A multi-model identification method for the fiber stretching process based on the EM algorithm

Fan Guo¹, Yongsheng Ding¹²*, Lei Chen¹², Lihong Ren¹², Kuangrong Hao¹²

1. College of Information Science and Technology, Donghua University, Shanghai 201620 China (*e-mail: ysding@dhu.edu.cn)
2. Engineering Research Center of Digitized Textile & Apparel Technology, Ministry of Education, Shanghai 201620 China

Abstract: In the fiber production process, the stretching process plays a key role in the quality of the final fiber product. Due to the fiber stretching process with inborn nonlinearity, the performance of a single controller and an optimizer may be compromised or even unsatisfactory. Thus, we consider a multi-model identification method for the fiber stretching process. The dynamic transitions among different operating points are achieved by the change of the operating conditions in the fiber stretching process. To excite all of the nonlinearity character in the fiber stretching process, the transitions among different operating conditions is achieved. The structure of each sub-models, operating points, operating range are assumed. Based on the input output data of the process, a linear parameter varying (LPV) model is built by applying a probability identification method. To achieve the smoothly connected among the different operating conditions, an exponential function is used. Then a global LPV model is constructed by synthesizing the local models. Simulated results show that the LPV method has the effectiveness in solving the inherent nonlinearity of the fiber stretching process.

Keywords: LPV model; Multiple model; Expectation maximization algorithm; System identification; Stretching process

1. INTRODUCTION

For modelling complex nonlinear systems, it is often difficult to obtain a single nonlinear model that accurately describes the plant in all regimes. Even though a global nonlinear model can be achieved, it poses significant challenges for the design of controller and estimator, since the performance of a single controller and an optimizer may be compromised or even unsatisfactory. To circumvent the difficulties, the multi-model is presented. Each local linear model is identified at given operating points. Furthermore, the state and parameter estimation of the global model is accomplished using this local model in a moving domain estimator.

In recent decades, the LPV model identification method has been extensively studied in the process identification areas, due to a LPV model capability of approximating certain nonlinear model. (Banerjee, et al., 1997; Bamieh and Giarr, 2002; Xu, Zhao, and Zhu, 2009; Jin, Huang, and Shook, 2011). Banerjee et al. (1997) proposed the issue of estimating the states of a nonlinear system that operates in multiple regimes and makes transitions between them. Bamieh and Giarr (2002) proposed a LPV identification method using the process data and the scheduling variables are assumed to be directly obtained. The least squares method or recursive least squares method is used to provide formulae for corresponding identification problem under the given condition. In the whole operating range, the LPV identification method demands the input signal to be manipulated fully. In practice, this demand can be high price or even unrealistic, because input persistent excitation signal can not be accomplished in the whole operating range.

Xu, Zhao, and Zhu (2009) put forward an identification program for the nonlinear process around certain selected operating points is needed to approximate a whole LPV model. In addition, the cubic spline functions are used to depict the validity functions of each local model for the LPV identification model, in some cases, it is difficult to find the appropriate orders for each local model. Jin and Huang (2011) introduced an identification method of LPV models under the framework of the expectation maximization (EM) algorithm. The EM algorithm is an iterative optimization procedure, which is used to identify the LPV models. The identify methods are required for the LPV local model parameters and effect width of the exponential functions are estimated simultaneously. To avoid an abrupt converting among the operating points, the transition dynamics of the process is implemented by the smooth transition among the operating points and a smooth validity function for connecting multiple local models. To smoothly interpolate the predictions among different local models, an exponential function is utilized. In addition, the centres of the exponential functions are given a priori and need not be estimated. The EM algorithm is applied in the time varying process modelling on time series forecasting and smoothing. Shumway and Stoffer (2011). Kadlec, Gabrys, and Strandt (2009) developed a soft sensor in terms of its counterpart with the linear model structure, furthermore, calculated the weights assigned to each sub-model by the EM algorithm under the concept of LPV. Gopaluni (2010) presented application of the EM algorithm to estimate the parameters in nonlinear state space models under the unknown model structure.
In the fiber production process, the stretching process is one of the important parts, and it plays a key role in the quality of the final fiber product. Due to the fiber stretching process with inborn nonlinearity, the performance of a single controller and an optimizer may be compromised or even unsatisfactory. Thus, a multiple LPV model approach is applied to identify the stretching process of carbon fiber. The simulation results demonstrate that the proposed method can obtain better approximate results for the fiber stretching process.

