular multiple criteria decision analysis (MCDA) method, which starts with the purpose of evaluating relative efficiency rather than choosing a specific course of action that is the general case in the traditional decision analysis [11]. Huang et al. [12] and Zhou et al. [13] have developed DEA-related MCDA studies.

This paper proposes a PID control loop performance assessment and diagnosis based on DEA-related MCDA

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approach to process operating data, without explicit model. The decision making unit (DMU) with multi-input and multi-output rather than a single value is proposed to assess and diagnosis control performance. PID control loop performance was assessed by a relative index, greater practicality than the absolute index. The proposed method can be used for online implementation of closed loop and provide the improvement potential with unambiguous physical meaning by optimized slack variables.

The rest of this paper has been organized as follows: The assessment algorithm based on data envelopment analysis (DEA)-related MCDA is provided in Section II. Section III presents the details of the diagnosis of control loop performance and tuning direction of controller parameters. Control loop performance assessment, sorting and ranking are described in Section VI. Section V presents a simulation case study about PID control loop performance assessment and diagnosis based on DEA-related MCDA. Finally, the conclusions are given in Section VI.

II. THE ALGORITHM OF CONTROL LOOP PERFORMANCE ASSESSMENT BASED ON DEA-RELATED MCDA

We consider the typical closed negative feedback control system as indicated in Fig.1. The process output variable is given as \( y(k) \), with the controller signal \( u(k) \), the disturbance driving white noise \( w(k) \), and the sample interval \( k \). The closed loop difference equation is given as

\[
y(k) = \frac{r(k) d(q^{-1}) c(q^{-1}) g(q^{-1}) + w(k) h(q^{-1})}{1 + c(q^{-1}) g(q^{-1})}
\]

(1)

The structure of the discrete model PID controller applied in this study is given as

\[
c(q^{-1}) = c_1 + c_2 q^{-1} + c_3 q^{-2}
\]

(2)

where \( c_1 = K_p + K_i + K_d \), \( c_2 = -(K_p + 2K_d) \) and \( c_3 = K_d \).

The objective to select the type and tuning parameters of the controller defined by (2) is usually to minimize IAE or ISE of the system response. IAE and ISE defined as (3).

\[
y(k) = u(k) g(q^{-1}) + w(k) h(q^{-1})
\]

(3)

\[
u(k) = e(k) c(q^{-1})
\]

\[
e(k) = r(k) d(q^{-1}) - y(k)
\]

The relationship among process output variable \( y(k) \), controller signal \( u(k) \), set-point value \( r(k) \) and error \( e(k) \) can be described as (3).

\[
\int_{0}^{\infty} e^2(t) dt = \int_{0}^{\infty} e^2(t) dt
\]

(4)

The collinearity (highly correlated) between the inputs and outputs of DEA will not affect the stability and reliability of the DEA model [14]. If a new added criterion has complete linear correlation with existing criteria, i.e., the inputs and outputs of DEA, the analysis results will remain unchanged. So, it is not necessary to consider the collinearity problem when choosing input or output criteria for DEA-related MCDA.

This paper selects the average absolute value of deviation \( |e(k)| \) and average absolute value of deviation change rate \( |\Delta e(k)| \) as the input criteria of the DEA-related MCDA for PID control loop performance assessment. The process variable \( y(k) \) should not be directly used as the output criteria because the control system usually cannot guarantee that \( y(k) \) is proportional to \( |e(k)| \) and \( |\Delta e(k)| \). We consider selecting performance indicator ISE, which can reflect the controlled performance of \( y(k) \) and establish a dominant relationship between \( |e(k)| \) and \( |\Delta e(k)| \), as the output criterion of the DEA-related MCDA for PID control loop performance assessment. Taking into consideration that ISE is passive value, it should adopt the reciprocal ISE as the output criteria. In order to ensure that the input criteria increase does not lead to the output criteria reduction, here we choose \( \frac{1}{\text{ISE}} \) as the output criterion for PID control loop performance assessment.
B. Choice of CLDMU

DMU is used to transfer inputs to outputs. DEA efficiency is the efficiency of the capacity of a certain DMU with respect to the other DMUs. Control loop decision making unit (CLDMU) should have the following characteristics: (a) all CLDMUs have the same goal or task. The controlled variable of the control system is controlled and performance is the best. (b) all CLDMUs have the same external environment. (c) all CLDMUs have the same type of input and output criteria, and the same dimension.

