Fault Isolation based on General Observer Scheme in Stochastic Non-linear State-Space Models Using Particle Filters

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Abstract—We utilize the particle filter algorithm to develop a fault isolation approach based on general observer scheme (GOS) in nonlinear and non-Gaussian systems. The proposed fault isolation scheme is based on a set of parallel particle filters each sensitive to all faults except one. The performance of the proposed approach is compared to an alternative approach called dedicated observer scheme (DOS) with respect to the measurement and state noise variances. The proposed scheme is also illustrated through an implementation on a benchmark polyethylene reactor system.

I. INTRODUCTION

FDI is a significant and challenging problem in the modern chemical engineering discipline. As the demand of high quality chemical products increases, more and larger chemical processes in order to meet that global energy demand. As consequence the control systems associated with those processes become more and more complicated. While automation can enhance safety, improve reliability, and increase profitability [15]. At the same time, it increases vulnerability of the processes to control system failures which have direct impact to the safety of human beings, economy, and environment pollution.

If the abnormal process behaviours are well diagnosed and well dealt with, the US petrochemical industry alone could save annually up to $10 billion [12]. [16] reported that the same industry looses over $20 billion per year due to inappropriate reaction to abnormal process behavior. So, in order to meet safety standards and reduce the environmental impacts, it is important that the faults are diagnosed as soon as possible before they lead to disasters [10].

A few incidents that have direct impacts to economy, environment, and human lives are listed below [10].
- Bhopal disaster, India, 1984
- Piper Alpha disaster, Scotland, 1988
- British Petroleum (BP) disaster, Gulf of Mexico, 2010

However, these incidents can not be completely prevented but at least the consequences of faults could be avoided using a suitable fault diagnosis system. The FDI system should have the ability to detect any variation from the nominal behavior of the process and give enough time to take corrective action before the disaster can take place [19]. As consequence, the FDI community is putting more attention in developing a reliable fault diagnosis system.

The main objective of any FDI method is to monitor the system operation and to raise an alarm when any change occurs in the process and to determine the location and time of the change [19]. Essentially, the model-based fault detection and isolation schemes are carried out using two steps. The first step is the residual generation which is not easy to design specially when the process has unmeasured state variables [14]. Usually the residuals are generated assuming that the model being used is linear and the noise is Gaussian [7]. Furthermore, suboptimal filters such as extended Kalman filters (EKF), and unscented Kalman filters (UKF) are used when the system being model is assumed to be nonlinear. These filters are not often satisfactory and usually lead to high missed alarm and false alarm rates. In this work, an algorithm called a particle filter is proposed in the design of the model-based fault isolation. It is based on the sequential monte carlo method (SMC) and does not need any linearization of the process or the Gaussianity of noise [15].

Recently, more attention has been paid to FDI problems in nonlinear systems due to the increasing demand for higher safety and reliability of chemical plants. As well as the growth in computational capabilities which has made statistical intensive methods such as sequential Monte Carlo techniques more practical. For more details of the use of the SMC on FDI problem, one can read ([14], [15], [9], [17], [11]). In this paper, we utilize the power of particle filter to compare the robustness of the FDI approach based on general observer scheme against the dedicated observer scheme in non-linear/non-Gaussian stochastic systems.

This paper is structured as follows. In Section II, the model-based fault isolation problem is formulated. In Section III, the particle filter filter algorithm used to generate the residuals is discussed. A brief description of general observer scheme used for fault isolation, is given in section IV. In Section IV-A, the performance of the the proposed algorithm is tested on a poly ethene reactor systems. Lastly, some conclusions and future work is presented in Section V.

II. PROBLEM STATEMENT

This work is an extension to our previous work [2] where we assumed that there are $N$ possible known faults that may occur in the process and there are $N+1$ models $\{M_i\}_{i=1}^{N}$, where $M_0$ corresponds to the nominal process model and $M_i$, each sensitive to all faults except one. The performance of the proposed approach is compared to an alternative approach called dedicated observer scheme (DOS) with respect to the measurement and state noise variances. The proposed scheme is also illustrated through an implementation on a benchmark polyethylene reactor system.
for \( i = 1, 2, \ldots, N \), represents the \( i \)th faulty model. The FDI approach considered in [2] uses a bank of particle filters running in parallel where each model \( \{ M_i \}_{i=1}^N \) is designed to be sensitive to a known single fault and excited by all output measurements \( y_i \) which is known as dedicated observer scheme (DOS) [18]. In this work we consider a different approach know as general observer scheme (GOS) where every model is excited by all outputs except one which is the sensor to be monitored. The standard design procedure of any FDI approach consists of the following two steps:

- Fault detection (FD): which takes a decision on the occurrence of any deviation in the nominal model, \( M_0 \), to a corresponding known faulty models \( \{ M_i \}_{i=1}^N \) and determine the time of occurrence.
- Fault isolation (FI): which determines which \( \{ M_i \}_{i=1}^N \) of the possible known faulty models has happened.