The main contributions of this paper are as follows: 1) Based on a probability identification method, using the input–output data a LPV model is built. Then a global LPV model is constructed, using the way of synthesized the local models. 2) The multiple model approach is used to build the identification model for the fiber stretching process, which provides a method to design the multiple controller and optimizer for the stretching process, it is beneficial to facilitate the system maintenance, and to improve the system stability.

The remainder of this paper is organized as follows: Section 2 elaborates the mathematical formulation for the multiple LPV model identification method and provides a relevant identification procedure. In Section 3, we discuss the stretching process model of carbon fiber. Section 4 show the identification results on the stretching process of carbon fiber. Section 5 draws the conclusion.

2. THE MULTI-MODEL IDENTIFICATION METHOD BASED ON THE EM ALGORITHM

2.1 The EM algorithm

After the first being introduced by Dempster, Laird, and Rubin (1977), the EM algorithm has obtained significant attention in different disciplines. As well known, In the EM algorithm, the expectation step and the maximization step is iterated. In the EM algorithm, a complete data set consists of an observed data set and a missing data set. Using all available data or the model of the observed posterior, we can obtain the distribution function of the missing observation. Furthermore, we can calculate the current best estimated parameter values, and derive the expectation of the distribution function of the complete data with missing observation, which is the function, it is formulated by (McLachlan and Krishnan, 2008):

\[ Q(\Theta | \theta^k) = E_{y|x} \{ \log[p(C_{obs} | C_{mis}, \Theta)] \}. \]

The \( Q \) function pertain to \( \Theta \) is to maximize parameter sets

\[ \Theta = \arg \max_{\theta} Q(\Theta | \theta^k). \]

Then, the expectation step and maximization step iterate until convergence.

2.2. The LPV model identification problem based on the EM algorithm

Due to the capability of the ARX model in approaching LPV model, We select the autoregressive exogenous (ARX) model as LPV model. The structure of the ARX model is adopted to this paper, which is expressed as (Jin and Huang, 2011):

\[ y_i = x_i^T \theta_i + e_i, \]

where \( x_i \in R^r, y_i \in R \) in (3) represent the \( k \) th of the system regressor and output, \( H_k, k = 1, ..., N \) is the scheduling variable. We adopt Gaussian noise \( e_i \) with zero mean and variance \( \sigma^2 \) as the interference noise \( e_i \in R, e_i \sim N(0, \sigma^2) \). The regressor \( x_i \) can be expressed in the following form:

\[ x_i = [y_{k-1}, y_{k-2}, ..., y_1, u_{k-1}, u_{k-2}, ..., u_1]^T. \]

To achieve the smooth transition between the different operating points, an exponential function \( w_{ki} \) is adopted:

\[ w_{ki} = \exp\left(-\frac{(H_i - H_{i-1})^2}{2\sigma^2}\right). \]

According to the total probability formula, we can obtain:

\[ p(y_N, y_1, y_N, u_N, u_N, H_N, H_N | \Theta) = \prod_{k=1}^{N} p(y_k | y_{k-1}, ..., y_1, u_{k-1}, ..., u_1, H_k, ..., H_N, \Theta) \]

From (7), we can further express as:

\[ \frac{1}{2\pi\sigma} \exp\left(-\frac{(y_k - x_k^T \theta)^T(y_k - x_k^T \theta)}{2\sigma^2}\right). \]

Then, the expectation step and maximization step iterate until convergence.

The formula (8) is carried to take logarithmic, and can be derived as:

\[ \sum_{k=1}^{N} \log \frac{1}{\sqrt{2\pi\sigma}} + \sum_{k=1}^{N} \frac{(y_k - x_k^T \theta)^T(y_k - x_k^T \theta)}{2\sigma^2}. \]

