Closed loop Identification of Quadruple Tank System using an Improved Indirect Approach

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Abstract: The well known, indirect method of closed loop identification is difficult to apply for multivariable systems owing to its two step procedure of first identifying the loop sensitivities which are then used in the second step to estimate the plant dynamics. In this paper, we propose an improved indirect method of closed loop identification that exploits (i) simple proportional output feedback control law and (ii) inherent dynamic relationships that results from the implementation of this control law to estimate the plant dynamics in a single step. In the proposed methodology, we used state-space parameterization of the open loop dynamics for both the plant and the loop sensitivities so as to accommodate multivariable systems. The use of the simple proportional output feedback control law in the proposed methodology obviates the need for inverse filtering using the loop sensitivities which could be a problem for large multivariable systems. Since, the proposed methodology involves a single open loop identification between the dither and the output, other problems of noise-output correlation are also not seen. The proposed methodology is demonstrated by a simulation study on a benchmark Quadruple Tank system proposed by Johansson (2000). The simulation results presented in this paper shows the effectiveness of the proposed method for system identification.

1. INTRODUCTION

The closed loop identification methodologies have been eminently proposed to shorten experimental times, to accommodate open loop unstable systems and to provide control relevant models. On the other hand, a closed loop condition presents some additional complications for system identification. The fundamental problem is the correlation that can be generated by such a control law can directly be used to estimate the plant dynamics. Such approaches for multivariable systems have been proposed earlier in Gudi et al. (2004) and Gudi and Rawlings (2006). While the two step method does have some attractive properties, the inverse filtering step could be daunting for relatively larger multivariable systems.

In this paper, we propose an alternate and improved indirect method of closed loop identification. Unlike majority of approaches available in the literature, we work with state realization of input-output models. This facilitates easy treatment of multi-variable as well as non-square systems. In this approach, the inverse filtering step of the indirect two step method is obviated by implementing simple proportional output feedback control law. We show that the relationship that can be generated by such a control law can directly be used to generate an estimate of the plant dynamics. The proposed identification method is validated on a benchmark quadruple tank system proposed by Johansson (2000).

The rest of the paper is organized as follows: The proposed identification methodology is described in section-2. Simulation study on quadruple tank is presented in section-3. The major conclusion reached from the analysis of the proposed method is described in section-4.
2. PROBLEM STATEMENT

The identification problem that we seek to address in this paper is shown in Figure 1. The system under consideration can be a large multivariable plant where there are many inputs to manipulate and many outputs to control. Each of the individual loop outputs is assumed to be affected by noise and unmeasured disturbances. For the purpose of state estimation, it is assumed that the existing multi-loop/multivariable controller is replaced by a suitable proportional feedback controller. The dither signals are assumed to be applied either at the Setpoints or at the controller output depending on the methodology. The identification problem is to generate an estimate of the multivariable plant dynamics using the identification data related to dither and the output, and also by exploiting inherent relationships of the closed loop. We elaborate our approach in the next subsection:

2.1 Improved Indirect Closed Loop Identification

It is proposed to capture the dynamics of the process under consideration using an innovation form of state space model, which is described by the following set of equations

\[
x(k+1) = \Phi x(k) + \Gamma u(k) + L e(k) \tag{1}
\]

\[
y(k) = C x(k) + e(k) \tag{2}
\]

where \( x \in \mathbb{R}^n \) represents state vector, \( u \in \mathbb{R}^m \) represents vector of manipulated inputs, \( e \in \mathbb{R}^r \) represent the innovation sequence and \( y \in \mathbb{R}^q \) represents vector of measurements. The innovation sequence is a zero mean white noise process with Gaussian distribution. The proposed model is equivalent to a MIMO ARMAX transfer function model of the form

\[
y(q) = G(q) u(q) + H(q) e(q) \tag{3}
\]

where

\[
G(q) = C \left[ qI - \Phi \right]^{-1} \Gamma \tag{4}
\]

\[
H(q) = C \left[ qI - \Phi \right]^{-1} L + I \tag{5}
\]

In this work, it is proposed to carry out identification in closed loop by replacing the existing controller by a proportional output feedback controller of the form

\[
u(k) = K (r(k) - y(k)) \tag{6}
\]

where \( K \) represents feedback controller gain. To carry out identification in closed loop, a dither signal is added either at the input or at the setpoint. Thus, the manipulated inputs with the dither signals can be expressed as follows

\[
u(k) = K (r_d(k) - y(k)) + u_d(k) \tag{7}
\]

where, Let \( r_d(k) \) and \( u_d(k) \) represent setpoint dither and input dither signals, respectively. The identification framework for this scenario is shown in Figure 1. Combining equations (1), (2) with equation (7), we have

\[
x(k+1) = [\Phi - \Gamma KC] x(k) + \Gamma [u_d(k) + Kr_d(k)] + (L - \Gamma K) e(k) \tag{8}
\]

