A Combined LQR-MPC robust strategy for the control of a Fluid Catalytic Cracking Unit

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Abstract— In this paper, the model predictive control of uncertain process systems with stable and unstable outputs that allows input saturation is tested through the simulation of the control of a Fluid Catalytic Cracking (FCC) Unit. A sub-set of the manipulated inputs is allocated to the control of the unstable outputs through a state feedback control law while the complementary sub-set of inputs are let free to control the remaining stable outputs through a conventional MPC framework. In the approach proposed in this work, it is assumed that all the outputs are controlled inside zones so that the unstable output set-points can become free manipulated inputs in place of the inputs selected to control the unstable outputs. The new controller allows the saturation of the inputs related to the unstable outputs through the manipulation of the free inputs and the set-points of the unstable outputs. The approach is extended to the case where model uncertainty can be represented by a finite set of linear models.

I. INTRODUCTION

The FCC system is a typical multivariable chemical process that, depending on the process configuration, can exhibit a complex dynamic behavior with several integrating outputs and nonlinearity. In industry, the control of FCC is usually based on model predictive control (MPC), which calculates at each time step a sequence of manipulated inputs that optimizes the predicted behavior of the system, usually subject to constraints on the inputs and on the outputs.

In the unconstrained case, MPC becomes equivalent to the Linear Quadratic Regulator (LQR) for which the solution to the optimization problem is a state feedback control law that can be expressed as $u(k+i) = K(k)x(k+i)$, $i \geq 0$. In the constrained case, the approach does not usually lead to a linear control law and because the control moves are not only a function of the actual state of the plant, they are denominated free control moves. In real MPC implementations, stability can be achieved for the nominal system but not in the general case when there are uncertainties associated with the unstable modes of the system.

In [1], it is proposed a strategy that is an extension of the LQR approach to the constrained case. In their work, conservative LMI constraints on the inputs and outputs are included so that the solution to the constrained case becomes a state feedback control law as in the unconstrained case. With their approach, the closed-loop stability can be achieved for the uncertain stable and unstable systems. Although the approach has opened the gate to several developments in the field of robust MPC and has been improved over time, it still suffers from some limitations as: the conservative way the constraints are implemented reduces the attraction domain of the controller; the zone control strategy cannot be directly addressed and the large number of decision variables impacts the computational burden of the robust MPC of large systems.

In this paper, the MPC with free control moves and the state feedback control strategies are integrated. In the proposed approach a sub-set of the manipulated inputs of the system is allocated to control the unstable outputs through a state feedback control law, while the other manipulated inputs are left free to control the remaining stable outputs of the system through a MPC framework.

Focusing on the implementation of the proposed approach on real systems of the process industry, the so-called zone control strategy [2] is considered here, and it is assumed that an upper layer in the control structure defines economic targets for some of the inputs manipulated by the MPC. In this strategy, the set-points of the stable outputs are treated as additional decision variables that can be varied inside the output zones, while the set-points of the unstable outputs actually become new manipulated inputs in place of the inputs chosen to control the unstable outputs.

With the proposed approach, the unstable outputs are controlled internally through the state feedback control law with variable set points, while the stable outputs are treated as the controlled outputs in the MPC framework. In the state space representation of the system, the closed-loop with the state feedback can be represented as a stable sub-system and the effect of the free inputs that are manipulated by the MPC on the unstable outputs can be included. Then, the unstable outputs are also added to the set of controlled outputs of the MPC.

Based on this formulation, the resulting controller can benefit from the advantages of both, the MPC, where constraints can be easily implemented and the state feedback control law, where robustness to model uncertainty of unstable systems has been successfully implemented. As a result, the method can deal with a larger class of model uncertainties on the unstable modes than the free control moves strategy, and unlike the state feedback control strategy, the input target and zone control strategy can be considered. Also, the temporary saturation of the inputs manipulated by the LQR is allowed and the
domain of attraction of the controller is maximized by the introduction of a set of slack variables.

In this work, it is assumed that the model state is measured as in the realigned state model in which the state is composed of the past measured outputs and inputs of the system [2].

The proposed robust controller is tested through the simulation of the control of a Fluid Catalytic Cracking unit.