Moreover, data length affects the statistical confidence level for the control loop performance assessment. The longer is the data length, the higher is the confidence level of the criteria for control loop performance assessment. The control loop is required to have same characteristics because the error of the controller is selected for control loop performance assessment. For instance, if the delay $r$ is known, the minimum length of the data is typically selected as $N \geq 150r$ [7]. That is, sampling number $N$ is necessary for CLDMU.

In order to ensure DEA efficiency and reliability, the relationship between the number of CLDMU and the sum of the number of input and output criteria should also follow certain rules: the number of CLDMU should not less than three times the sum of the number of input and output criteria. In addition, the sum of the number of input and output criteria of CLDMU can be up to a maximum of 10 [15].

C. Assessment Algorithms

Suppose there are $n$ CLDMUs. Each CLDMU has two input criteria $[e(k)]$ and $[de(k)]$, and the output criterion is $1 - \frac{1}{\text{ISE}},$ where $[e(k)]$ and $[de(k)]$ are the input criteria of CLDMU, and $1 - \frac{1}{\text{ISE}}$ is the output criterion of CLDMU. Then DEA-related MDCA based on the input-oriented DEA cross-model (with slack variables and non-Archimedean infinitesimal $e$) can be described as (5) [10].

$$\begin{aligned}
\min & \{\theta - e(I_1^+ + I_2^+)\} \\
\sum_{i=1}^{n} \lambda_i x_i + s^- = \theta y_i, \sum_{i=1}^{n} \lambda_i y_i - s^+ = y_i \\
s^- \geq 0, s^+ \geq 0, \lambda_i \geq 0, i = 1, 2, \ldots, n
\end{aligned}$$

(5)

where $x_i = ([e(k)]_{de(k)})$, $y_i = (1 - \frac{1}{\text{ISE}})$, $s^-$ and $s^+$ are the slack variables, $s^- = (s^{1-}, s^{2-})$ represents the redundant amount of input $[e(k)]$ and $[de(k)]$, $s^+ = (s^{1+})$ means the insufficient amount of output $1 - \frac{1}{\text{ISE}}, I_1 = (1, 1)^+,$ $I_2 = (1, 1)^-,\text{ and } \theta,$ represents the efficiency value of CLDMU. In actual applications, $e = 10^{-6}$ [10].

The self-evaluation value ($\theta = E_s(1 \leq i \leq n)$) of DEA-related MDCA can be calculated by (5). If $E_s$ is equal to 1, then CLDMU is effective; if $E_s$ is less than 1, then CLDMU is non-effective. However, the number of the CLDMUs which $E_s$ is equal to 1 may be multiple, therefore (5) cannot be used for the ranking of the effective CLDMUs. Equation (5) can also calculate the redundant amount of input $s^-$ and the insufficient amount of output $s^+$ for the non-effective CLDMUs, which can be applied to guide and improve the input and output of the non-effective CLDMUs.

The ranking of performance is used to determine the priority of process control loops. Meanwhile, such ranking results can guide the operator and the manager to improve the efficiency of operation and management. Therefore, the cross-evaluation DEA-related MCDA is selected to rank, which can avoid using extreme and unreasonable weight distribution for various input and output criteria, in order to effectively distinguish the pros and cons of various CLDMUs [16].

