Variability Matrix: A Novel Tool to Prioritize Loop Maintenance

Marcelo Farenzena*, Jorge O. Trierweiler*, Sirish L. Shah**

* Department of Chemical Engineering, Federal University of Rio Grande do Sul (UFRGS)
Porto Alegre, RS, CEP: 90.040-040, BRAZIL (Tel: +555133084072; e-mail: {farenz,jorge}@enq.ufrgs.br)
** Department of Chemical and Materials Engineering, University of Alberta, Edmonton, AB, T6G 2G6, Canada (e-mail: sirish.shah@ualberta.ca)

Abstract: It is now common knowledge that as many as 40% of the control loops in most industrial processes have considerable potential for improving control performance by reducing variability. Because of the large number of control loops in an industrial plant, controller performance monitoring is indispensable, but equally important is how to prioritize their maintenance. It is well known that variance reduction in a loop occurs by transferring variability to other variables or loops. The focus of this study is to propose a methodology to prioritize loop maintenance based on the potential improvement of each loop and the variability transfer among them. The central point of this work is the Variability Matrix (VM), an array that shows the impact of performance improvement of a given loop on the whole plant. Based on the VM, a methodology to translate this array into a potential loop economic benefit metric is also introduced. The VM can be quantified in the ideal scenario where plant model and controller are available and also when they are not, thus allowing the application of these ideas in industry. The efficacy of proposed methodology is illustrated by successful application to two case studies.

1. INTRODUCTION

The main requirement for a control system is to ensure process stability and robustness. This is the key reason for the widespread industrial interest in performance assessment methodologies and tools. A typical plant has hundreds or thousands of controllers and most of them have potential for improvement (Bialkowski, 1993). Many good reviews on assessment of control loops are available in the literature (Huang and Shah, 1999, Jelali, 2006). A common problem in controller performance monitoring is how to prioritize loop maintenance. The answer should not only be based on the performance potential, but also on the economic benefits that can be realized in improving the performance of each loop.

The main motivation for improving the performance of the plant is simple: reduction in process variability allows achieving a more profitable operating point, closer to the constraints, as shown in Fig. 1. In scenario I, the process has large variability and therefore the setpoint or the target has to be significantly far away from the economically optimal operating point. If the variability is reduced, due to controller or process improvement (scenario II) the process operating point can be moved to a more profitable setpoint (scenario III).

The literature is relatively sparse in terms of quantification of economic benefits due to improvement of controller performance. Muske (2003) proposed the idea of potential reduction in control loop variability. The economic benefit is quantified based on the shift in the mean operation toward a product specification or process constraint. The variance reduction can be based on a fixed or user-specified benchmark, e.g. minimum variance benchmark or a desired rise time or settling time benchmark. Craig and Henning (2000) proposed another methodology to quantify the economic benefit of Advanced Process Control (APC) projects. The authors mention that the whole part of the benefits come from the steady-state optimization. They assume that the variance of the products can be reduced by 35% to 50%. Mascio and Barton (2001) proposed a methodology to quantify the control quality in economic terms based on the Taguchi Framework.

All available methodologies agree that reduction in variability means shifting the operating point to a more profitable point. The main drawback is that they consider each loop as an isolated case, i.e. if performance of one loop is improved then the whole plant will not suffer its effect.

All modern industrial plants have significant interaction among loops due to tighter heat integration. Because of this,
one cannot assume that the variance reduction in one loop will occur without impacting other loops adversely. Typically, variability is transferred from loops where it should be reduced to loops that have the room or the buffer to accommodate large fluctuations (e.g. level loops). In many cases, if one variable has its variability reduced and its operating point shifted, then it is likely that other interacting or complementary loops will have their variability increased, shifting the operating point away from the constraints. This implies that “part of the profit” realized by variability reduction in a given loop “will be offset” by the loops where the variability increases. This is why a control loop should not be considered in isolation and the potential economic benefit should be computed by analyzing the whole plant and not only a specific loop. The common idea that the improvement of a given controller performance will increase the performance of the whole plant is not always true. Sometimes in an interacting system, the coupling between the channels can help or hinder overall performance. For example, decrease in the variability of a given controller can also reduce the variability in other loops in which case, one can say that the interaction helps. In other cases, the interaction may affect performance of associated loops adversely.

