Sensitivity analysis with optimal input design and model predictive control for microalgal bioreactor systems

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Abstract: Microalgae have been suggested as a promising feedstock for producing biofuel because of their potential of lipid production. In this study, a first principles ODE model for microalgae growth and neutral lipid synthesis proposed by Surisetty et al. (2010) is investigated for the purpose of maximizing the rate of microalgae growth and the amount of neutral lipid. The model has 6 states and 12 parameters and follows the assumption of Droop model which explains the growth as a two-step phenomenon; the uptake of nutrients is first occurred in the cell, and then use of intracellular nutrient to support cells growth. In this study, optimal input design using D-optimality criterion is performed to compute the system input profile and sensitivity analysis is also performed to determine which parameters have a negligible effect on the model predictions. Furthermore, model predictive control based on successive linearization is implemented to maximize the amount of neutral lipid contents.

Keywords: Microalgae, optimal input design, Droop model, sensitivity analysis, model predictive control

1. INTRODUCTION

Microalgae are photosynthetic microorganisms, which have an ability to produce large amounts of oil that can be used directly as a high value bioactives or be used to synthesize biodiesel. The oil content in microalgae ranges from 15% to 77% depending on species and culture conditions [Yusuf, 2007]. Although the oil production rate in microalgae is strain dependent, it has several advantages as a feedstock for biodiesel because of high growth rate and the ability of producing high amounts of lipid [Miao and Wu, 2006, Minowa et al., 1995]. However, biodiesel from microalgae is not economically competitive compared to biodiesel from conventional plant sources or petrodiesel [Yusuf, 2007]. For the economic competitiveness, optimal model-based control strategy is required to increase the rate of microalgae growth and the amount of stored oil.

However, there are some difficulties in applying model-based control to microalgal bioreactor systems. Microalgal systems are highly nonlinear and some of the parameters and states are difficult to measure directly or estimate. Furthermore, metabolism inside the cells makes process response very slow and its effect on process is difficult to understand.

In order to explain cell growth in bioreactor systems, more than 50 phenomenological models have been proposed in the literature. The first model describing micro-organisms growth was proposed by Monod and this model is based on the assumption that cell growth is proportional to the consumption rate of extracellular limiting nutrient [Monod, 1950]. However, some discrepancies appear in experimental data for photosynthetic algal systems. For algal systems, Droop proposed a new approach explaining cells growth as a two-step phenomenon; the uptake of nutrients is first occurred in the cell, and then use of intracellular nutrient to support cells growth [Droop, 1968]. The Caperon-Meyer model improves the Droop model by introducing the minimum extracellular nutrient concentration for nutrient uptake rate [Caperon and Meyer, 1972].

Recently, Surisetty et al. suggested an updated model that modified the Caperon-Meyer model by introducing the simultaneous effect of carbon and nitrogen concentration to the growth rate [Surisetty et al., 2010]. This model can describe the cell growth better, but has many parameters to identify. Towards implementing a model-based control strategy, the objective of this study is to design the optimal input profile to get the maximum information about the effect of parameters on the process outputs and to determine the parameters which have significant effects on model prediction by performing the sensitivity analysis.

Furthermore, model predictive control (MPC) based on successive linearization is implemented for optimization and control of the amount of neutral lipid contents in the microalgae. Most of the model based control algorithms are based on the linear models, since various of the techniques are available for identification and optimization [Zoltan Kalman, 2007]. However, most of the chemical processes are highly nonlinear with wide operating ranges. Hence, the development of nonlinear MPC (NLMPC) techniques, which use nonlinear models for prediction, was...
motivated. Among these techniques, successive linearization of the nonlinear model (nonlinear quadratic dynamic matrix control (NLQDMC)) has seen significant industrial application, mainly due to its mild computational requirement[Lee and Ricker, 1994]. In this study, NLQDMC technique is implemented to see its applicability for microalgal bioreactor systems.