The self-evaluation value ($\theta = E_s(1 \leq i \leq n)$) of CLDMU can be calculated by solving the linear programming (6).

$$\begin{aligned}
\min & \{y^T (y - x^T \omega) \leq 0, j = 1, 2, \ldots, n, \}
\min & \{y^T (y - x^T \omega) \leq 0, j = 1, 2, \ldots, n, \}
y^T (y - x^T \omega) = 1, i \in \{1, 2, \ldots, n\}, k \in \{1, 2, \ldots, n\}
z \geq 0, \omega \geq 0,
\end{aligned}$$

(6)

The cross-evaluation value can be calculated by the optimal solution $\tilde{z}$ and $\tilde{\omega}$ to (7).

$$E_a = \frac{y^T \tilde{z}}{x^T \tilde{\omega}}$$

(7)

where $E_a$ are self-evaluation values and $E_s(k 
eq i)$ are cross-evaluation values. The $k$th row of $E$ is the evaluation values of various CLDMUs to CLDMU. The larger the value, the more excellent the CLDMU. Therefore, the average value $\mu_i$ of the $k$th row of $E$ can be described as the (8).

$$\mu_i = \frac{1}{n} \sum_{k=1}^{n} E_{ik}, i = 1, 2, \ldots, n$$

(8)

To measure the merits of the CLDMU, $\mu_i$ can be regarded as the overall evaluation of the various CLDMUs to CLDMU. The greater $\mu_i$ means CLDMU more excellent.

III. DIAGNOSIS OF CONTROL LOOP PERFORMANCE AND TUNING DIRECTION OF CONTROLLER PARAMETERS

Dependent on the control performance assessment results based on DEA-related MCDA, the following diagnosis can be made for $n$ CLDMUs in the exclusion of the faults of detecting elements, actuators, and other hardware and control software.

If $s^- = s^+ = 0$, from the view of parameter setting point, CLDMU corresponding control loop should be close to the optimal parameter setting. But in practical applications, the $K_p$
value determined by (9) is necessary to diagnose whether the input and output have satisfied the best combination.

\[
K_i = \frac{1}{\theta_i} \sum_{i=1}^{n} \lambda_i, i = 1, 2, \ldots, n
\]  
(9)

where the \( \theta_i \) is the self-evaluation value and \( \lambda_i \) is the weight.

A. Optimal Parameters

The \( K_i = 1 \) represents the input of \( CLDMU_i \) is neither too small nor too large. The controller parameters of the \( i \)th loop have reached the optimization. Tuning controller parameters will not help to improve performance of the \( i \)th control loop.

B. Increasing Proportional and Derivative Action

If \( K_i > 1 \) and \( s^- + s^+ \) is large (typically greater than 0.01 or other specific threshold value which can be defined by a specific application), the controller parameters of the \( i \)th control loop need to be re-tuned. And the \( K_i \) value will decrease if the proportional and derivative action of the \( i \)th controller is increased. Therefore, the performance of the \( i \)th control loop becomes better by appropriately increasing the proportional and derivative action.

C. Decreasing Proportional and Derivative Action

If \( K_i > 1 \) and \( s^- + s^+ \) is small (generally less than 0.01 or other specific threshold value which can be defined by a specific application), the controller parameters of the \( i \)th control loop need to be re-tuned. And the \( K_i \) value will decrease if the proportional and derivative action of the \( i \)th controller is decreased. Therefore, the performance of the \( i \)th control loop becomes better by appropriately decreasing the proportional and derivative action.

IV. CONTROL LOOP PERFORMANCE ASSESSMENT, SORTING AND RANKING

According to the results of the self-evaluation DEA-related MCDA for PID control loop performance assessment, the control loop performance can be assessed and sorted by self-evaluation efficiency values of relative DMUs. The PID control performance can be sorted as poor, better, and the best.

