Application of the IHMPC to an industrial process system

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Abstract: This paper addresses the application of a new MPC to a distillation system where isobutane and light butenes are separated from butane and heavier compounds. This system is located in the alkylation unit of an oil refinery. The MPC considered here is based on the infinite horizon MPC extended to the case where the system has stable and integrating modes. The controller is developed based on a particular state space model in the incremental form, which considers the existence of time delays. The proposed controller provides nominal stability to the closed loop system. Practical tests in a distillation system show that the performance of the new controller, which can be extended to consider robustness to model uncertainty is similar to the performance of the conventional MPC with finite prediction horizon.

Keywords: Predictive control; Stability; Infinite horizon; Integrating system.

1. INTRODUCTION

One of the key issues in the application of MPC to industrial processes is the requirement that the closed loop system should remain stable for a large set of tuning parameters and any possible control structure in terms of active controlled outputs and available manipulated inputs. Rawlings & Muske (1993) have demonstrated that, in the regulator operation of stable systems, the infinite horizon MPC preserves stability even in the presence of constraints in the inputs and states. These ideas have been extending to the case of output tracking of stable systems (Odloak, 2004) and to systems with stable and integrating modes (Carrapiço & Odloak, 2005; González et al., 2007). However, a recent review by Qin & Badgwell (2003) points out that these developments have not been incorporated into the available MPC technology. Thus, the main scope of this work is to report the application of an infinite horizon MPC with nominal stability to an industrial system of small dimension but that presents the typical ingredients of a practical application: time delay, measured and unmeasured disturbances and integrating modes.

The state space model considered here is an extension of the model developed by Gouvêa & Odloak (1997) and Rodrigues and Odloak (2003) and implemented by Porfirio et al. (2003) to include time delays and integrating modes. To represent systems with time delays, it is assumed that

\[ \{ i,j \} \in \text{int} \min \{ / \} \theta \geq \Delta t, \]

where \( m \) is the control horizon of the MPC, \( \Delta t \) is the sampling time and \( \theta_{ij} \) is the time delay associated with output \( y_i \) and input \( u_j \). In the state matrix defined in (3), one has

\[
A = \begin{bmatrix}
0 & I_{ny} & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & I_{ny} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & I_{ny} & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & I_{ny} & \Psi((np+1)\Delta t) \ I'((np+1)\Delta t) \\
0 & 0 & 0 & \cdots & 0 & I_{ny} & \Delta t \\
0 & 0 & 0 & \cdots & 0 & 0 & F \\
0 & 0 & 0 & \cdots & 0 & 0 & I_{ny}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
S(\Delta t)^T \\
S(2\Delta t)^T \\
\vdots \\
S((np+1)\Delta t)^T \\
\end{bmatrix}
\begin{bmatrix}
[D^xFN] \ [D'F]^T 
\end{bmatrix}
\]

\[ C = \begin{bmatrix}
I_{ny} & 0 & \cdots & 0
\end{bmatrix}
\]
\[\Psi((np+1)\Delta t) = \begin{bmatrix} \Phi_{1}((np+1)\Delta t) & 0 & \cdots & 0 \\ 0 & \Phi_{2}((np+1)\Delta t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_{np}((np+1)\Delta t) \end{bmatrix}\]

Where \(f_{i,j,g} = r_{i,j,g} \Delta t - \theta_{i,j} \), \(r \) is a stable pole of the system. States \(y(k/k), y(k+1/k), \ldots, y(k+np/k) \) correspond to the output predictions calculated at time \(k \) based solely on past control actions and disturbances.

\[I'(t) = \begin{bmatrix} (np+1)\Delta t - \theta_{i} & 0 & \cdots & 0 \\ 0 & (np+1)\Delta t - \theta_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (np+1)\Delta t - \theta_{np} \end{bmatrix} \in \mathbb{R}^{np \times np}\]