The main contribution of this work is the introduction of the notion of the Variability Matrix (VM). This array shows how the variability will transfer between the loops and the impact of one specific loop on the variances of all other interacting or complementary loops. The potential economic benefit of each loop can be quantified based on VM.

This paper is structured as follows: section 2 introduces the concept of Variability Matrix. In section 3, practical issues in computing the VM are discussed. The methodology to quantify the economic benefit of each control loop and prioritize loop maintenance is shown in section 4. The complete methodology is illustrated by successful application on two case studies (section 5). The paper ends with concluding remarks.

2. VARIABILITY MATRIX: CONCEPTS AND DEFINITION

2.1 Preliminary Definitions

To quantify the economic impact, it is interesting to consider the classification of control loops into the following two categories:

**Main Loops**: Loops that directly control the products specification. Their performance improvement affects the product variability, which can be directly translated into profitability.

**Auxiliary Loops**: Loops that do not directly control product quality, but can indirectly affect the product variability.

2.2 Variability Matrix Structure

The structure of the variability matrix consists of the following:

**Rows**: The rows show the influence of each loop on the same final product. The number of rows is the same as the products or the number of main loops.

**Columns**: Shows the influence of a specific loop on all other loops that may impact or influence the specification of the final product. The number of columns is the same as the number of control loops implemented in the plant. The first columns correspond to the main loops and the adjacent set of columns corresponds to the auxiliary loops as shown in Figure 2.

![Figure 2: Schematic representation of Variability Matrix](image)

Fig. 2: Schematic representation of Variability Matrix

In Fig. 2 $M_n$ is the main loop $i$ and $Aux_j$ is the auxiliary loop $j$. The total number of loops in the plant is $l$ and it has $m$ main loops. For example, column 1 ($M_{n1}$) shows the impact of variability reduction in main controller 1 on all other main loops. Row 1 shows the impact on the variability of $M_{n1}$ when the performance of all other loops is changed.

2.3 VM Computation

This section discusses the methodology for computing each element $VM(i,j)$ of the Variability Matrix. In the first scenario, the following assumptions are taken: (I) the plant model ($G$) is available; (II) the controller model ($C$) is also available; and (III) the controlled variables ($y$) and control outputs ($u$) are available. For the sake of simplicity, we consider that the setpoint is fixed and set to zero.

Based on the previous assumptions, the procedure to quantify the VM is described below:

1. Read process data $y_j$ ($j=1...l$) and $u_j$ ($j=1...l$) with all loops closed (with actual performance);
2. Select main and auxiliary loops;
3. Compute the actual variance for each main loop $(\text{var}_{act,i}, i=1...m)$;
4. For each loop $j$ ($j=1...l$)
   1. Calculate the best performance achievable (see section 3.2) for loop $j$;
   2. Apply the controller;
   3. Calculate the new variance for each main loop $i$ ($\text{var}_{best,i,j}, i=1...m$)
   4. Compute the elements of VM $j^{th}$ column using eq. 1.

$$ VM(i,j) = \frac{\text{var}_{act,i} - \text{var}_{act,i,j}}{\text{var}_{act,i}} $$

(1)

This structure for VM elements was chosen because for two main reasons: 1) it provides a direct measure of the
variability improvement potential for each loop; and 2) it is dimensionless, a fact that allows the comparison of the impact of two or more loops in the plant. For example, consider the VM of:

\[
\begin{bmatrix}
0.3 & 0 & -1.2 \\
-0.7 & 0.9 & -1.5
\end{bmatrix}
\]

(2)

Initially, we can verify that this plant has 2 main loops and one auxiliary loop. From this VM, by examining column 1, we can conclude that: if the performance of main controller 1 is improved, its variance will decrease 30%; however, it has a negative and strong impact on another loop: its variance will increase by 70%. Is this healthy for the process? Clearly the answer to this question depends on the economic impact of each main loop. In column 2, the main loop 2 has potential reduction in variability of 90%. This controller has no influence on the main loop 1 variance; furthermore improving the performance of the auxiliary loop (3rd column) will lead to variability increase in both main loops.

