Zone Model Predictive Control Algorithm Using Soft Constraint Method

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Abstract: A zone model predictive control algorithm is proposed and developed through the soft constraint method. The estimation of zone violation is avoided; as a consequence the selection of the approximate setpoint when the control variable violates its zone constraint is skipped. To further improve control performance, zone trajectory method is proposed and a parameter is provided to trade off the response performance and model accuracy. The effective performance is proved by the simulation results. The stability of the algorithm is also analyzed.

Keywords: Model Predictive Control; Zone Control; Soft Constraint; Stability Analysis

1. INTRODUCTION

Although in industrial control applications, the controlled variable usually has a specific set point. It is common that many of the controlled variables have range limits rather than set point. This kind of process variable is treated as zone variable in most industrial MPC controller such as RMPCT, DMCPplus and HIECON, which all provide zone and setpoint options for CVs to meet industrial need (Richalet, et al., 1978; Qin and Badgwell, 1997; Morari and Lee, 1999).

Zone control is also necessary for over-specified processes, whose process model can be cast at steady state by the following form (Muske and Rawlings, 1993)

\[
\begin{bmatrix}
    y_1 \\
    \vdots \\
    y_m \\
\end{bmatrix} = \begin{bmatrix}
    a_{i1} & \cdots & a_{in} & u_i & d_i
\end{bmatrix} + \begin{bmatrix}
    \vdots \\
    \vdots \\
    \vdots \\
\end{bmatrix}
\]

(1)

where \( a_{ij} \) is steady gain , \( d_i \) is disturbance. When the number of outputs exceeds the number of inputs, all the set points cannot be met at the same time. If one of the set points is changed into a zone specification, the outputs specifications are relaxed slightly. The probability that the process will meet all of its specifications increases. Moreover, because the output’s change within zone is ignored, the need to coordinate the movement of inputs is largely eliminated, which decreases its sensitivity to model mismatch and improves its robust performance, especially for the process whose outputs and inputs variables are interacted with each other strongly.

In conventional dynamic model control, zone control cannot be solved directly. But the receding optimization formulation of model predictive control provides the possibility to realize zone control. Zhou (2001) used setpoint approximation method to implement zone control, but the limit was that it still needed estimation of zone violation concomitant with the selection of the approximate setpoint value.

In this paper, a zone model predictive control algorithm using the soft constraint method is proposed to achieve better control performance and to avoid the mentioned problem. To further improve control performance, zone trajectory method is proposed which provides a tuning parameter to trade off the response performance and model accuracy. The stability of the algorithm is analyzed finally.

2. ZONE CONTROL ALGORITHM

Consider a stable multi-input multi-output system represented by the following model (Garcia, et al., 1989)

\[
y(k+j|k) = \sum_{i=1}^{N-1} H_j \Delta u(k+j+i) + H_N \Delta u(k+j-N) + d(k+j|k)
\]

\[
d(k+j|k) = d(k|k) - \sum_{i=1}^{N-1} H_j \Delta u(k+i) - H_N \Delta u(k-N)
\]

(2)

where

- \( y(k+j|k) \) = Predicted output vector at time \( k+j \)
- \( y(k) \) = Actual output vector at time \( k \)
- \( u(k) \) = Actual input vector at time \( k \)
\(d(k+j)\)=Predicted disturbance vector at time \(k+j\)

\(N\)= model horizon length
\(r\)= number of inputs
\(s\)= number of outputs

For setpoint control, the optimization problem at every sampling time is solved (Cutler and Ramaker, 1979; Garcia and Prett, 1986; Garcia, et al., 1989): Find the optimal sequence of \(M\) future manipulated variable moves \(\Delta u(k), \ldots, \Delta u(k+M−1)\) so that the prediction of the manipulated variables and controlled outputs satisfy the criteria which minimizes the sum of squared deviations of the predicted CV values from a time varying reference trajectory over \(P\) future time steps. The formulation of optimization problem is:

\[
\begin{align*}
\text{min} & \quad J_u(k) = \sum_{j=1}^{P} \sum_{k=1}^{M-1} [y(k+j)|k] - w(k+j)|k]^2 \\
\text{s.t.} & \quad u^− \leq u(k+j) \leq u^+ \\
& \quad \Delta u^− \leq \Delta u(k+j) \leq \Delta u^+ , \quad \forall j = 1, M
\end{align*}
\]

where

\(w(k+j)|k)\)=reference trajectory value at time \(k+j\)

A zone region is defined by the minimum and maximum values of a controlled variable’s desired range of values. One way to simply implement zone control is to use setpoint approximation method: when the CV is predicted to lie within its zone, its weight coefficient of matrix Q is set to zero so the controller will ignore it; when the CV is predicted to violate its zone limits, its weight is non-zero and a soft constrained controller will avoid it; when the CV is predicted to violate its zone constraint is skipped.

Soft Constraints are used to prevent the controller from introducing transient errors by defining soft constraints on the controlled outputs at intervals from the current interval to predictive horizon. When the controlled variable has a set point instead of a zone region, both the upper and lower limits of the zone are set equal to the set point. Through soft constraint method, the estimation on the zone violation is avoided; as a consequence the selection of the approximate setpoint when the control variable violates its zone constraint is skipped.