The process dynamics and the known possible faults being monitored on the system can be designed using the following general discrete stochastic nonlinear state space model:

\[
\begin{align*}
    x_k^i &= f^i(x_{k-1}^i, u_{k-1}^i, v_k^i, \theta^i) \\
    y_k &= g^i(x_k, u_k^i, \omega_k^i, \theta^i) \quad \text{for } i = 0 \text{ to } N
\end{align*}
\]

(1)

(2)

\( f^i \) and \( g^i \) represent the state and measurement dynamic functions, respectively. \( k \) denotes a time instant. \( x_k \) is the hidden state vector while \( y_k \) is the measurements vector. The hidden state vector is assumed to have a known initial probability density function \( p(x_0^i) \). The state and measurement noise sequences are defined respectively as \( v_k^i \) and \( \omega_k^i \) with known probability density functions with zero mean. The vector \( \theta^i \) represents a vector of constant values as well as other process measurement which are assumed to be constant. The measurement and the state noises are assumed to enter the process in a linear manner while in the classical FDI approaches are assumed to enter in linear fashion. Therefore the measurement equation (Eq.2) can be written as,

\[
y_k = g^i(x_k, u_k^i, \omega_k^i, \theta^i)
\]

(3)

The fault can be simply detected and isolated by generating the residuals which are the differences between the process output and the predicted output. The one step-ahead predictions from (Eq.3) can be written as,

\[
\hat{y}_k^i = g^i(x_{k|k-1}^i, u_k^i, \theta^i)
\]

(4)

where \( x_{k|k-1}^i \) is the one step-ahead prediction of the state, \( \hat{y}_k^i \) is the one-step ahead prediction of the output. Then the prediction error or the residual can be simply written as

\[
\hat{r}_k^i = y_k - \hat{y}_k^i.
\]

(5)

In the case that there is no fault the residual will be equal to zero or more precisely the measurement noise encountered in the process \( \theta^i \). The residuals are usually evaluated using on of the statistical techniques i.e. cumulative sum test statistic (CUSUM), sequential probability ratio test (SPRT), generalized log-likelihood ration (GLR), or log-likelihood ratio (LLR).

### III. PARTICLE FILTER

The main idea behind the particle approximations is to generate a number of samples of random variables from an importance density function with the same or larger density function. The density function is then approximated using a sum of Dirac delta functions weighted appropriately. For instance, a target density function \( p(x) \) with a random variable \( x \) can be approximated by sampling \( x^{(i)} \) from an important density function \( q(x) \) and then approximating the target density as

\[
p(x) = \sum_{i=1}^N w^{(i)} \delta(x - x^{(i)})
\]

(6)

where \( N \) is the number of particles generated and \( w^{(i)} \) are appropriate weights. This idea can be easily extended to find the density function of the hidden states given a series of measurements up to the current time instant. outputs, \( p(x_t|y_{1:t}, \theta) \) is called a filter. Applying Bayes’ rule in a straightforward manner, one can derive recursive expressions for the density function of the filter. The following predictor density function can be derived using Bayes’ rule,

\[
p(x_t|y_{1:t-1}, \theta) = \int p(x_t|x_{t-1}, \theta) p(x_{t-1}|y_{1:t-1}, \theta) dx_{t-1}
\]

(7)

Now using the predictor and (7), one can write the following expression for the filter,

\[
p(x_t|y_{1:t}, \theta) = \frac{p(y_t|x_t, \theta)p(x_t|y_{1:t-1}, \theta)}{\int p(y_t|x_t, \theta)p(x_t|y_{1:t-1}, \theta) dx_t}
\]

(8)

The filter density can be evaluated recursively by substituting (7) in (8). The above integrals needed to estimate the filter density are often intractable, and need to be approximated. Although numerous approximations are available, in this paper a particle filter approach is used. The basic idea behind particle filters is to approximate a density function using dirac-delta functions. The filter density at \( t-1 \), could be approximated as

\[
p(x_{t-1}|y_{1:t-1}, \theta) = \sum_{i=1}^N w_{i|t-1}^{(i)} \delta(x_{t-1} - x_{t-1}^{(i)})
\]