\(\theta_{i} \) is the time delay associated with the integrating mode related to output \(y_{i}.\)

\[D^{f} \in \mathbb{R}^{np \times mu} \text{ and } D^{i} \in \mathbb{R}^{np \times nu}, F = \text{diag}\left(r_{1,1,1} \cdots r_{1,nu,1} \cdots r_{mu,1,1} \cdots r_{nu,mu,1} \cdots r_{ny,nu,1}, \ldots, r_{ny,mu,1} \cdots r_{nu,ny,1} \right), \]

\[F \in \mathbb{C}^{nu \times nu}, D^{f} = \text{diag}\left(d_{1,1,1}^{f} \cdots d_{1,nu,1}^{f} \cdots d_{mu,1,1}^{f} \cdots d_{nu,mu,1}^{f} \cdots d_{ny,nu,1}^{f}, \ldots, d_{ny,mu,1}^{f} \cdots d_{nu,ny,1}^{f} \right), D^{i} \in \mathbb{C}^{nu \times nu}, N = \begin{bmatrix} J_{1} \\ \vdots \\ J_{ny} \end{bmatrix}, J_{i} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, J_{ny} = \begin{bmatrix} 0 & 0 \cdots & 0 \\ \vdots & \vdots \ddots & \vdots \\ 0 & 0 \cdots & 1 \end{bmatrix}, J_{i} \in \mathbb{R}^{nu \times mu}, \quad i = 1, 2, \ldots, ny\]

In the above matrix, it is assumed that either output \(y_{i}\) integrates only input \(u_{i}\), or if this output integrates other inputs, the time delays between the integrated inputs and the output are the same for all the inputs. If this condition is not satisfied, the model proposed above is not observable. The step response of the system can be calculated by the following equation

\[S(t) = D^{f} + \Psi(t)D^{i}N + I'(t)D^{i}\]

where \(t \) is supposed to be larger than any time delay included in the process model.

2. THE INFINITE HORIZON MPC

The infinite horizon MPC considered here is based on the following control cost

\[V_{c} = \sum_{j=0}^{\infty} \left[ e(k+j|k) - \delta_{j} - j\Delta t \delta_{i} \right]^{T} Q \left[ e(k+j|k) - \delta_{j} - j\Delta t \delta_{i} \right]
+ \sum_{j=0}^{\infty} \Delta u(k+j|k) \Delta u(k+j|k) + \delta_{j}^{T} S_{i} \delta_{j} + \delta_{j}^{T} S_{j} \delta_{j}\]

where \(e(k+j|k) = \hat{y}(k+j|k) - y_{op} \) and \(\hat{y}(k+j|k) \) is the output prediction at time \(k+j \) computed at time \(k \) and considering the future control actions. Weight matrices \(Q, R, S_{i}, S_{j} \) are assumed positive definite.

The control objective defined in (4) can be expanded as follows

\[V_{c} = V_{c}^{(1)} + V_{c}^{(2)} + \sum_{j=0}^{\infty} \Delta u(k+j|k) \Delta u(k+j|k) + \delta_{j}^{T} S_{i} \delta_{j} + \delta_{j}^{T} S_{j} \delta_{j}\]

where

\[V_{c}^{(1)} = \sum_{j=0}^{\infty} \left[ e(k+j|k) - \delta_{j} - j\Delta t \delta_{i} \right]^{T} Q \left[ e(k+j|k) - \delta_{j} - j\Delta t \delta_{i} \right]
\]

\[V_{c}^{(2)} = \sum_{j=0}^{\infty} \left[ e(k+np+j|k) - \delta_{j} - (np+j)\Delta t \delta_{i} \right]^{T} Q \left[ e(k+np+j|k) - \delta_{j} - (np+j)\Delta t \delta_{i} \right]\]

It can be shown that

\[V_{c}^{(1)} = \left[ \tilde{C} x(k|k) + \tilde{C} \Delta u_{i} - y_{i}^{op} - T \delta_{i} - \tilde{I} \delta_{i} \right]^{T} \tilde{Q} \left[ \tilde{C} x(k|k) + \tilde{C} \Delta u_{i} - y_{i}^{op} - T \delta_{i} - \tilde{I} \delta_{i} \right]\]

where

\[\tilde{C} = \begin{bmatrix} C \\ CA \\ CAB \\ \vdots \\ CA^{nu} \end{bmatrix}, \tilde{Q} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \]

\[\Delta u_{i} = \begin{bmatrix} \Delta u(k|k) \\ \Delta u(k+1|k) \\ \vdots \\ \Delta u(k+m-1|k) \end{bmatrix}, \tilde{Q} = \text{diag}\left( Q \quad Q \quad \cdots \quad Q \right) \in \mathbb{R}^{np \times np \times np}, \]

\[y_{i}^{op} = \begin{bmatrix} y_{i}^{op} \quad y_{i}^{op} \quad \vdots \quad y_{i}^{op} \end{bmatrix} \in \mathbb{R}^{np \times np}, \tilde{I} = \begin{bmatrix} I_{y_{i}} & I_{y_{i}} & \cdots & I_{y_{i}} \end{bmatrix} \in \mathbb{R}^{np \times np \times np}, \]