Equation 8 can be expressed as the following form of state space model for the closed loop system

\[
x(k+1) = \Psi x(k) + \Gamma [u_d(k) + Kr_d(k)] + \Omega e(k) \tag{9}
\]

\[
y(k) = C x(k) + e(k) \tag{10}
\]

where,

\[
\Psi = \Phi - \Gamma KC \quad \text{and} \quad \Omega = L - \Gamma K \tag{11}
\]

Based on the closed loop model, following observations can be made

- Signals \( r_d(k) \) and \( u_d(k) \) are uncorrelated with \( e(k) \) and direct method can be used for estimation of matrices \( (\Psi, \Gamma, \Omega, C) \).
- Introduction of output feedback proportional controller preserves the order of the state space model. It only modifies the pole locations. This feature can be particularly useful when the process under consideration is open loop unstable.
- Closed loop model is also an innovation form of state space model with the Kalman gain matrix modified.

Any standard state space identification approach, such as prediction error method (PEM) or subspace identification methods, can now be used for estimating matrices \( (\Psi, \Gamma, \Omega, C) \). Let \( (\hat{\Psi}, \hat{\Gamma}, \hat{\Omega}, \hat{C}) \) represent estimates of model matrices for the closed loop system. Then, the matrices appearing in the open loop model can be recovered as follows

\[
\hat{\Phi} = \hat{\Psi} + \hat{\Gamma} K \hat{C} \quad \text{and} \quad \hat{L} = \hat{\Omega} + \hat{\Gamma} K \tag{12}
\]

In practice, the dither is applied either at the input or at the setpoint. Thus, the identification step consists of either identifying a model between \( \{y(k)\} \) and \( \{u_d(k)\} \) or between \( \{y(k)\} \) and \( \{Kr_d(k)\} \).

In a large dimensional MIMO system, the number of parameters required to be estimated can be large for a state space model and, as a consequence, a relatively larger data set is necessary to keep the variance errors low. In such a scenario, it is possible to work with parameterization such as ARMAX or Box-Jenkins for the closed loop model. The closed loop model under the proportional feedback controller (7) can be written as

\[
y(q) = G_c(q) [u_d(k) + Kr_d(k)] + H_c(q) e(k) \tag{13}
\]

Transfer function matrices \( G_c(q) \) and \( H_c(q) \) can be estimated using PEM method. A state realization of the form (9-10) can be constructed using estimated \( G_c(q) \) and \( H_c(q) \) such that

\[
\hat{G}_c(q) = \hat{C} \left[ qI - \hat{\Psi} \right]^{-1} \hat{\Gamma} \tag{14}
\]

\[
\hat{H}_c(q) = \hat{C} \left[ qI - \hat{\Psi} \right]^{-1} \hat{\Omega} + I \tag{15}
\]

Using matrices \( (\hat{\Psi}, \hat{\Gamma}, \hat{\Omega}, \hat{C}) \) and relations (12), matrices for the open loop system \( (\hat{\Phi}, \hat{\Gamma}, \hat{L}, \hat{C}) \) can be recovered.
### Simulation Case Study

In this work, we validate the proposed methodology using simulation study on the benchmark quadruple tank system (Johansson - 2000). Depending on the operating point, the quadruple tank system can behave as a minimum phase and non-minimum phase system. In our simulations, we have considered minimum phase quadruple tank system. A schematic diagram of the process is shown in Figure 3.

The first principal model of the quadruple tank system are given by the following equations (Johansson - 2000)

\[
\frac{dh_1}{dt} = -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_2}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1
\]

\[
\frac{dh_2}{dt} = -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_2}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2
\]

Where, \( A_i \): cross-sectional area of tank \( i \);
\( a_i \): cross-sectional area of the outlet hole of tank \( i \);
\( h_i \): liquid level of tank \( i \);
\( V_i \): volume of tank \( i \);
\( k_1, k_2 \): pump constants
\( \gamma_1, \gamma_2 \): valve positions in the range of 0 – 1
\( v_1, v_2 \): voltage applied to pump -1 and pump -2 respectively
\( i \): 1, 2, 3, 4

In above equations, the four state variables are the levels of the corresponding tanks denoted as \( Z = [h_1 \ h_2 \ h_3 \ h_4]^T \), the manipulated variables are the pump voltages denoted as \( U = [v_1 \ v_2]^T \) and the outputs are the levels of bottom two tanks (tank-1 and tank-2) denoted as \( Y = [h_1 \ h_2]^T \).