A. Poor or Unacceptable Performance

The performance of the \( i \)th control loop is poor or unacceptable when the self-evaluation efficiency value of \( CLDMU_i \) calculated by (5) is less than \( 1 - \Delta \) and greater than 0, i.e., \( 0 < \theta_i = E_i < 1 - \Delta \), and \( s^- + s^+ \geq \Omega \) (\( \Delta \) and \( \Omega \) are small positive figures, determined based on a specific application). That is to say the performance of the \( i \)th control loop is poor and unacceptable. The input of the \( i \)th control loop can be reduced down to \( \theta_i x_i \) by tuning the controller parameters, which means that the average absolute deviation of the control loop can be reduced down to the \( \theta_i \) times of the original, and the original output \( y_i \) can keep unchanged.

B. Good or Acceptable Performance but Need Improvement

The performance of the \( i \)th control loop is good or acceptable when the self-evaluation efficiency value of \( CLDMU_i \) calculated by (5) is greater than or equal to \( 1 - \Delta \) but less than \( 1 - \Delta \) (\( \Delta \) is a small positive figure and less than \( \Delta \), and it can be determined based on a specific application), and \( 0 < s^- + s^+ < \Omega \), even if \( CLDMU_i \) is DEA non-effective. That is to say the performance of the \( i \)th control loop is good but needs to be improved. The input of the \( i \)th control loop can be reduced \( s^- \), which means that the absolute mean value of the control loop can be reduced \( s^- \), and the original output \( y_i \) can keep unchanged. In case the input of the \( i \)th control loop keeps unchanged, the performance of the \( i \)th control loop, as output \( y_i \), can be improved additional \( s^- \).

C. Better Performance

The performance of the \( i \)th control loop is better and close to the best but does not achieve the best, when the self-evaluation efficiency value of \( CLDMU_i \) is near to 1, i.e., \( 1 - \Delta \leq \theta_i = E_i \leq 1 \), and \( s^- + s^+ = 0 \). That is to say the performance of the \( i \)th control loop is better and close to the best performance (higher efficiency of DMU), but has not yet reached the best performance (the highest efficiency of DMU). Meanwhile, it is not necessary to do anything for improvement.

D. The Best Performance

The performance of the \( i \)th control loop reaches the best when the self-evaluation efficiency value of \( CLDMU_i \) is equal to 1, i.e., \( \theta_i = 1 \), and \( s^- + s^+ = 0 \). That is to say the performance of the \( i \)th control loop is DEA effective. The efficiency value of \( CLDMU_i \) has reached the highest, and the performance of the \( i \)th control loop has reached the best. The output \( y_i \) of \( CLDMU_i \) on the basis of the original input \( x_i \) has obtained the optimal.

E. Control Loop Performance Ranking

The ranking of control loop performance can be determined by the cross-evaluation DEA-related MCDA with \( \mu_y \). The greater \( \mu_y \), the more excellent the performance of \( CLDMU_i \) will be ranked.

DEA control loop performance assessment and diagnosis process is shown in Fig.2.
Chemical process can be represented by the approximate form as first-order-plus-dead-time (FOPDT) process or second-order-plus-dead-time (SOPDT) process. The FOPDT model is selected for this simulation study as follows.

\[ G(s) = \frac{K_p e^{-ts}}{(T_1s + 1)} + w(s) \]  

(10)

where \( K_p = 1 \), \( T_1 = 20 \), \( \tau = 5 \), and \( w(s) \) denotes the white noise driving signal.

The classical Ziegler-Nichols (ZN) critical proportional method [17] is used to tune the PID controller parameters \( K_p, K_c, \) and \( K_d \). The critical proportional gain \( K_p = 6.935 \) and the critical concussion cycle \( T_u = 18.5 \) can be obtained firstly. The PID parameters \( K_p = 4.1610, K_c = 0.25, \) and \( K_d = 2.3125 \) are calculated by the empirical equation of ZN critical proportional method.

### B. Choice of DMU

The 40 response curves have been generated as 40 DMUs by utilizing the empirical equation of Z-N critical proportional method. The interval of the critical proportional change is 0.03, and the change range of the critical proportional is from 0.23 to 1.4.