In order to develop the infinite sum defined in (6), one needs to consider an expression for the calculation of the output prediction at time steps beyond time \(np \). Using the model expressions defined in (1) to (3), the output prediction at time step \(np+1 \) can be written as follows:
In order to guarantee that \( V_k^{(2)} \) will be bounded, it is necessary to force the state components related to the integrating modes to be zero at the end of the control horizon:

\[
\dot{x}(k + m + k) - y_p - \delta_i^m = 0 \quad (8)
\]

\[
\dot{x}(k + m + k) - \delta_i^m = 0 \quad (9)
\]

If constraints (8) and (9) are satisfied, \( V_k^{(2)} \) can be written as follows:

\[
V_k^{(2)} = \sum_{i=1}^{N} \dot{x}(k + m + k)^T (F^{i-1})^T (\Psi((np + 1)\Delta t))^T Q \Psi((np + 1)\Delta t) F^{i-1} \dot{x}(k + m + k)
\]

\[
V_k^{(2)} = \dot{x}(k + m + k)^T \hat{Q} \dot{x}(k + m + k) \quad (10)
\]

where \( \hat{Q} \) satisfies

\[
F^T \hat{Q} F - \hat{Q} = (\Psi((np + 1)\Delta t))^T Q \Psi((np + 1)\Delta t)
\]

and

\[
x(k + m + k) = F^m x(k / k) + F_u \Delta u_k
\]

\[
F_u = \begin{bmatrix} F^{m-1} B^{d1} & F^{m-2} B^{d1} & \cdots & B^{d1} \end{bmatrix}, \quad B^{d1} = D^1 F N
\]

Substituting (7) and (10) in (4), the control objective becomes

\[
V_k = \begin{bmatrix} \Delta u_k & \delta_i^m & \delta_i^m \end{bmatrix} H \begin{bmatrix} \Delta u_k \\ \delta_i^m \\ \delta_i^m \end{bmatrix} + 2C'_j \begin{bmatrix} \delta_i^m \\ \delta_i^m \end{bmatrix} + c
\]

where

\[
H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & H_{23} \\ H_{13}^T & H_{23}^T & H_{33} \end{bmatrix}
\]

\[
C_j = \begin{bmatrix} x(k / k)^T \tilde{C} \tilde{Q} \tilde{C} - y_1^m \tilde{Q} \tilde{C} + x(k / k)^T F^{m-1} \tilde{Q} F_u \end{bmatrix}
\]

\[
H_{11} = \tilde{C}^T \tilde{Q} \tilde{C} + F_u^T \tilde{Q} F_u + \tilde{R}
\]

\[
c = x(k / k)^T \tilde{C} \tilde{Q} \tilde{C} x(k / k) - 2y_1^m \tilde{Q} \tilde{C} x(k / k) + y_1^m \tilde{Q} y_1^m + x(k / k)^T F^{m-1} \tilde{Q} F_x x(k / k)
\]

\[
\tilde{R} = \text{diag}(R, R, \ldots, R) \in \mathbb{R}^{n_u \times n_u}
\]

It is easy to show that for the model defined in (1), the constraint defined in (8) can be written as follows

\[
x(k / k) - y_p + m\Delta t \dot{x}(k / k) + \tilde{D} \Delta u_k - \delta_i^m = 0
\]

where

\[
\tilde{D} = [D^0 + m\Delta t D^1 + \cdots + D^0 + \Delta t D^m]
\]

Analogously, the constraint defined in (9) becomes

\[
x(k / k) + \tilde{D}^\top \Delta u_k - \delta_i^m = 0
\]