2. MICROALGAL BIOREACTOR MODEL

A microalgal bioreactor model employed in this study is summarized in Table 1. This model contains 3 inputs, 12 parameters and the 6 system states. Equations $f_1,f_6$ represent the material balance for the continuous bioreactor systems and six states are 1) $x$ : functionally active biomass concentration (g/ml), 2) $S_1$ : nitrogen source concentration in culture media (g/ml), 3) $S_2$ : carbon source concentration in culture media (g/ml), 4) $Q$ : total concentration of intracellular nitrogen (g/ml), 5) $I_0$ : total algal oil stored in cells (g/ml) and 6) $V$ : total reaction volume (ml). Microalgae growth rate ($\mu$) depends on the nitrogen cellular quota (g) and carbon source concentration in culture media and follows the Michaelis-Menten relationship. The uptake rate of the nitrogen source to the cells ($\rho$) is the function of nitrogen concentration in culture media with a minimum external nitrogen concentration ($S_0$) at which uptake occurs[Caperon and Meyer, 1972]. Since the algal oil production increases in nitrogen starvation conditions, it is assumed that the oil production rate ($\pi$) is only affected by carbon source concentration in culture media. The system inputs are flow rate of nitrogen rich feed ($f_1$), flow rate of carbon rich feed ($f_2$) and outlet flow rate ($f_0$). It is further assumed that fed-batch operation is performed with $f_0=0$.

The nominal parameter values used for the calculation of the optimal input profile and sensitivity analysis were selected from the literature[Suri et al., 2010]. Table 2 shows the nominal parameter values and known quantities.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_m$</td>
<td>Maximum growth rate</td>
<td>0.15</td>
<td>1/h</td>
</tr>
<tr>
<td>$q_m$</td>
<td>Maximum cell quota for supporting growth</td>
<td>0.027</td>
<td>g/g</td>
</tr>
<tr>
<td>$K_q$</td>
<td>Half saturation constant of nitrogen quota</td>
<td>0.5</td>
<td>g/g</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Maximum uptake rate</td>
<td>0.08</td>
<td>1/h</td>
</tr>
<tr>
<td>$K_s$</td>
<td>Half saturation constant of glucose for growth</td>
<td>0.014</td>
<td>g/ml</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Threshold substrate concentration</td>
<td>1.8-5</td>
<td>g/ml</td>
</tr>
<tr>
<td>$1/Y_{xs}$</td>
<td>Inverse of biomass to substrate yield</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>$k_m$</td>
<td>Maintenance constant</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$1/Y_{ps}$</td>
<td>Inverse of product to substrate yield</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>$\pi_m$</td>
<td>Maximum oil production rate</td>
<td>0.05</td>
<td>1/h</td>
</tr>
<tr>
<td>$K_o$</td>
<td>Hall saturation constant for oil production</td>
<td>0.01</td>
<td>g/ml</td>
</tr>
<tr>
<td>$1/Y_{xq}$</td>
<td>Inverse of biomass to substrate yield</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

3. OPTIMAL EXPERIMENTAL DESIGN

For the parameter sensitivity analysis which is used to determine key parameters in the process, optimal input signal was calculated using D-optimality criterion[Yao et al., 2003]. Following matrix is a sensitivity matrix whose entries are the partial derivatives of the state variables ($z$) with respect to the parameters at specific time points.
By computing $\partial y / \partial P$, the output sensitivity matrix $(Z)$ can be expressed as in Eq. (4) which represents the effect of parameter values on the system outputs. The sampling time and total calculation time was assumed to be 1 hour and 15 days, respectively. Then, $Z$ becomes a $1440\times 12$ matrix.

$$Z = \begin{pmatrix}
\left( \frac{\partial y_1}{\partial P_1} \right)_{t_1} & \cdots & \left( \frac{\partial y_1}{\partial P_2} \right)_{t_1} \\
\vdots & \ddots & \vdots \\
\left( \frac{\partial y_f}{\partial P_1} \right)_{t_f} & \cdots & \left( \frac{\partial y_f}{\partial P_2} \right)_{t_f}
\end{pmatrix}
$$

(4)

The entries of $\partial z / \partial P$ is computed integrating the sensitivity equations in Eq. (2) by ODE solver which is obtained using chain rule, where $f$ is the vector values function listed in Table 1.

$$\frac{d}{dt} \left( \frac{\partial z}{\partial P} \right) = \frac{\partial f}{\partial z} \frac{\partial z}{\partial P} + \frac{\partial f}{\partial P}$$

(2)

Then, $\partial y / \partial P$ is also computed using chain rule as in Eq. (3).

$$\frac{\partial y}{\partial P} = \frac{\partial h}{\partial P} + \frac{\partial h}{\partial z} \frac{\partial z}{\partial P}$$

(3)