In order to drive the outputs back into its zone region more slowly to avoid overshoot consequently, zone trajectory is introduced for each controlled output as follows:

\[
\begin{align*}
\text{min} & \quad J_u(k) = \sum_{j=1}^{P} \sum_{k=1}^{M-1} [y(k+j)|k] - w(k+j)|k]^2 \\
\text{s.t.} & \quad u^− \leq u(k+j) \leq u^+ \\
& \quad \Delta u^− \leq \Delta u(k+j) \leq \Delta u^+ , \quad \forall j = 1, M
\end{align*}
\]

For zone control, the deviation between the output predictive value and zone limits \([y^−, y^+]\) is defined as

\[
e(k+j)=
\begin{cases} 
  [y(k+j)|k] - y^+, \text{if } y(k+j)|k] \geq y^+ \\
  y^+ - (y(k+j)|k], \text{if } y(k+j)|k] \leq y^+ \\
  0, \text{if } y(k+j)|k] \leq y^+ \text{ and } y(k+j)|k] \geq y^+
\end{cases}
\]

The optimization problem of zone control can be formulated as:

\[
\begin{align*}
\text{min} & \quad J_u(k) = \sum_{j=1}^{P} \sum_{k=1}^{M-1} [y(k+j)|k] - w(k+j)|k]^2 \\
\text{s.t.} & \quad u^− \leq u(k+j) \leq u^+ \\
& \quad \Delta u^− \leq \Delta u(k+j) \leq \Delta u^+ , \quad \forall j = 1, M
\end{align*}
\]

In the above problem formulation, the zone limits are treated as soft constraints by adding a slack variable. At the same time the slack variables are also included in the objective function to be minimized.
by the trade-offs that inherently exist between speed of response and model accuracy or inputs movement. A smaller value gives faster response and consequently large MV movement, which requires a more accurate model for stable control. A larger value, on the contrary, gives slower response with smaller MV movement and works well with a less accurate model.

The controller is obliged to keep the CV within the constraints defined by the zone trajectory, but it is allowed to follow any figure within these constraints. The sensitivity to model error is decreased and the robustness is improved.

3. STABILITY ANALYSIS

Alex Zheng and Manfred Morari (1995) analyzed the closed-loop stability for constrained MPC with setpoint control. Zone Control also has the similar property when using soft constraint method. Assume:

(a) There is no model mismatch
(b) Predictive horizon is infinite
(c) Steady-state gain matrix of the model has full row rank.

then the closed-loop system is asymptotically stable if and only if the optimization problem (7) is feasible at the first sampling time.

Proof: If the optimization problem is not feasible, then the controller is not defined.

At sampling time k, the optimal solution is
\[ \Delta u^*(k+i|k), i = 0, \ldots, m - 1 \]
\[ \varepsilon^*(k+i|k), i = 1, \ldots, \infty \]

At sampling time k+1, the solution (18) is a feasible solution but may not be the optimal solution.

\[ \Delta u(k+i|k+1) = \Delta u^*(k+i|k) \]
\[ \varepsilon(k+i|k+1) = \varepsilon^*(k+i|k) \]

Define \( \Delta u_k = \Delta u^*(k|k) \), \( \varepsilon_k = \varepsilon^*(k|k) \)

The above feasible control input yields:
\[ J_{k+1}^* \leq J_{k+1} = J_k - \varepsilon_k^T Q \varepsilon_k - \Delta u_k^T S \Delta u_k \]

\[ J_{k+1}^* \leq J_{k+1} = J_k - \varepsilon_k^T Q \varepsilon_k - \Delta u_k^T S \Delta u_k \]

Therefore, the sequence \( \{J_k^*\} \) is non-increasing, its low boundary is zero. Consequently, the sequence \( \{J_k^*\} \) converges. So

\[ \lim_{k \to \infty} J_{k+1} = \lim_{k \to \infty} J_k^* = 0 \]

This together with \( Q, S > 0 \) implies that \( \varepsilon_k \to 0 \) and \( \Delta u_k \to 0 \) as \( k \to \infty \). Since the steady-state gain matrix of the model is bounded, \( y(k) \) approaches the steady-state value asymptotically.

4. SIMULATION

(1) Consider the two-input three-output system:

\[
G(s) = \begin{bmatrix}
1.77 e^{-28s} & 5.88 e^{-27s} \\
60 s + 1 & 50 s + 1 \\
5.72 e^{-34s} & 6.9 e^{-15s} \\
60 s + 1 & 40 s + 1 \\
4.42 & 7.2 \\
44 s + 1 & 19 s + 1
\end{bmatrix}
\]

with the following input constraints
\(-0.5 \leq u_1, u_2 \leq 0.5, |u_0|, |\Delta u| \leq 0.03\)

and the following initial conditions
\[ y_1 = y_2 = y_3 = 0, u_1 = u_2 = 0 \]

Choose T=5s,N=100,M=4,P=30,Q=I,S=I,\( \alpha = 0.95 \)

If all of the controlled outputs have set points
\[ y_1 = 0.59, y_2 = 0.64, y_3 = 0.67 \]

Because the degree of freedom is insufficient, it is physically impossible to keep all output at setpoint or within range. When the set point for \( y_3 \) is replaced by zone limit [0.65, 0.7], all output specification would be met.
5. CONCLUSION

Estimating the violation of zone output limits in the setpoint approximation method is simply through examining its output predictive initial value, but it can not always keep zone output in its zone limit while correcting other outputs errors. Using the soft constraint method, zone specification is directly imposed as constraints in optimization formulation, while correcting other CV errors, it will not violate zone output limits, but its computing burden is larger than the setpoint approximation method. The tuning parameter provided by zone trajectory method enables a flexible way to achieve better performance and reach a tradeoff between performance and model accuracy.

REFERENCES