Therefore, the infinite horizon MPC that is implemented here is based on the following optimization problem:

\[
\min_{\Delta u_k, \delta_i^m} V_k
\]

subject to

(12), (13) and

\[
\Delta u(k + j) \in U, \quad j \geq 0,
\]

where

\[
U = \begin{bmatrix} \Delta u(k + j) \\ \Delta u(k-1) \end{bmatrix} \begin{bmatrix} -u_{\max} & \leq \Delta u(k + j) \leq u_{\max} \\ \Delta u(k + j) = 0; \quad j \geq m \\ u_{\min} \leq u(k + j) \leq \Delta u(k + j) \leq u_{\max}; \quad j = 0, 1, \ldots, m-1 \end{bmatrix}
\]

Carrapiço & Odloak (2005) showed that the problem defined in (14) produces a nominally stable MPC if it is solved in a two step approach. In the first step, the objective is to minimize slack \( \delta_i^m \), which is related to the integrating modes. Then, in the second step, the objective is to minimize \( V_k \) while \( \delta_i^m \) is kept at the same value computed in the first step.

In practical terms, the two step approach would be equivalent to adopting the value of the slack weight \( S_2 \) large enough to force the controller to minimize \( \delta_i^m \) before considering the other control objectives. Thus, the IHMPC, which is implemented here, is obtained through the solution to the problem defined in (14) with suitable tuning parameters that produce the nominal stability of the controller.

3. PROCESS OVERVIEW AND CONTROL STRATEGY

A schematic representation of the de-isobutanizer distillation column where the infinite horizon MPC was implemented is illustrated in Figure 1. The system is part of an alkylation unit in the PETROBRAS/Cubatão oil refinery. The feed stream distillation column comes from the FCC unit and consists of a mixture of isobutane, 1-butene, cis-2-butene, trans-2-butene, n-butane and n-pentane. The top product, which is sent to the alkylation reactor, is composed mainly of isobutane and light butenes. The bottom stream that is
composed of n-butane and heavy butenes is sent to storage and sold as a special product.

The feed flowrate is defined by the refinery production plan and usually remains constant over long periods of time. The feed temperature is the main disturbances to the control system. A recycle stream of isobutane and steam are used as sources of heat to the reboiler. The pressure in the top drum is controlled by manipulating the bypass of the top condenser. In the original regulatory strategy, a PID controller of the temperature of tray 68 cascades the steam flowrate to the reboiler and there was no control on the level of liquid in the top drum. The main control objective is to keep the composition of the top product composition at desired values.

As shown in Figure 1, in the control strategy implemented in the IHMPC there are two manipulated inputs: \( u_1 \) (ton/h) is the steam flowrate to the reboiler and \( u_2 \) (m³/d) is the reflux flowrate. The feed temperature \( d_1 \) (°C) is a measured disturbance. The outputs of the distillation column are: \( y_1 \) (%) the level of liquid in the top drum, \( y_2 \) (°C) the temperature of tray 68 and \( y_3 \) (%) the percentage of flooding in the column. The flooding is calculated based on the measured values of some variables of the process. The two degrees of freedom (inputs) are used to control the liquid level in the drum at a fixed set-point and the other two outputs are controlled by zone: the column flooding has to be kept below an upper limit and the temperature in tray #68 has to be kept above a minimum value. Two other outputs that will be included in this control strategy in the near future are the volumetric ratio iC4/(Σ olefin components) in the top product and the volumetric fraction of iC4 in the bottom stream. Step tests were performed in the distillation column and the resulting transfer function model is the following:

\[
\begin{bmatrix}
    y_1(s) \\
    y_2(s) \\
    y_3(s)
\end{bmatrix} = \begin{bmatrix}
    2.3 s^{-2} \\
    4.7 e^{-7} s + 1.4 \times 10^{-3} e^{-2 s} \\
    9.3 s + 1.9 e^{-s} + 1 \\
    10.1 s + 1 \\
    6.8 s + 1 \\
    61 \times 10^{-3} e^{-3 s} + 6.6 s + 1 \\
    6.6 s + 1
\end{bmatrix} \begin{bmatrix}
    u_1(s) \\
    u_2(s)
\end{bmatrix} + \begin{bmatrix}
    0.2 s \\
    0.4 \delta e^{-3 s} \\
    11.6 s + 1 \\
    0.2 \delta e^{-3 s} \\
    12.3 s + 1
\end{bmatrix} d_1(s)
\]

During the identification tests, it was observed that the control valve of the reflux flow rate has shown an erratic behavior probably due to stickiness. Although, this problem could be easily repaired, we found it interesting to perform the evaluation test of the proposed IHMPC in these conditions, as this scenario may be frequently found in industry. So, the sticking reflux control valve becomes an unmeasured disturbance to the MPC controller. The transfer functions represented above relates the outputs to the set points to the regulatory flow control loops.