In complement with the VM, the concept of the complementary VM arises (CVM). It is not necessary for all controllers to have fast performance, many loops have to play the role of accommodating or buffering disturbances. Based on this assumption, we define the Complementary Variability Matrix (CVM). The values are computed with actual loop variance (\(\text{var}_{\text{act},i}\)) and the variance of the loop with the worst performance acceptable (\(\text{var}_{\text{wor},i}\)). The structure is the same as shown before, and the elements are computed as follows:

\[
CVM(i, j) = \frac{\text{var}_{\text{act},j} - \text{var}_{\text{wor},i,j}}{\text{var}_{\text{act},i}}
\]

(3)

The same procedure as considered earlier can be used to evaluate the Complementary Variability Matrix (CVM). Only step 4.1 is replaced by the slowest accepted performance (see Smith, 2002) and the worst accepted performance (\(\text{var}_{\text{wor}}\)) should be quantified.

The proposed computational steps may not be easily applicable in an industrial setting, because the required information (controller and process model) is generally unavailable. The algorithm to compute VM where the controller and plant model are not available is shown in section 3.

2.4 VM Dependence of the System Parameters

From a preliminary inspection, VM seems to be analogous to static the RGA (Skogestad and Postlethwaite, 2005), where only the process static gains have impact in the analysis. However, VM is not only a function of process gains, but also depends on process behavior (dynamics and time delays), disturbance patterns and correlation among the disturbances, controller structure (e.g. PI, PID, MPC, among others), closed loop performance, and best performance achievable. The VM values are specific for each process: even two systems where the models and controllers are the same can have a completely different VM, because of the disturbance pattern.

2.5 Some Peculiar Behaviour

Intuitively, the diagonal elements of the VM should have a positive sign and the off-diagonal elements negative sign, i.e.: improving the performance of a given controller will reduce its variability; and transfer variability to the other loops, increasing their variability. However, this may not always be the case:

**Proposition 1:** Diagonal elements of VM can have negative sign, i.e. the performance improvement of a given controller can increase its variability.

**Proof:** Consider a SISO system with linear PI type controller that is affected by an output disturbance \(y\). Suppose that the disturbance is a pure white-noise random signal. Considering that \(y\) is random, it is not possible to predict its future values based on the past values. In this case increasing loop gains will likely increase \(y\) variability. In this case, the diagonal VM element will have a negative sign.

**Proposition 2:** Off-diagonal elements can have positive sign, i.e. the performance increase of a given controller can also decrease the variability in other interacting loops. Typically this happens when interactions help in accommodating disturbances.

**Proof:** Consider the triangular system shown in Fig. 3.

![Fig. 3: Schematic representation of the triangular system](image)

Consider the case when \(C\) reduces the output variability when it is compared with the open loop case (i.e. \(\sigma^2(d_1 + y_{1,p}) < \sigma^2(d_1)\)), and upon improving \(C\) performance, \(y\) will also decrease its variability.

\[
\sigma^2(d_1) > \sigma^2(d_1 + y_{1,p}) > \sigma^2(d_1 + y_{1,p2})
\]

(4)

Where \(p_1\) and \(p_2\) are the controllers performance and \(p_2 > p_1\) (i.e. closed loop performance in the second scenario \(p_2\) is faster than \(p_1\)). Considering the case when:

\[
G_{11} = G_{21}
\]

\[
d_1 = d_2
\]

(5)

Then \(y_{1,p} = y_{2,1}\). From the loop 2 and \(y_{2,1}\), it is clear that improving the performance of loop 1, will also have the effect of reducing the variability of \(y\). This will occur as \(y_{2,1}\) will help offset the effect of \(d_1\) (in the same way as \(y_{1,p}\) offsets \(d_1\)). Thus leads to:
\[ \sigma^2(d_1) > \sigma^2(d_2 + y_{2,1,\rho_1}) > \sigma^2(d_2 + y_{2,1,\rho_2}). \]  

(6)

3. PRACTICAL ISSUES IN COMPUTING VM

3.1 Computing the VM

This section presents the methodology to evaluate VM in industrial settings where process and/or controller models may not be available.