(9)

where \( w_{i|t-1}^{(i)} \) are weights proportional to the filter density at \( x_{t-1}^{(i)} \) and \( \delta(\cdot) \) is a dirac-delta function. Substituting (9) in (7), an approximation of the predictor can be obtained as follows,

\[
p(x_t|y_{1:t-1}, \theta) = \sum_{j=1}^N p(x_t|x_{t-1}^{(j)}, \theta) w_{t-1|t-1}^{(j)}
\]

(10)
Similarly, substituting (10) in (8), one can approximate the
filter density function ([13])
\[ p(x_t | y_{1:t}, \theta) = \frac{p(y_1 | x_{1:t-1}, \theta)}{\sum_{i=1}^{N} p(y_1 | x_{1:t-1}, \theta)} \]
where \( x^{(i)}_{t-1} \) are chosen from an importance sampling function
\[ p(x_{t-1} | y_1, \theta) \], and therefore weights are given by
\[ w^{(i)}_t = \frac{p(y_t | x^{(i)}_t, \theta)}{\sum_{j=1}^{N} p(y_t | x^{(j)}_t, \theta)} \]

Particle Filter Algorithm

1. **Initialization**: Generate \( N \) samples of the initial state
   \( x_1 \) from an initial distribution, \( p(x_1) \). Set \( w^{(i)}_1 = \frac{1}{N} \) for
   \( i \in \{1, \ldots, N\} \). Set \( t = 2 \).
2. **Prediction**: Sample \( N \) values of \( x_t \) from the distributions
   \( p(x_t | x_{t-1}, \theta) \) for each \( i \).
3. **Update**: Using (12), find the weights of filter density,
   \( w^{(i)}_t \).
4. **Resampling**: Resample \( N \) particles from the set
   \( \{ x^{(1)}_t, \ldots, x^{(N)}_t \} \) with the probability of picking \( x^{(i)}_t \)
   being \( w^{(i)}_t \). Assign \( w^{(i)}_t = \frac{1}{N} \) for all \( i \).
5. Set \( t = t + 1 \). Repeat the above steps (2), (3), and (4)
   for \( t \leq T \).

IV. PROPOSED ALGORITHM

In this work, the fault detection algorithm used identify
any changes in the model is taken from [2] by monitoring
the vector \( \theta \) which incluse process parameters and other
process variables that are assumed to be constant.

Once the fault is successfully detected, then the fault must
be isolated in order to locate a particular fault from others
within a monitored system. Basically, fault is detected using
a single residual set, however, model-based fault isolation
can be accomplished using a bank of residuals based on one
of the following two frameworks:

- Structure residual
- Directional residual

In this work we used the structure residual approach in
isolating the faults. The main idea behind this approach
is to use a bank of structured residuals instead of one
residual. Those residuals are designed in such a way that
each sensitive to some faults while insensitive to other faults.
Basically, two steps are required to design and implement
this approach. First, is to appoint the relationship between
the residuals based on the sensitivity and insensitivity to
different faults that may occur in the process. Second, is to
design residual generators based on the relationship specified
in the first step [1]. The structure residual approach can be
designed in two different ways: dedicated residual scheme
and general residual scheme.

A. Dedicated residual scheme

In dedicated residual scheme which was introduced by [5], one measurement is fed into each residual generator which are designed to be only sensitive to single faults [4] as shown in Fig.1 or to be sensitive to all faults except one [18]. It is also well known in literature as dedicated observer scheme (DOS). There are two restrictions arise in this type of multiple observer/filter state estimation based FDI scheme. First, since each observer/filter in the scheme is driven by only one output measurement, the states of the process should be completely observable through each sensor or actuator, which is not always the case in practical applications. Second, multiple and simultaneous faults are difficult to identify specially in large processes [8]. If all possible faults to be isolated, a dedicated residual set can be designed according to the following fault sensitive condition:
\[ r_i(t) = G(f_i(t)); \quad i \in \{1, 2, \ldots, N\} \]
where \( G(\cdot) \) stands for a function relation and \( N \) is
the number of fault to be isolated within the process.
The following threshold logic as in [1] will be used to
decide if there is any fault occur in the system:
\[ r_i(t) > \xi_i \quad \Rightarrow \quad f_i(t) \neq 0 \]
where \( \xi_i (i = 1, 2, \ldots, N) \) are predetermined thresholds
for each residual \( r_i(i = 1, 2, \ldots, N) \). The threshold
values selected in a way that the false alarm and missed
alarm rates are minimized.
Furthermore, in [3], the authors have extended the
work done by Clarks to actuator fault isolation using
exactly the same principkle.