4. PRACTICAL RESULTS

Figures 2 and 3 show the typical responses of the industrial system with IHMPC when a step disturbance is introduced in the set point of the liquid level. The tuning parameters of the controller are the following: \( m = 2 \), \( \Delta t = 1 \), \( Q = \text{diag}(6,4,1) \), \( R = \text{diag}(0,1,5) \), \( S_1 = S_2 = \text{diag}(1,1,1) \times 10^3 \).

In this case, the column flooding (\( y_3 \)) was controlled at a fixed set point of 91%, while the temperature in tray #68 (\( y_2 \)) was kept above the minimum constraint (52°C). Fig. 3 clearly shows that there is a sticking problem in the valve of the reflux flow rate (\( u_2 \)), where the process variable (PV) has a significant delay in comparison to the corresponding set point (SP). The consequence is a continuous cycling of this variable with a period of about 30min. This disturbance is transferred to the controlled variables of the system, but the IHMPC can cope with this situation quite nicely as the amplitude of the resulting oscillation is largely attenuated. Concerning the tuning parameters of the proposed IHMPC, they can be borrowed from the conventional MPC, except the prediction horizon, which is infinite, and the slack weights \( S_1 \) and \( S_2 \). Typically, these parameters should be two or three orders of magnitude larger than the output weights. The main point related to the slack weights is that they should be large enough to make the hessian matrix \( H \) defined in (11) positive definite. If this condition is not satisfied, the integrating outputs may become unbounded or the stable outputs may show offset.

![Fig. 1. Schematic diagram of the de-isobutanizer column.](image)

There was a question if the proposed IHMPC would amplify this sort of periodic disturbance, as the controller includes equality constraints (12) and (13) related to cancellation of the integrating modes. Apparently, the inclusion of slacks \( \delta_i \) and \( \delta_i^\prime \) greatly reduced this problem. To verify if the gain in stability associated with the use of an infinite prediction horizon would result in a loss of performance, the proposed controller was compared with the conventional MPC. For this purpose, a finite horizon MPC was also implemented in this distillation column. Although, in practice, one cannot repeat exactly the experiment reported above but considering the conventional MPC, Figures 4 and 5 show the responses of the conventional MPC for a similar step disturbance in the set point of the liquid level. The tuning parameters for the two controllers are the same except, for the prediction horizon, which is infinite in the IHMPC and the slack weights, which do not exist in the conventional MPC. Observing figs. 4 and 5, one may conclude that there is no significant difference between the performances of the two controllers and consequently, there is no practical disadvantage in implementing the infinite horizon MPC that introduces nominal stability.
Another practical experiment performed with the IHMPC is shown in figures 6 and 7. In this case, the set point to the flooding percentage ($y_3$) in the column is successively decreased along a series of step changes, while the set point to liquid level in the reflux drum ($y_1$) is fixed and the column temperature ($y_2$) is controlled by zone. Although the performance of the controller can be considered satisfactory, the sticking problem in the reflux control valve seems more serious and heavily affects the behavior of the system, mainly the reflux flow rate ($u_2$) and the column flooding. It is not represented here, but the same kind of behavior is observed when the system is controlled with the conventional MPC.

5. CONCLUSIONS

A MPC with infinite prediction horizon was successfully implemented in an industrial distillation column and has been in continuous operation for several months. The proposed controller can be applied to systems with stable and integrating outputs. The IHMPC was compared to the conventional finite horizon MPC and the performances of the two controllers seem quite similar. The new controller has some additional parameters related to the weighting of slack variables that are introduced in the control problem in order to guarantee that this control problem will remain always

REFERENCES

Fig. 4–Outputs for the conventional MPC. Step change in the liquid level set-point.

Fig. 5–Inputs for the conventional MPC. Step change in the liquid level set-point.

Fig. 6–Outputs for the IHMPC. Step changes in the set point to column flooding.

Fig. 7–Inputs for the IHMPC. Step changes in the set point to column flooding.