The formulas (7-9) are used to illustrate the effectiveness of the method by the introduction of hidden variable \( l_k \). The observed data set can be written as \( C_{obs} = \{y_k, y_{k-1}, u_k, H_k, H_{i-1}\} \).
the missing data set can be represented as \( C_{\text{mis}} = \{ I_k \} \). Thus, we can calculate the \( Q \) function as:

\[
Q(\Theta | \Theta^{\text{old}}) = E_{C_{\text{obs}}(\Theta^{\text{old}}, C_{\text{mis}})} \{ \log p(C_{\text{obs}}, C_{\text{mis}} | \Theta) \} = E_{C_{\text{obs}}(\Theta^{\text{old}}, C_{\text{mis}})} \{ \log p(y_{N,M} | y_{1:M}, u_{1:M}, \Theta_{\text{old}}, \Theta_{\text{mis}} | \Theta) \}
\]

\[
= \sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \log p(y_{1:k} | y_{1:k}, u_{1:k}, \Theta_{\text{mis}} | \Theta)
\]

\[
+ \sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \log p(I_k = i | H_k, o_i)
\]

\[
+ \sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \log C_{1}, C_{2},
\]

(10)

where

\[
C_1 = p(H_k | y_{1:k}, ..., y_{N,M}, x_{1:k}, ..., x_{N,M}, H_{1:M}, ..., H_k, \Theta)
\]

and

\[
C_2 = p(x_k | y_{1:k}, ..., y_{N:M}, x_{1:k}, ..., x_{N,M}, H_{1:M}, ..., H_k, \Theta)
\]

are not dependent on \( \Theta \). The term \( p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \) in (10) represents the probability of the \( k \) th data belongs to \( i \) th sub-model. The term \( p(I_k = i | H_k, o_i) \) in (10) is an optimization problem of nonlinear function with constrains. In cite the term \( p(I_k = i | H_k, o_i) \) and \( p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \) are calculated as \( a_{ki} \). However, the term \( p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) = a_{ki} \) is simplified, without considering the effects of other observed variables of the model. Now, using Bayes’ rule, we can calculate the probability \( p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \) as:

\[
p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) = p(I_k = i | y_{1:k}, x_{1:k}, H_k, \Theta_{\text{old}})
\]

\[
= \frac{p(y_{1:k} | x_{1:k}, H_k, \Theta_{\text{old}} | I_k = i) p(I_k = i)}{\sum_{i=1}^{M} p(y_{1:k} | x_{1:k}, H_k, \Theta_{\text{old}} | I_k = i) p(I_k = i)}
\]

\[
= \frac{p(y_{1:k} | x_{1:k}, \Theta_{\text{old}}) p(I_k = i | H_k, \Theta_{\text{old}})}{\sum_{i=1}^{M} p(y_{1:k} | x_{1:k}, \Theta_{\text{old}}) p(I_k = i | H_k, \Theta_{\text{old}})}
\]

(11)

where

\[
p(y_{1:k} | x_{1:k}, \Theta_{\text{old}}) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{(y_{1:k} - x_{1:k}^T \theta)^T (y_{1:k} - x_{1:k}^T \theta)}{2\sigma^2} \right)
\]

and \( p(I_k = i | H_k, \Theta_{\text{old}}) \) can be calculated using \( a_{ki} \).

The parameter vector of the local LPV model, \( \theta \), and the interference noise \( \sigma \), the first term in (10) can be derived as:

\[
\theta = \sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) x_k^T y_k
\]

\[
\sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) x_k^T x_k
\]

(12)

The local model \( a \) can be calculated as:

\[
\max_{a_{1:M}} \sum_{k=1}^{N} \sum_{i=1}^{M} p(I_k = i | \Theta^{\text{old}}, C_{\text{obs}}) \log p(I_k = i | H_k, o_i).
\]

s.t. \( 0 < a_{\text{min}} < a_i < 1, i = 1, 2, ..., M \leq a_{\text{max}} \)

(14)

The validity width \( a \) value of the local model is calculated by a constrained nonlinear optimization function named ‘fmincon’.