The parameters for the minimum phase quadruple tank model are given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1, A_3 )</td>
<td>28</td>
<td>cm²</td>
</tr>
<tr>
<td>( A_2, A_4 )</td>
<td>32</td>
<td>cm²</td>
</tr>
<tr>
<td>( a_{1, 0} ), ( a_{2, 0} )</td>
<td>0.071</td>
<td>cm²</td>
</tr>
<tr>
<td>( a_{2, 0} ), ( a_{4, 0} )</td>
<td>0.057</td>
<td>cm²</td>
</tr>
<tr>
<td>( K_c )</td>
<td>1</td>
<td>V/cm</td>
</tr>
<tr>
<td>( g )</td>
<td>981</td>
<td>cm/s²</td>
</tr>
<tr>
<td>( k_1, k_2 )</td>
<td>3.35, 3.35</td>
<td>cm/s³/Vs</td>
</tr>
<tr>
<td>( \gamma_1, \gamma_2 )</td>
<td>0.70, 0.60</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.1 Linear Simulations

The purpose of the linear simulation studies is to demonstrate that unbiased estimated of the open loop input-output model can be generated using the proposed indirect approach.

A linearized perturbation model is obtained from a truncated Taylor series expansion of the nonlinear differential equations described in Equations 16, 17, 18 and 19, around a steady state operating point \((Z, U, Y)\) described in Table 1. The descretized linear perturbation model (with sampling period of \( T = 3.5 \) sec) for the quadruple tank system are as follows

\[
z(k + 1) = Az(k) + Bu(k) + w(k)
\]

\[
y(k) = Hz(k) + v(k)
\]

where

\[
z = Z - \bar{Z}, \quad u = U - \bar{U}, \quad \text{and} \quad y = Y - \bar{Y}
\]
Here, $w(k) \in \mathbb{R}^n$ represents zero mean state noise with Gaussian distribution and $v(k) \in \mathbb{R}^r$ represents zero mean measurement noise with Gaussian distribution.

For the purpose of testing the ability and accuracy of the proposed method, we used linearized discrete time model given by the Equations 20 and 21. For the linear simulations, $w(k)$ and $v(k)$ are assumed to be zero mean, normally distributed white noise sequences with the standard deviation as $\sigma_w_1 = \sigma_w_2 = \sigma_w_3 = \sigma_w_4 = 0.01$ and $\sigma_v_1 = \sigma_v_2 = 0.05$ respectively. The controller is a simple proportional controller with a gain, $K = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

The dither excitation is given at the controller output and setpoints are kept at zero. The dither signals are generated as random binary signals with an amplitude of 0.25 and frequency band $[0, 0.05\omega_N]$ where $\omega_N = \pi/T$. We define the signal to noise ratio as the ratio of variance of dither signal and variance of noise. In our simulations, the signal to noise ratio is

$$SNR = \begin{bmatrix} 94.97 & 103.03 \\ 95.08 & 103.14 \end{bmatrix}$$

and total of 1000 data points are used for model identification.

The linearized discrete time quadruple tank model considered in our simulation is the 4th order minimum phase model. We used state-space parameterization of the open loop dynamics for both the plant and loop sensitivities so as to accommodate multivariable systems. The prediction error method has been used for the identification. It has been assumed that the correct model order is known a priori. The estimated model matrices are as follows:

$$\hat{\Phi} = \begin{bmatrix} 1.14 & 0.09 & 0.13 & -0.23 \\ 0.09 & 1.03 & -0.02 & 0 \\ 0.09 & 0.08 & 0.78 & -0.01 \\ 0.37 & 0.23 & 0 & 0.60 \end{bmatrix}$$

$$\hat{\Gamma} = \begin{bmatrix} 0.06 & 0.05 \\ -0.07 & 0.02 \\ -0.00 & 0.06 \\ 0.01 & 0.06 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 2 & -2.01 & -1.066 & 0.18 \\ 2.09 & 1.84 & 0.10 & 0.96 \end{bmatrix}$$

Even when the model order is assumed to be known, estimated ($\hat{\Phi}$, $\hat{\Gamma}, \hat{C}$) will be significantly different from (A,B,H). Therefore, it is reasonable to validate the estimated model by comparing

$$G(q) = C[qI - \hat{\Phi}]^{-1} \hat{\Gamma}$$

with

$$G_T(q) = H[qI - A]^{-1} B$$

This can be done by comparing (i) poles of the linearized discrete time model and estimated model, (ii) steady state gain matrices, (iii) step responses and (iv) Nyquist plots. The comparison of poles of the linearized discrete time model and the poles of the estimated model are shown in Table 2. As evident from this table, the estimated poles are quite close to the poles of the linearized discrete time model.