II. METHODOLOGY

A. The process model for the combined LQR-MPC

Consider a controllable system that can be represented by a realigned state space model in the incremental form as in [3]. Then, the model considered here can be written as follows:

\[
\begin{align*}
x(k + 1) &= Ax(k) + B\Delta u(k) \\
y(k) &= Cx(k)
\end{align*}
\]  

(1)

where \( x \in \mathbb{R}^{n_x} \), \( u \in \mathbb{R}^{n_u} \), \( \Delta u(k) = u(k) - u(k - 1) \), \( y \in \mathbb{R}^{n_y} \) and \( A, B \) and \( C \) are matrices of appropriate dimensions.

Here, one considers systems, as the FCC system, with integrating and stable outputs, which are quite common in the process industry. It is assumed that the system dynamics can be partitioned into two sub-systems corresponding to the stable and unstable outputs respectively. Then, it will be assumed that the system represented in (1) can be partitioned into the stable sub-system:

\[
\begin{align*}
x_s(k + 1) &= A_s x_s(k) + B_s \Delta u(k) \\
y_s(k) &= C_s x_s(k)
\end{align*}
\]  

(2)

where \( x_s \in \mathbb{R}^{n_{xs}} \) and \( y_s \in \mathbb{R}^{n_{ys}} \), and the unstable sub-system:

\[
\begin{align*}
x_u(k + 1) &= A_u x_u(k) + B_u \Delta u(k) \\
y_u(k) &= C_u x_u(k)
\end{align*}
\]  

(3)

where \( x_u \in \mathbb{R}^{n_{xu}} \) and \( y_u \in \mathbb{R}^{n_{yu}} \).

Suppose now that a sub-set of the inputs \( \tilde{u} \in \mathbb{R}^{n_u} \) is selected to stabilize the unstable outputs. This set of inputs defines the input matrix \( B_{u,1} \). Then, under the condition that the pair \( (A_u, B_{u,1}) \) is controllable, suppose that the unstable outputs of the system are controlled through the following state feedback control law:

\[
\Delta \hat{u}(k) = F [x_u(k) - x_u^{sp}(k)]
\]  

(4)

where \( x_u^{sp}(k) \) is the set-point of the corresponding unstable state at time step \( k \) and \( F \) is the state feedback gain calculated by extending the method presented in [1] to a state space model in the incremental form.

Also, assume that the complementary sub-set of the inputs \( \bar{u} \in \mathbb{R}^{n_u-n_{yu}} \) are used as free manipulated inputs by a MPC that controls the remaining stable outputs of the systems.

Based on the input sub-sets defined above, (3) can also be written as

\[
x_u(k + 1) = A_u x_u(k) + B_{u,1} \Delta \tilde{u}(k) + B_{u,2} \Delta \bar{u}(k)
\]  

and using (4), the above equation can now be represented as follows:

\[
x_u(k + 1) = [A_u + B_{u,1}F]x_u(k)
\]

\[
-B_{u,1}Fx_u^{sp}(k) + B_{u,2} \Delta \bar{u}(k)
\]  

(5)

Similarly, considering the unstable states feedback, the stable sub-system (2) can be written as follows:

\[
x_s(k + 1) = A_s x_s(k) + B_{s,1}Fx_u(k)
\]

\[
-B_{s,1}F x_u^{sp}(k) + B_{s,2} \Delta \bar{u}(k)
\]  

(6)

When the system model is written in the output realigned form as in [3], the vector of set-points of the unstable states can actually be expressed as \( x_u^{sp}(k) = I_n \tilde{y}_u^{sp}(k) \) where:

\[
\tilde{I} = \begin{bmatrix}
I_{n_{yu}} & \cdots & I_{n_{yu}} & 0_{n_{yu},n_{yu}} & \cdots & 0_{n_{yu},n_{yu}}
\end{bmatrix}_{n_{yu} \times n_{yu}}^T
\]

and \( n_{yu} \) are the orders of the difference equation model or transfer function corresponding to the system.