B. General residual scheme

An alternative approach to dedicated residual scheme
is the general residual scheme which is also known as
general observer scheme (GOS). In this approach every
residual is designed to be sensitive to all faults except one
[1] i.e.
\[ \begin{align*}
  r_1(t) &= G(f_2(t), \ldots, f_N(t)) \\
  \vdots \\
  r_i(t) &= G(f_1(t), \ldots, f_{i-1}(t), f_{i+1}(t), \ldots, f_N(t)) \\
  \vdots \\
  r_N(t) &= G(f_1(t), \ldots, f_{N-1}(t))
\end{align*} \]
The isolation task can be archived using simple threshold
testing according to the following logic:
\[ r_i(t) \leq \xi_i \quad \Rightarrow \quad f_i(t) \neq 0 \]
In this paper, we will utilize the particle filter approxima-
tion to examine the robustness of the above two schemes
in terms of state and measurement noise variances using a
polyethylene Reactor System.
A. Application to A Polyethylene Reactor System

1) Process Description: The example considered in this paper is taken from [6]. Ethylene, comonomer, hydrogen, inert, and catalyst are fed to the reactor at a temperature of $T_{\text{feed}}$ as shown in Fig.3. The unreacted gases are cold down using a cold-water heat exchanger which are then fed back to from the top of the reactor. The cooling rates are adjusted by mixing both the cold and warm water streams. All the definition of all parameters and process variables used in equations (17) and (18) are given in [6] and the steady state data of the reactor process is given in Table I. Two manipulated inputs are considered in this study which are the feed temperature $T_{\text{feed}}$, and the inlet flow rate of ethylene $F_M$. A mathematical model that describe the dynamic behaviour of the reactor system are derived using mass and energy balances can take the following form:

\[
d\ln[M_1] = \frac{d[M_1]}{dt} = \frac{F_{\text{in}} - [\ln]}{V_g} \cdot b_t
\]
\[
d\ln[M_1] = \frac{d[M_1]}{dt} = \frac{F_{\text{in}} - [\ln]}{V_g} \cdot b_t - R_{M1}
\]
\[
dY_1 = \frac{dY_1}{dt} = F_c \cdot a_c - k_d \cdot Y_1 - \frac{R_{M1} \cdot M_{\text{w1}} \cdot Y_1}{B_w}
\]
\[
dY_2 = \frac{dY_2}{dt} = F_c \cdot a_c - k_d \cdot Y_2 - \frac{R_{M1} \cdot M_{\text{w1}} \cdot Y_2}{B_w}
\]
\[
dT = \frac{dT}{dt} = H_f + H_{g1} - H_w - H_{\text{pol}}
\]