**Table 2. Comparison of poles between linearized discrete time model and estimated model in linear simulation**

<table>
<thead>
<tr>
<th>Poles of the</th>
<th>Poles of the</th>
</tr>
</thead>
<tbody>
<tr>
<td>discrete time model</td>
<td>est. model</td>
</tr>
<tr>
<td>-0.9620</td>
<td>0.9396</td>
</tr>
<tr>
<td>-0.9457</td>
<td>0.9351</td>
</tr>
<tr>
<td>-0.8899</td>
<td>0.8835</td>
</tr>
<tr>
<td>-0.8637</td>
<td>0.8727</td>
</tr>
</tbody>
</table>

The step response comparison for linear simulation is shown in Figure 4. The step response comparison shows that the dynamics are identified accurately, as there is negligible mismatch in steady state gain.

The Nyquist response comparison for linear simulation is shown in Figure 5. The Nyquist response comparison shows that the frequency response of the linearized discrete time model and estimated model matches quite well over a wide frequency range. The bias error is negligible and therefore, identified model agree with the plant dynamics fairly well.
3.2 Non-linear Simulation

In order to test the performance of the proposed methodology in a more realistic scenario, we have used 4th order non-linear quadruple tank model described by the Equations 16, 17, 18 and 19. The minimum phase quadruple tank dynamics are considered and the steady state operating point and parameters are used as per the Table 1. We used the state-space parameterization and the prediction error method for the identification of the dynamics of the non-linear model.

During non-linear simulation, it has been assumed that \( w(k) \) and \( v(k) \) are the zero mean, normally distributed state noise and measurement noise respectively. The standard deviations are kept identical to linear simulation as described above. As reported in the linear simulation, the dither excitation is given at the controller output and setpoints are kept at zero. The dither signals are generated as random binary signals with an amplitude of 0.5 and frequency band \([0 \, 0.05\omega_N]\). The controller gain and signal to noise ratio are kept identical to linear simulation.

We validate the non-linear simulation results by comparing (i) poles of the linearized discrete time model and the poles of the estimated model, (ii) step responses of linearized discrete time model and estimated model, (iii) Nyquist responses of linearized discrete time model and estimated model, (iv) output responses of the non-linear first principal based model and predictions of the estimated model. The estimated model matrices during non-linear simulations are as follows:

\[
\hat{\Phi} = \begin{bmatrix}
1.10 & -0.06 & -0.15 & -0.24 \\
-0.06 & 1 & -0.19 & -0.02 \\
-0.09 & 0.07 & 0.78 & 0.01 \\
0.40 & -0.22 & -0.03 & 0.50 
\end{bmatrix}
\]

\[
\hat{F} = \begin{bmatrix}
0.06 & 0.05 \\
0.07 & -0.01 \\
0 & -0.06 \\
0 & 0.06 
\end{bmatrix}
\]

\[
\hat{C} = \begin{bmatrix}
1.94 & 2.07 & 1.08 & 0.14 \\
2.14 & -1.77 & -0.07 & 0.94 
\end{bmatrix}
\]

The comparison of poles between linearized discrete time model and estimated model are shown in Table 3. In this case also the estimated poles are different than the poles of the linearized discrete time model. This can be attributed to the fact that the true plant dynamics is nonlinear.

<table>
<thead>
<tr>
<th>Poles of the discrete time model</th>
<th>Poles of the est. model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9620</td>
<td>0.9617</td>
</tr>
<tr>
<td>0.9457</td>
<td>0.9326</td>
</tr>
<tr>
<td>0.8899</td>
<td>0.7999</td>
</tr>
<tr>
<td>0.8637</td>
<td>0.7075</td>
</tr>
</tbody>
</table>

The step response and Nyquist response comparison between linearized discrete time model and estimated model are shown in Figure 6 and Figure 7, respectively. While the poles of the estimated model are different when compared with the poles of the linearized discrete time model, the step responses and the frequency responses show a reasonably good match with the those of the linearized first principle model.

Step responses and Nyquist response comparison shows that the estimated model is able to capture the dynamics of the non-linear quadruple tank system.

The comparison between measured and predicted outputs along with the input signal from independent validation data set are shown in Figure 8 and Figure 9, respectively. It shows that the estimated model captures the dynamic behavior of the non-linear quadruple tank system with a reasonable accuracy.
Fig. 9. Manipulated Inputs

4. CONCLUSION

The proposed improved closed loop identification methodology estimates the model in a single step using the relationship generated by implementing simple output feedback proportional controller. Since, the proportional controller does not increase the order of the plant, it becomes relatively easy to extract the model from the closed loop. The state-space parameterization and prediction error method used in our approach facilitates the identification of Multi Input Multi Output (MIMO) system. The concept of using dither signal as an identification input eliminates the correlation problem of closed loop identification. The proposed methodology has been validated on a representative case study on Quadruple Tank Process and it has been shown to function satisfactory.

5. REFERENCES