3. THE STRETCHING PROCESS MODEL OF CARBON FIBER

In material manufacturing industry, fiber production process is a common process. The raw materials, liquid, colloid to tiny threads is the basic purpose for the shape of producing fiber (Carroll, Givens and Piefer, 1998). The fiber production can be further applied to other compound material productions, such as the manufacturing or weaving process. The fiber production processes are formed by a series of special equipment for different purposes. Due to its complexity, it demands different links to cooperate with each other. A general fiber production process consists of seven sections, namely raw material polymerization, spinneret components, coagulation bath, stretching units, fabricating, drying, and wire collection (Bazbouz and Stylios, 2008). After extruded from the spinneret and emerged into the coagulation bath, the raw material becomes dense and hard, it is gradually solidified and obtained an original silk shape. To obtain the quality of the fiber production, the threads are completely pulled through the different stretching units. Furthermore, after conducting some other procedures, such as oiling, drying and wire collection, the whole process of the production fibers is completed (Ismail and Rahman et al., 2008). The stretching process is an important part of the production process, which makes the fibers gradually stretch in the roller using a plurality of different rotational speeds of the rotating motor at a difference stretch ratio under different working environmental conditions. The change of the roller speeds causes the changing of the fiber molecular chains structure, rearranges the inherent molecular structure, and then increases strength and toughness of the final fiber products (Carroll, Givens and Piefer, 1998). Generally speaking, the stretching process consists of the several stretch stages, and each fiber stretch unit has to achieve different values of elongation. In addition, the number of the stretch stages can affect the stretch ratio of the fiber products (Bazbouz and Stylios, 2008). In the whole stretching process, the filament is stretched by changing the speed of the rollers (Chung Feng Jeffrey Kuo, 2008). Winding roller is the basic equipment in a stretching unit, which is shaped like a cylinder having a different diameter. A stretching unit is comprised of two winding rollers at least. The stretching process is a major part of the fiber production process, and the stretching effect is directly related to the fiber precursor and the final fiber...
product (Ismail and Rahman et al., 2008). The fiber is sequentially wound on a take-up roll, according to the direction of traction, and then, is stretched at different rotational speeds on the roller machine. The stretch ratio is determined by the rotational speed corresponding to the respective winding roller. In the production process, different working environments can also influence the fibers products (Liang and Ding, et al., 2012). Each section includes a plurality of rollers, a unit consists of two rollers. The stretching ratio is defined as the speed proportion in one unit. The stretch ratio plays a significant role in the fiber stretching process, which is closely related to the process stability and synchronization, and the relationship between the each stage stretch ratio and the quality of the carbon fiber product. Therefore, it is difficult to achieve synchronization between control of the fiber stretching process and stretching ratio of the fiber.

For any spinning line points, the change of the polymer is in the "steady state" and continuous refers to any point of the polymer through a spinning production line with a constant, and its parameter values do not change over time. That is, velocity, temperature and other parameters in the whole spinning line points have different locations. However, variation between different points is continuous, and their parameters do not change over time in the fixed position. Thus the stable distribution of the spinning line is taken shape, and called "steady state spinning ".

The stretching process on the carbon fiber production is as shown in Fig. 1. In Fig. 1, two stretching units and stretching environment is illustrated. The raw strand is derived from the coagulating bath, and filaments are uniaxially stretched into the solidified filaments. Winding wound around the two rollers, there have different rotational speeds. Therefore, the filament may be subjected to a degree of stretching, and the stretching ratio of the rotational is determined by the speed of the two winding roller. Here, the stretching ratio is defined as the speed ratio of the two winding rollers.

The tension \( F \) of raw filament is the function of the strain (Koc and Knittel, et al., 2002):

\[
F = E A (L - L_0) / L_0,
\]

where \( E \) represents Young's modulus, \( A \) represents the raw material radius, and they are related to the temperature of coagulation bath. \( L \) represents the length under stress, \( L_0 \) represents the nominal length under without stress. \( L - L_0 \) represents the tensile elongation under tension, and is considered to between the front motor and the driving motor speed. We find out that the tension change is directly related to the angular velocity of the drive motor. That is, the speed and the stretch ratio are treated with to maintain a constant value in the traditional control process, unless disturbed. Therefore, we can give the dynamic expression between the motor voltage and the filament strain. Formulation (16) consists of two models, one is the motor used for driving the stretching roller, and the other is represented the dynamic of the stretching process.

\[
J_0 \frac{dV(t)}{dt} = -fV(t) + (F(t) - \dot{F}_0(t))R^2 + nuR, \\
L \frac{dF(t)}{dt} = E A (V(t) - V_0) + \dot{F}_0(t)V_0 - F(t)V(t).
\]  

(16)

State variables: \( X = \begin{bmatrix} V \\ F \end{bmatrix} \),

System input: \( u \),

Controlled variable: \( Z = F \),

State estimation: \( Z = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X \).