where

\[
b_t = V_p \cdot C_v \cdot \sqrt{[M_1] + [\ln]} \cdot RR \cdot T - P_v
\]
\[
R_{M1} = [M_1] \cdot k_p \cdot \exp \left( -\frac{E_a}{T} \left( 1 - \frac{1}{T_f} \right) \right) \cdot (Y_1 + Y_2)
\]
\[
C_{pg} = \frac{[M_1]}{[\ln]} \cdot C_{pm1} + \frac{[\ln]}{[\ln]} \cdot C_{pIn}
\]
\[
H_f = F_M \cdot C_{pm} \cdot (T_{\text{feed}} - T_f) + F_{\text{in}} \cdot C_{pIn} \cdot (T_{\text{feed}} - T_f)
\]
\[
H_{g1} = F_g \cdot (T - T_f) \cdot C_{pg}
\]
\[
H_{\text{pol}} = H_{\text{real}} \cdot M_{\text{w1}} \cdot R_{M1}
\]
\[
T_{\text{pol}} = C_{pg} \cdot (T - T_f) \cdot R_{M1} \cdot M_{\text{w1}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_g$</td>
<td>500</td>
<td>m³/s</td>
</tr>
<tr>
<td>$V_p$</td>
<td>0.5</td>
<td>m³/s</td>
</tr>
<tr>
<td>$k_d$</td>
<td>17</td>
<td>kg/s</td>
</tr>
<tr>
<td>$k_{cw}$</td>
<td>$7 \times 10^3$</td>
<td>kg/s</td>
</tr>
<tr>
<td>$\dot{Q}_d$</td>
<td>$85 \times 10^{-3}$</td>
<td>kW</td>
</tr>
<tr>
<td>$C_{pIn}$</td>
<td>(8000, 4700)</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$C_{pol}$</td>
<td>(13, 4700)</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$C_{pg}$</td>
<td>7.5</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$C_{pg}$</td>
<td>(10^2, 10^3)</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$C_{pg}$</td>
<td>(0.85, 10^3)</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$T_{f}$</td>
<td>1.0001</td>
<td>s^-1</td>
</tr>
<tr>
<td>$T_{f}$</td>
<td>0.0001</td>
<td>s^-1</td>
</tr>
<tr>
<td>$M_{\text{w1}}$</td>
<td>2.105 $\times 10^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td>$M_w$</td>
<td>5.314 $\times 10^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td>$M_c$</td>
<td>6000.5</td>
<td>kg</td>
</tr>
<tr>
<td>$C_{mp}$</td>
<td>(1.4 $\times 10^2$, 10^3)</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$R_{\text{eff}}$</td>
<td>$9.44 \times 10^{-3}$</td>
<td>(4.10^8)</td>
</tr>
<tr>
<td>$T_{f}$</td>
<td>2.5</td>
<td>10^3</td>
</tr>
<tr>
<td>$T_{\text{pol}}$</td>
<td>500, 250, 250, 250, 250, 250</td>
<td>K</td>
</tr>
<tr>
<td>$RR$</td>
<td>8.20675 $\times 10^{-5}$</td>
<td>m²K/m³</td>
</tr>
<tr>
<td>$R$</td>
<td>0.114</td>
<td></td>
</tr>
</tbody>
</table>
B. Simulation Results and Discussion

In this section, we assume that there are three different possible faults \( \{ f_1, f_2, & f_3 \} \) may occur in the process as shown in Table II.

<table>
<thead>
<tr>
<th>Fault</th>
<th>Steady-State</th>
<th>Faulty-State</th>
<th>Time Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1: ( F_{In} )</td>
<td>5</td>
<td>7 mol/s</td>
<td>400 – 600</td>
</tr>
<tr>
<td>F2: ( F_g )</td>
<td>8500</td>
<td>9000 mol/s</td>
<td>400 – 600</td>
</tr>
<tr>
<td>F3: ( T_{feeds} )</td>
<td>293</td>
<td>305 K</td>
<td>400 – 600</td>
</tr>
</tbody>
</table>

The two fault scenarios are simulated using data corrupted with different measurement and state noise variances in order to examine the robustness of the GOS against the DOS in isolating these two faults. In case 1, the measurement and state noise variances in this simulation are assumed to be

\[
Q_{\nu 1} = 10^{-3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

and

\[
Q_{\omega 1} = 10^{-3} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},
\]

respectively and the faulty biased flow rate sensor, \( F_{In} \), is reading a value of 7 mol/s instead the true value given in Table I. The polyurethane reactor was simulated for 1000 samples, and the fault was introduced at \( k = 400 \) and removed at \( k = 600 \). By comparing the residuals, the plot in Fig 4 shows how does the DOS failed to isolate the faults while the plot in Fig 5 shows how did the GOS was able to isolate the fault clearly with the above measurement and state noise variances.

In case 2, the same fault scenario was used and the simulation was carried out using a smaller state and measurement noise variances i.e.

\[
Q_{\nu 2} = 10^{-5} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

and

\[
Q_{\omega 2} = 10^{-5} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},
\]

respectively. Fig 6 and Fig 7 show the ability of the both schemes in isolating the fault correctly with smaller noise
Fig. 6. The plot shows the residuals generated using particle filter approach for the biased flow rate sensor \( F_\text{In} \) for the polyethylene reactor process based on DOS using \( Q_{12} \) and \( Q_{02} \).

Fig. 7. The plot shows the residuals generated using particle filter approach for the biased flow rate sensor \( F_\text{In} \) for the polyethylene reactor process based on GOS using \( Q_{12} \) and \( Q_{02} \).

The performance of the proposed algorithm is directly proportional to the number of samples used in the particle filter. However, increasing the sampling size will increase the computational load.

V. CONCLUSIONS

A general model-based fault isolation approach for stochastic non-linear non-Gaussian systems has been developed using general observer scheme (GOS). The simulation results show excellent performance of the proposed approach against the dedicated observer scheme (DOS) in highly non-linear system. In future, we intend to extend to develop an algorithm capable of isolating actuator faults.

REFERENCES