The first analyzed scenario is where a Model Predictive Controller is implemented. In this case, the controller model is not available, because most industrial MPCs are “closed box solutions”. However, the plant model is available. In this case, setpoint variations in MPC controllers are quite common, because of the optimization layer. In this scenario, the controller model can be extracted (identified) using the Asymptotic Method (Zhu, 1998) or Subspace Identification (Overschee and Moor, 1996).

A second scenario contemplates the case where only low order controllers (PI and PID) are present and setpoint activity is available in all loops. For this case, the following steps are contemplated: (I) identify the controller order and parameters (C) using structured target factor analysis (STFA) (Fotopoulos et al., 1994); (II) estimate the time delay (Tuch et al., 1994); (III) identify the process model (G) using Subspace Identification (Overschee and Moor, 1996); (IV) identify the disturbance model (d) using Subspace Identification; (V) with G, C, and d available, the VM can be estimated applying the methodology shown in section 2.3.

Based on our limited experience, we can affirm that the VM is not extremely dependent on the accuracy of the plant and controller. Even for a visible mismatch in the plant model, the obtained results are fairly good, comparing with the case where accurate controller and plant models are available.

3.2 Best and Worst Controller Performances

A natural question that arises is: how can the best and worst performance be computed for a given system? The answer clearly depends on the controller that is implemented on the process.

For MPC controllers, the best achievable performance can be computed using the methodology proposed by Trierweiler and Farina (2003). If the desired performance is attainable, this methodology provides the tuning parameters for the chosen performance. Otherwise, if it’s not achievable, the best achievable performance is quantified. In this work, we assume that the “best performance” is based on the open and closed loop rise time ratio, and a convenient value for this ratio is 3.

For low order (PI and PID) decentralized controllers, the best performance can be estimated using the methodology proposed by Faccin and Trierweiler (2004). The worst performance can be evaluated based on the methodology to tune buffer tank controllers (Smith, 2002).

4. QUANTIFYING THE ECONOMIC BENEFITS BASED ON VM

The economic benefits of improving control performance of each loop can be computed in two ways. The first method considers that the best performance can be achieved. In this case the VM can be used as follows. We represent the column j of the VM as VMj. The economic benefit can be easily quantified using the relationship:

\[ \text{CLEB} = D \cdot VM \]  

(7)

where CLEB is the Control Loop Economic Benefit array. It has the same number of elements as the number of loops in the plant (l).

\[ \text{CLEB} = [D \cdot VM_1 \ D \cdot VM_2 \ \cdots \ D \cdot VM_l] \]  

(8)

Where D is the array that translates variability reduction into $ per unit time.

\[ D = [D_1 \ \ D_2 \ \ \cdots \ \ D_m] \]  

(9)

where m is the number of main loops in the plant. This array can be quantified as a function of plant throughput increase, utilities reduction, etc. This value can be provided by the commercial department of the plant or the optimization layer weights used in MPC design.

However, as previously mentioned, not all controllers need to have high or tight tuning and the economic benefit, considering the worst performance of each one, can also be quantified. This vector is defined as Complementary Control Loop Economic Benefit:

\[ \text{CCLEB} = [D \cdot CVM_1 \ D \cdot CVM_2 \ \cdots \ D \cdot CVM_l] \]  

(10)

For example, suppose a plant where the VM and D are:

\[ VM = \begin{bmatrix} 0.7 & -0.6 & 0.8 \end{bmatrix} \]

(11)

\[ D = [100 \ \ 50] \]

(12)

the CLEB is then be computed as:

\[ \text{CLEB} = [55 \ \ -20] \]

(13)

The CLEB indicates that improvement in loop 1 performance means increase the plant profitability. However, the opposite behavior is expected when loop 2 performance is improved.