The optimal input ($u^*$) can be calculated by solving optimization problem of Eq. (5), which maximizes the determinant of $(Z^T Z)$. Among various statistical criteria, D-optimal criteria seeks to minimize $|X^T X|^{-1}$, or maximize the determinant of the information matrix $X^T X$ of the design. Because the input switch frequency was chosen as 12 hours within 15 days of total calculation time, the number of optimization variables ($u_1, \ldots, u_f$) were 52 and the optimization problem was solved using “Pattern search tool” available in Matlab.

$$u^* = \arg \max_{u \in U} |Z^T Z|$$

(5)

4. PARAMETER SENSITIVITY ANALYSIS

For the optimal model-based control of microalgal bioreactor systems, reducing the number of parameters to estimate is important. Generally, not all parameters have a significant effect on the model predictions [Yao et al., 2003]. The sensitivities of the parameters on the system outputs are computed to determine the parameters with a negligible effect on the outputs.

The most generally used sensitivity analysis method is a gradient-based method as in Eq. (6). This method also called local sensitivity analysis because it is calculated at nominal parameter values ($p^*$) [Kim et al., 2010].

$$S_{ij} = \frac{\partial y_i}{\partial P_j} \bigg|_{p^*}$$

(6)

However, in reality, the nominal parameter values are highly uncertain or unknown, and only a reasonable range is known during model development stage. Hence, the result of a single local sensitivity analysis will not be enough to identify dominant parameters. Thus, parameter-independent, global sensitivity analysis techniques have received great interest. For the calculation of global sensitivity, local sensitivities are averaged with multiple parameter
choices selected randomly within parameter ranges as in Eq. (7).

\[
\tilde{S}_{ij} = \frac{1}{N_{\text{sample}}} \sum_{k=1}^{N_{\text{sample}}} S_{ij}(k)
\]

In this study, global sensitivity analysis was performed with 1000 randomly selected values of parameters. Figure 2 shows the results of the global sensitivity analysis and can be found that the dominant parameters are \(u_m\), \(K_q\), \(K_s\), \(S_0\).

Fig. 2. Global sensitivity analysis

5. MODEL PREDICTIVE CONTROL

For the purpose of control and optimization of the amount of neutral lipid, model predictive control based on successive linearization is implemented on the nominal model in Table 1 to control the output (mass fraction of lipid, \(I_p/X\)) in the microalgae bioreactor systems. To control the mass fraction of lipid, the feed rates of nitrogen and carbon are manipulated. The objective function is the predicted deviation of the output from the set point plus some penalty on the input movement size measured in terms of the quadratic norm as in Eq. (8).

\[
\sum_{i=1}^{P} \left[ \begin{array}{c} r(k+1) - y(k+1) \\ r(k+2) - y(k+2) \\ r(k+p) - y(k+p) \\ \end{array} \right]^{T} Q \left[ \begin{array}{c} r(k+1) - y(k+1) \\ r(k+2) - y(k+2) \\ r(k+p) - y(k+p) \\ \end{array} \right] + \sum_{i=0}^{m-1} \left[ \begin{array}{c} \Delta u(k+1) \\ \Delta u(k+2) \\ \Delta u(k+m-1) \\ \end{array} \right]^{T} R \left[ \begin{array}{c} \Delta u(k+1) \\ \Delta u(k+2) \\ \Delta u(k+m-1) \\ \end{array} \right] \]

In Eq. (8), \(y(k+i|k)\) is the output \(y(k+i)\)(mass fraction of lipid) calculated from information available at time \(k\), \(u(k+i)\) is the input \(u(k+i)\)(flowrate of nitrogen and carbon) calculated from information available at time \(k\), \(\Delta u(k+i)\) is selected as 0.55 in this study because the microalgae species we tried to cultivate is known to have 55% of lipid contents. \(Q\) and \(R\) are weighting matrices in objective function and selected as 15 and 3, respectively, to give more weight on reducing the deviation of the output from the set point than input movement size. The prediction horizon \(P\) was chosen as 15 and the control horizon \(M\) was chosen as 6.