Table 1

The steady state values of the stretching process of carbon fiber

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Steady state values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension ( F )</td>
<td>Output1</td>
</tr>
<tr>
<td>Velocity ( V )</td>
<td>Output2</td>
</tr>
<tr>
<td>Voltage ( U )</td>
<td>Input</td>
</tr>
<tr>
<td>Modulus of elasticity ( E )</td>
<td>211.1N/tex</td>
</tr>
<tr>
<td>Radius of Motor roller ( R )</td>
<td>0.1m</td>
</tr>
<tr>
<td>Moment of inertia ( J_0 )</td>
<td>0.02N.M.s^2</td>
</tr>
<tr>
<td>Length of span between two driving rollers ( L )</td>
<td>6.09m</td>
</tr>
<tr>
<td>Coefficient of friction roller ( f )</td>
<td>0.685</td>
</tr>
<tr>
<td>Filaments cross-sectional area ( A )</td>
<td>1.13( \mu )m^2</td>
</tr>
<tr>
<td>Ratio between driving Motor and roller ( n )</td>
<td>1</td>
</tr>
</tbody>
</table>

4. SIMULATION RESULTS

In the stretching process of the fiber, the filament tension is changed by operating the motor voltage. Fig. 2 displays the trend between the tension and the voltage. The input voltage is a random binary signal, and the output signal is noise-free. Based on the practical operating experience in the fiber stretching process, we choose the following operating points to obtain linear local sub-models:

\( H_1^0 = 5, H_2^0 = 6, H_3^0 = 8 \).

The following test simulation is performed: First period: 200 seconds, at working centre point 5, and 100 seconds varies linearly in time from point 5 to point 6. Second period: 250 seconds, at working centre point 6, and 100 seconds varies linearly in time from point 6 to point 8. Third period: 250 seconds, at working centre point 8.
From Fig. 2, we can notice that an operation range of the input voltage from 5 to 10 V, and the nonlinear relationship between the voltage and tension. It is difficult to design a controller for this nonlinear system. To avoid the difficulties, we use the multiple model approach to identify the stretching process of the carbon fiber. In accordance with the stretching process of the carbon fiber, the mechanism model is shown in (16). The parameters of the nonlinear model are listed in Table 1. The actual output tension results under the input data of the process are as shown in Fig. 3. The real data of the process input-output data are shown in Fig. 4. We can see from Fig. 5, the simulated data and the prediction data in self-validation section achieve the desired fitting result. The weighting function results for each local model are shown in Fig. 6.

To further verify the capability and effectiveness of the approximate method of the multiple sub-models, we use the data under different voltage from that of the training data to achieve a fair validation, namely cross validation. Due to the operating range and operating points are fixed, in addition, the operating point can not be changed dramatically in the fiber stretching process. Thus, the data are generated under other operating centres in cross validation:

\[ H_1^0 = 5.5, H_2^0 = 6.5, H_3^0 = 7.5. \]

![Fig. 2. The relationship between the input voltage and the output tension.](image)

![Fig. 3. Output tension under the input voltage.](image)

![Fig. 4. The input-output data of stretching process.](image)

![Fig. 5. Self-validation result.](image)

![Fig. 6. Weighting function results of each local model under the self-validation.](image)

Cross validation results between the process data and the model data are as shown in Fig. 7. Fig. 8 displays the weighting function of the local model under different voltages.
5. CONCLUSIONS

This paper considers the several different LPV model approach to identify the stretching process of the carbon fiber with two rollers. According to the mechanism model of the stretching process, we select input variable voltage as the scheduling variable, the tension as the control variable, and the ARX model as local sub-model. Under the certain working range, the multiple model identification method is validated. The cross validation is considered under the different operating points, to reflect the usefulness of the multiple model identification method. This identified method is beneficial to approximate the stretching process with the nonlinear.

ACKNOWLEDGEMENTS

This work was supported in part by the Key Project of the National Nature Science Foundation of China (No. 61134009), the National Nature Science Foundation of China (No. 61473077, 61473078), Cooperative research funds of the National Natural Science Funds Overseas and Hong Kong and Macao scholars (No. 61428302), Program for Changjiang Scholars from the Ministry of Education, Specialized Research Fund for Shanghai Leading Talents, Project of the Shanghai Committee of Science and Technology (Nos. 13JC1407500), and Innovation Program of Shanghai Municipal Education Commission (No. 14ZZ067).

REFERENCES