In order to identify the relationship between the control parameters tuning and the efficiency value, the optimized PID control loop is added as the 41\textsuperscript{st} CLDMU. The method proposed by Chien et al. [18] is adopted to tune the PID parameters, and the optimized PID parameters tuning results are \( K_p = 3.80, K_c = 11.90, \) and \( K_d = 2.10 \).

### C. Simulation Results

The results of control loop performance assessment and diagnosis for 41 control loops are shown in Table I.

### D. Analysis

Due to the paper length, we discuss only the optimal situation. The other situation can be analyzed similarly.

#### Diagnosis of control loop performance and parameter tuning direction

The following diagnosis can be made for the 41 control CLDMUs with \( K \) and \( s^{-ss} \) as shown in Table I.

#### The optimal parameters

The \( K \) values of CLDMU\(_{22} \), CLDMU\(_{23} \), and CLDMU\(_{41} \) are equal to 1, which indicates that the corresponding control loops do not need to tune PID parameters. The inputs are neither too small nor too large, having reached the best settings. The response curves of the 22\textsuperscript{nd}, 23\textsuperscript{nd} and 41\textsuperscript{st} control loops with overshoot, setting time and damping ratio are illustrated in Fig 3.

#### Control loop performance assessment and sorting

According to results calculated by the DEA-related MCDA for 41 control loops in Table I, Fig.4 can be drawn with the limited line 1-\( \Delta_1 \) and 1-\( \Delta_2 \). Further assessment and sorting of 41 control loops performance can be made as below.

### Table I. DEA Control Loop Performance Assessment and Diagnosis Parameters Results

<table>
<thead>
<tr>
<th>DMU</th>
<th>( \theta )</th>
<th>( s^{-ss} + s^ss )</th>
<th>( K )</th>
<th>( \mu )</th>
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Figure 3. The response curves of the 22\textsuperscript{nd}, 23\textsuperscript{nd}, 24\textsuperscript{th} and 41\textsuperscript{st} control loops.

#### The best performance

The self-evaluation values of the control loops 22, 23, 24 and 41 are \( \theta = E_\text{in} = 1 \), and \( s^{-ss} = 0 \). It means that the performance is perfect. The response curves of the 22\textsuperscript{nd}, 23\textsuperscript{nd}, 24\textsuperscript{th} and 41\textsuperscript{st} control loops with overshoot, setting time and damping ratio are shown in Fig. 3.
Ranking of control performance assessment

According to $\mu$ in Table I, the ranking result of PID control loop performance from high to low is shown in Fig.5. Compared with the others, the control loop response curve of the method proposed by Chien et al. showed the best performance.

E. Consistency Verification with ISE Evaluation

In order to further verify the correctness of the control loop performance sorting, the comparison of efficiency values $\mu$ in Table I with the traditional performance indexes ISE are shown in Fig.6.

The result in the Fig.6 demonstrates that the efficiency values of the control loop are consistent with the control loop performance represented by the ISE. The correctness and effectiveness of the proposed method are verified.

VI. CONCLUSION

PID control loop performance evaluation method by DEA-related MCDA can realize the sorting and ranking of the control loop performance. Meanwhile, the optimized slack variables provide the improvement direction and the quantitative space for the control loop performance, and these results can be applied to determine the priorities and improving direction for the control loop PID parameters tuning. The results of the diagnosis and assessment of control loop performance can be mutually suplemental. The effectiveness and efficiency of the proposed method are verified by simulation examples.

The advantages of the proposed method include as follows:
(a) It is a data-driven approach, which can be implemented with available online closed loop operation data; (b) the explicit model is not required; (c) the control loop performance assessment and diagnosis can be achieved in parallel; (d) the input and output are not sensitive to the dimension and the coupling; (e) the performance sorting and ranking information can be obtained by DEA-related MCDA for control loop performance assessment.

REFERENCES