5. CASE STUDIES

5.1 Wood and Berry Distillation Column Model

The pilot-scale distillation column proposed by Wood and Berry (1973) is the first case study. The plant model is given by:

\[
\begin{bmatrix}
    x_D(s) \\
    x_B(s)
\end{bmatrix} =
\begin{bmatrix}
    12.8 e^{-4s} & -18.9 e^{-3s} \\
    16.7 s + 1 & 21 s + 1 \\
    6.6 e^{-7s} & -19.4 e^{-3s} \\
    10.9 s + 1 & 14.4 s + 1
\end{bmatrix}
\begin{bmatrix}
    R(s) \\
    S(s)
\end{bmatrix}
\]

(14)
where \( x_0 \) and \( x_g \) are the overhead and bottom products composition, and \( R \) and \( S \) are the reflux and steam flow rates, respectively. The time constants and time delays are expressed in minutes.

Two decentralized PI type controllers were applied in this case study. The disturbance was generated by passing a random signal through a first order transfer function with unitary gain and 50 minute time constant. The VM analysis of this case study is presented next under 3 scenarios: 1) controller and plant models are assumed to be available; 2) only plant model is available; 3) neither the plant model nor controller models are available. However, setpoint activity is assumed. This serves as good excitation for closed loop identification. For case 3, details of closed-loop based subspace identification method are not included here due to lack of space. The PI controllers were tuned to have a unitary gain and 50 minute time constant. The VM was computed using the methodology shown in section 2.3. In the first scenario, the controller and plant model were available. The VM was computed using the methodology shown in section 2.3.

\[
D = \begin{bmatrix} 100 & 30 \end{bmatrix}
\]

(15)

In the first scenario, the controller and plant model were available. The VM was computed using the methodology shown in section 2.3. The VM for this case is:

\[
VM = \begin{bmatrix} 0.57 & -0.17 \\ -0.18 & 0.41 \end{bmatrix}
\]

(16)

The CLEB for this case is:

\[
CLEB = \begin{bmatrix} 52 & -5 \end{bmatrix}
\]

(17)

Based on CLEB, loop 1 should have its performance improved (top composition), increasing the plant profitability. Loop 2 shows the opposite behavior, improvement in its performance is likely to result in decreased plant profitability.

In the second scenario, the controller model is assumed to be unavailable. Initially, using a scenario where two setpoint variations in each variable are available, the controller model was identified (see section 3.1). In this scenario, the VM was estimated to be:

\[
VM = \begin{bmatrix} 0.60 & -0.19 \\ -0.18 & 0.46 \end{bmatrix}
\]

(18)

Notice that the estimated VM closely matches the true VM shown in (16). In the third scenario, both controller and plant model were identified using closed loop data. The estimated VM for this scenario is:

\[
VM = \begin{bmatrix} 0.60 & -0.19 \\ -0.18 & 0.46 \end{bmatrix}
\]

(19)

Even for this case, where controller and plant model were first identified using subspace identification, a good estimate of VM was obtained.

### 5.2 Shell Benchmark Process

The Shell Control Problem benchmark was proposed by Prett and Morari (1987). The system is characterized by the high interaction among channels and large time delays.

It involves one heavy oil fractionator. It has three product draws, three side circulating loops and a gaseous feed stream. The system consists of seven measured outputs, three manipulated inputs and two unmeasured disturbances. In this case study, we will reduce the problem to a 3 input and 3 output case. The three controlled variables are: top end point (y1); side endpoint (y2); bottom reflux temperature (y3). The manipulated variables are: top draw (u1); side draw (u2); bottom reflux duty (u3). The system has also two disturbances: upper reflux (d1); intermediate reflux (d2). The process output can be written as:

\[
y = G u + G_d d
\]

(20)