Then, based on (5) and (6), a state space representation of the whole system in the incremental form is:

\[
\hat{x}(k + 1) = \hat{A}\hat{x}(k) + \hat{B}\Delta \hat{u}(k)
\]

\[
y(k) = \hat{C}\hat{x}(k)
\]  

(7)

where \( \hat{x}(k) = \begin{bmatrix} x_s(k) \\ x_u(k) \end{bmatrix} \), \( \Delta \hat{u}(k) = \begin{bmatrix} \Delta \bar{u}(k) \end{bmatrix} \),

\[
\hat{A} = \begin{bmatrix}
A_u + B_{u,1}F & 0 \\
B_{s,1}F & A_s - B_{s,1}F \tilde{I}_{n_s}
\end{bmatrix},
\hat{B} = \begin{bmatrix}
B_{u,2} \\ B_{s,2}
\end{bmatrix}
\]

and

\[
\hat{C} = \begin{bmatrix}
C_u & 0 \\ 0 & C_s
\end{bmatrix}.
\]
Observe that, as the zone control strategy is adopted, the model state has to be extended with \( y_u^{sp}(k) \) and \( \Delta y_u^{sp}(k) \) becomes a manipulated input of the MPC that controls the stable outputs. Also, one should emphasize that, in this model formulation, the unstable outputs are internally controlled through the state feedback control law.

B. Robust LQR-MPC for systems with stable and unstable outputs

In this section, the robust LQR-MPC for systems with stable and unstable outputs is presented. The system to be controlled is represented by the model defined in (7) where the uncertainty concentrates on matrices \( A, B_{u,1}, B_{u,2}, B_{s,1}, B_{s,2} \). For the case of the FCC system, the multi-model uncertainty is considered and the set of possible plants is defined as \( \Omega = \{ \Theta_1, \ldots, \Theta_N \} \) where each element of this set corresponds to a different model of the system that represents the process system at an operating point and is parameterized by \( \Theta_N = (A, B_{u,1}, B_{u,2}, B_{s,1}, B_{s,2}) \). Assume that \( \Theta_N \) corresponds to the nominal or most probable model of the system.

One first computes the state feedback control gain that will be used to control the unstable outputs of the system adapting the method proposed in [1] for the unconstrained case to the state space model of the system:

\[
x_{u,\Theta_N}(k+1) = A_{u,\Theta_N} x_{u,\Theta_N}(k) + B_{u,1,\Theta_N} \Delta u_{\Theta_N}(k) + B_{u,2,\Theta_N} \Delta v_{\Theta_N}(k)
\]

\[
y_{u,\Theta_N}(k) = C_{u,\Theta_N} x_{u,\Theta_N}(k)
\]

Note that the hard constraints on \( \hat{u} \) will be dealt with in the MPC framework. With the state feedback law considered above, the unstable outputs can be robustly stabilized. Now, to control the stable outputs, let us define for any given model \( \Theta_N \) the following cost function [4]:

\[
V_{k,\Theta_N} = \sum_{j=0}^{\infty} \left[ y_{u,\Theta_N}(k+j | k) - y_u^{sp}(k+j | k) - \delta_{y,k,\Theta_N} \right]^2_{Q_y} + \sum_{j=0}^{m-1} \left[ \hat{u}(k+j | k) - \hat{u}_{\text{des}, -k} - \delta_{\hat{u},k,\Theta_N} \right]^2_{Q_{\hat{u}}}
\]

\[
\delta_{y,k,\Theta_N} = T_{y,k} \delta_{y,k,\Theta_N} + \delta_{\hat{u},k} T_{\hat{u},k} \delta_{\hat{u},k,\Theta_N} + \lambda_{\hat{u}} T_{\hat{u},k} \lambda_{\hat{u},k}
\]

where \( Q_y, Q_{\hat{u}}, R, S_y, S_{\hat{u}} \) and \( S_{\lambda} \) are positive weighing matrices of appropriate dimensions and \( \hat{u}_{\text{des}, -k} \) is the free input target. In the cost defined in (8), \( \delta_{y,k,\Theta_N} \) and \( \lambda_{\hat{u}} \) are slack variables that extend the domain of attraction of the controller to the whole definition set of the states. One can note that the slack variables are meant to go to zero if the input targets and the output set-points are reachable. Then the robust LQR-MPC for systems with stable and unstable outputs is obtained through the solution to the following problem:

**Problem 1**

\[
\min_{\hat{u}, \delta_{y,k,\Theta_N}} \sum_{j=0}^{\infty} \left[ y_{u,\Theta_N}(k+j | k) - y_u^{sp}(k+j | k) \right]^2_{Q_y} + \sum_{j=0}^{m-1} \left[ \hat{u}(k+j | k) - \hat{u}_{\text{des}, -k} \right]^2_{Q_{\hat{u}}}
\]

subject to

\[
\delta_{y,k,\Theta_N} = T_{y,k} \delta_{y,k,\Theta_N} + \delta_{\hat{u},k} T_{\hat{u},k} \delta_{\hat{u},k,\Theta_N} + \lambda_{\hat{u}} T_{\hat{u},k} \lambda_{\hat{u},k}
\]

where \( \delta_{\hat{u},k} \) and \( \lambda_{\hat{u}} \) are slack variables that extend the domain of attraction of the controller to the whole definition set of the states. One can note that the slack variables are meant to go to zero if the input targets and the output set-points are reachable. Then the robust LQR-MPC for systems with stable and unstable outputs is obtained through the solution to the following problem:

**Problem 1**

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\]

subject to

\[
\delta_{y,k,\Theta_N} = T_{y,k} \delta_{y,k,\Theta_N} + \delta_{\hat{u},k} T_{\hat{u},k} \delta_{\hat{u},k,\Theta_N} + \lambda_{\hat{u}} T_{\hat{u},k} \lambda_{\hat{u},k}
\]

III. RESULTS AND DISCUSSION

The control of the Fluid Catalytic Cracking Unit with the robust LQR-MPC is tested here and in the control structure simulated here, the controlled variables are
\( y_1 \) (m/s) the CO boiler flue gas velocity, \( y_2 \) (kgf/cm\(^2\)) the spent catalyst valve differential pressure, \( y_3 \) (°C) the regenerator dense phase average temperature and \( y_4 \) (°C) the regenerator dilute phase average temperature. The manipulated inputs are \( u_1 \) (m/d) the gas oil feed flow rate, \( u_2 \) (°C) the riser feed temperature, \( u_3 \) (°C) the reaction temperature, \( u_4 \) (m\(^3\)/d) the riser feed naphtha flow rate and \( u_5 \) (kN\(\cdot\)m\(^3\)/d) the air flow rate.

To simplify the problem, only three models are considered and they are meant to correspond to different operating points of the system. These models are assumed to constitute the set of models \( \Omega \) that the robust controller will be based on. The transfer functions relating the inputs and the outputs corresponding to these models, based on experimental data of the real process plant, are as follows:

\[
G_s(s) = \begin{bmatrix}
3 \times 10^{-4} & 4.33 \times 10^{3} & 2.33 \times 10^{2} & 1.05 \times 10^{4} & -4.89 \times 10^{-4} \\
(3.9 - 0.6) \times 10^{-3} & 5 \times 10^{-1} & -3.33 \times 10^{-2} & -3.8 \times 10^{4} & 5.7 \times 10^{4} \\
(30 + 10)(10 + 1) & 22 + 1 & 14 + 1 & 48 + 1 \\
-2.43 \times 10^{-3} & 9 \times 10^{-2} & -9.2 \times 10^{-2} & -1.9 \times 10^{-3} & 1.2 \times 10^{-2} \\
-2.29 \times 10^{-3} & 3.32 \times 10^{-2} & -0.227 & -3.3 \times 10^{-2} & 3.78 \times 10^{-1}
\end{bmatrix}
\]

\[
G_s(s) = \begin{bmatrix}
3.4 \times 10^{4} & 5 \times 10^{-2} & 1.9 \times 10^{-2} & 1.2 \times 10^{4} & -4 \times 10^{-4} \\
(3.12 - 0.48) \times 10^{-3} & 5 \times 10^{-1} & -5.6 \times 10^{-2} & -3.10^{-4} & 6.3 \times 10^{-4} \\
(25 + 1)(8 + 1) & 25 + 1 & 18 + 1 & 42 + 1 \\
-2.9 \times 10^{-3} & 11 \times 10^{-2} & -6 \times 10^{-2} & -1.5 \times 10^{-3} & 1 \times 10^{-2} \\
-2.8 \times 10^{-3} & 3 \times 10^{-2} & -0.18 & -2.7 \times 10^{-2} & 4.5 \times 10^{-1}
\end{bmatrix}
\]

From the above transfer functions, one can notice that \( y_1 \), \( y_3 \) and \( y_4 \) are integrating functions of the five inputs of the system. Then, following the approach proposed here, \( u_1 \), \( u_3 \) and \( u_4 \) are selected to control these integrating outputs through a state feedback control while the stable output \( y_2 \) is controlled through the robust MPC defined in Problem P1, in which the actual manipulated inputs are \( y_1^p \), \( y_3^p \), \( y_4^p \), \( u_2 \) and \( u_5 \). The weighting matrices used to calculate the state feedback control gain are

\[
Q_{LQR} = \text{diag} \left( \begin{bmatrix} 10^{-2} \cdots 10^{-2} \end{bmatrix} \right)
\]

In the simulation shown in this section, the real plant \( \Theta_r \) is represented by \( G_s(s) \) and the nominal plant \( \Theta_N \) by \( G_s(s) \).

The constraints on the inputs and the initial output zones are the following:

**Input constraints:**

\[
u_{\min} = \begin{bmatrix} 6000 & 170 & 520 & 500 \end{bmatrix} \text{T},
\]

\[
u_{\max} = \begin{bmatrix} 9000 & 300 & 550 & 1600 \end{bmatrix} \text{T},
\]

\[
\Delta u_{\max} = \begin{bmatrix} 50 \end{bmatrix} \quad \Delta u_{\min} = \begin{bmatrix} 3 \end{bmatrix} \quad \Delta u_{\min} = \begin{bmatrix} 3 \end{bmatrix} \quad \Delta u_{\min} = \begin{bmatrix} 3 \end{bmatrix} \quad \Delta u_{\min} = \begin{bmatrix} 3 \end{bmatrix} \quad \Delta u_{\min} = \begin{bmatrix} 3 \end{bmatrix}
\]

**Output zones:**

\[
y_{\min} = \begin{bmatrix} 0 & 0.4 & 630 \end{bmatrix} \text{T},
\]

\[
y_{\max} = \begin{bmatrix} 30 & 1.1 & 725 \end{bmatrix} \text{T}.
\]

The tuning parameters of the robust LQR-MPC are the following:

\[
Q = \text{diag} \left( \begin{bmatrix} 10^{-3} & 10^{-3} & 10 & 10 \end{bmatrix} \right),
\]

\[
Q_{\pi} = \text{diag} \left( \begin{bmatrix} 1 & 0 \end{bmatrix} \right),
\]

\[
S_{\pi} = \text{diag} \left( \begin{bmatrix} 10^4 \end{bmatrix} \right) \quad \text{and} \quad S_{\pi} = \text{diag} \left( \begin{bmatrix} 10^3 & 10^3 & 10^3 \end{bmatrix} \right).
\]

At time instant \( t = 0 \) min, the system starts from the steady-state defined by \( y_0 = \begin{bmatrix} 1 & 1.2 \end{bmatrix} \text{T}, \text{and} \ u_0 = \begin{bmatrix} 7500 & 220 & 537 \end{bmatrix} \text{T} \). In the first part of the simulation, the proposed controller aims at driving the system to a new steady state defined by \( u_{2,\text{des}} = 225 \) and stabilizing the outputs inside the zones defined above. Figures 1 and 2 show that, with the proposed tuning parameters, the robust combined LQR-MPC successfully manages to drive the system to the input target while the controlled outputs stabilize inside their predefined boundaries. At time instant \( t = 150 \) min, the bounds of the output zones are moved to \( y_{\min} = \begin{bmatrix} 6 & 0.4 & 630 \end{bmatrix} \text{T} \) and \( y_{\max} = \begin{bmatrix} 30 & 1.1 & 700 \end{bmatrix} \text{T} \) while the input target remains unchanged. It can be seen in figures 1 and 2 that again the robust LQR-MPC manages to successfully drive the system to the input target while the controlled outputs stabilize inside their predefined boundaries.
Figure 1: Input targets (blue dashed line), inputs limits (black dashed lines) and inputs with the robust LQR-MPC (blue solid line)
In this work, it was addressed the control of a FCC process system through a robust controller that considers the zone control of the outputs and optimizing targets for the inputs of the process system, which presents stable and unstable outputs. The controller allows the saturation of the inputs that are used to control the unstable modes. In the new state space representation on which the controller is based, a sub-set of the manipulated inputs is assigned to the control the unstable outputs of the system through a state feedback control law, while the remaining inputs are let free to control the stable outputs of the system. In this configuration, the unstable outputs set points become manipulated variables of the system and are allowed to temporarily leave their predefined bounds when unfeasibility on the input constraints may occur. The proposed controller showed a good performance in the simulation study of the control of the industrial Fluid Catalytic Cracking Unit.

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REFERENCES