Where G is the plant model

\[
G = \begin{bmatrix} 4.05 & -27s \end{bmatrix} \begin{bmatrix} 50s + 1 \\ 5.39 & 60s + 1 \\ -18s & 72.5 \end{bmatrix} \begin{bmatrix} -28s \end{bmatrix} \begin{bmatrix} 50s + 1 \\ 6.9 & 50s + 1 \\ -15s \end{bmatrix} e^{-27s}
\]

(21)

and G_d is the disturbance model:

\[
G_d = \begin{bmatrix} 1.2 & e^{-27s} \\ 45s + 1 & 1.44 \end{bmatrix} \begin{bmatrix} e^{-27s} \\ 152 & e^{-15s} \end{bmatrix}
\]

(22)

(21)

Where the time constant and time delays are reported in minutes. The MPC from Matlab® (MPC toolbox V. 2.2.2) was applied in this study. The analysis for this case is reported under two scenarios: (I) where controller and plant models are available and (II) when both are unavailable.

The D vector for this case is (hypothetically set):

\[
D = \begin{bmatrix} 100 & 50 & 20 \end{bmatrix}
\]

(23)

The actual performance in this case was computed based on closed loop rise time when it is set equal to the open loop case. The desired performance for each channel is three times faster than open loop. The VM for this scenario is:

\[
VM = \begin{bmatrix} -0.39 & 0.10 & 0.26 \\ -0.12 & -0.16 & 0.28 \\ -0.24 & -0.60 & 0.32 \end{bmatrix}
\]

(24)

The VM shows that improving the performance of controller 1 and 2 will result in an increase in the variance of all main loops. Retuning loop 1 means increase its variance by 39% and increase in the variances of loops 2 and 3 by 12% and 24% respectively. On the other hand, improvement of loop 3 performance means decrease in its variance by 32% and...
corresponding reductions in variances in loop 1 and loop 2 by 26% and 28%, respectively. The CLEB for this case is:

\[
CLEB = \begin{bmatrix} -50 & -10 & 46 \end{bmatrix}
\] (25)

The answer to improving plant profitability lies not only in VM but also the CVM, i.e. some controllers should not have their performance improved, but rather detuned. The CVM for this case is:

\[
CVM = \begin{bmatrix} 0.15 & 0.13 & -0.12 \\ 0 & 0.28 & -0.13 \\ -0.30 & -0.29 & -0.44 \end{bmatrix}
\] (26)

The CCLEB for this case is:

\[
CCLEB = [9 \quad 21 \quad -27]
\] (27)

The maintenance list for this hypothetical case indicates that the most important controller to maintain or improve performance is loop 3. The second loop in the maintenance list should be loop 2 followed by loop 1.

In the second scenario, both controller and plant are assumed to be unavailable, only setpoint activity is assumed. In this case, the VM is estimated using the procedure shown in section 3.1. The estimated VM is:

\[
VM = \begin{bmatrix} -0.42 & 0.06 & 0.26 \\ -0.15 & -0.27 & 0.28 \\ -0.32 & -0.74 & 0.32 \end{bmatrix}
\] (28)

Both plant model and controller are identified using subspace identification from Matlab® (system identification toolbox version 6.1.1, function n4sid). Both models have 9 states.

Eq. 28 shows that the estimated VM compared with the original (eq. 24) is fairly good. We attribute the success of this fairly accurate VM estimation to the direct closed loop subspace identification method under reasonable level of setpoint activity.

6. CONCLUDING REMARKS

The main conclusions of the proposed work can be summarized as:

- industrial plants have many loops with considerable potential for performance improvement and therefore a methodology to prioritize loop maintenance is required;
- the concept of Variability Matrix was introduced in this work and has been shown to highlight the potential improvement in each loop and its impact on the whole plant;
- the methodologies to compute VM where neither the controller nor plant model are available has also been presented; in this scenario Subspace Identification can be used; even for this case the methodology has been shown to yield very good results based on closed loop identification;
- the proposed methodology was applied to two case studies providing good results;
- the proposed scenarios where the VM can be computed allows the application of these ideas in an industrial setting.

ACKNOWLEDGMENT

The first two authors wish to thank CAPES, PETROBRAS and FINEP for supporting this work.

REFERENCES