5.1 Multi-step prediction equation

In Eq. (8), predicted output \(y(k+i|k)\) is calculated by Eq. (9).

\[
\begin{bmatrix}
\tilde{y}(k+1|k) \\
\tilde{y}(k+2|k) \\
\vdots \\
\tilde{y}(k+p|k)
\end{bmatrix} =
\begin{bmatrix}
y(m_k - y(k)) \\
y(m_k - y(k)) \\
\vdots \\
y(m_k - y(k))
\end{bmatrix} +
\begin{bmatrix}
S_{0}^{u} \\
S_{1}^{u} \\
\vdots \\
S_{m}^{u}
\end{bmatrix} \Delta u(k) +
\begin{bmatrix}
S_{1}^{d} \\
S_{2}^{d} \\
\vdots \\
S_{P+1}^{d}
\end{bmatrix} \Delta d(k) +
\begin{bmatrix}
S_{1}^{f} \\
S_{2}^{f} \\
\vdots \\
S_{P+1}^{f}
\end{bmatrix} \text{feedforward term}
\]

In this study, feedback term is neglected because we assume that output from model prediction \((\tilde{y}(k|k))\) and output measured from the plant \((y_m(k))\) are the same. Feedforward term is also neglected because the disturbances are varying only infrequently.

The microalgal bioreactor model in Table 1 can be expressed as

\[
\begin{align*}
\dot{z} &= f(z, u) \\
y &= h(z)
\end{align*}
\]

By linearization of 1st order approximation of ODEs around an equilibrium at every sample time, a continuous-time state-space system is obtained

\[
\begin{align*}
\dot{z} &= A^c z + B^c u \\
y &= H^c z
\end{align*}
\]

where

\[
A^c = \begin{bmatrix}
\frac{\partial f_1}{\partial z_1} & \frac{\partial f_1}{\partial z_2} & \cdots & \frac{\partial f_1}{\partial z_n} \\
\frac{\partial f_2}{\partial z_1} & \frac{\partial f_2}{\partial z_2} & \cdots & \frac{\partial f_2}{\partial z_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m}{\partial z_1} & \frac{\partial f_m}{\partial z_2} & \cdots & \frac{\partial f_m}{\partial z_n}
\end{bmatrix},
B^c = \begin{bmatrix}
\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_n} \\
\frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m}{\partial u_1} & \frac{\partial f_m}{\partial u_2} & \cdots & \frac{\partial f_m}{\partial u_n}
\end{bmatrix},
H^c = \begin{bmatrix}
\frac{\partial h_1}{\partial z_1} & \frac{\partial h_1}{\partial z_2} & \cdots & \frac{\partial h_1}{\partial z_n} \\
\frac{\partial h_2}{\partial z_1} & \frac{\partial h_2}{\partial z_2} & \cdots & \frac{\partial h_2}{\partial z_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial h_m}{\partial z_1} & \frac{\partial h_m}{\partial z_2} & \cdots & \frac{\partial h_m}{\partial z_n}
\end{bmatrix}
\]

By converting continuous-time system to discrete-time system, Eq.(13) is obtained.

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k \\
y_k &= Hx_k
\end{align*}
\]
Then, dynamic matrix \((S_u)\) can then be calculated by the principle of superposition from the above equation at every sample time. Open loop prediction vector \(\hat{y}(k + i|k)\) is computed by integrating the ODEs every sampling time with fixed input using Matlab ODE Solver.

### 5.2 Control action and recursive update of memory

From section 5.1, predicted output \(y(k + i|k)\) is obtained. By substitution of it into Eq. (8), optimization problem can be formulated. For the calculation of optimization problem, objective function in Eq. (8) is changed to the following form of quadratic program:

\[
\min_{\Delta u} \left( \varepsilon^T(k) Q \varepsilon(k) \right) + 2 \varepsilon^T(k) Q S^u \Delta u(k) + \Delta u^T(k) (S^u^T Q S^u + R) \Delta u(k)
\]

where \(H\) : Hessian matrix, \(g\) : gradient vector, \(\Delta u\) : decision variable,

\[
\varepsilon(k) = R(k + 1|k) - Y(k + 1|k) - S^d \Delta d(k) - I_p (g(k) - \hat{y}(k/k))
\]

The Hessian \(H\) is a constant matrix while the gradient vector \(g\) must be updated at each time step. To the above minimization objective, the following input magnitude constraints is added.

\[
u_{\text{min}} \leq u(k + l|k) \leq u_{\text{max}}
\]

\[
|\Delta u(k + l|k)| \leq \Delta u_{\text{max}}, \quad l = 0, \ldots, m - 1
\]

In this study, \(u_{\text{min}}\) is selected as 0, \(u_{\text{max}}\) is selected as 50, and \(\Delta u_{\text{max}}\) is selected as 10.

To improve the quality of prediction developed from a linearized model, iteration between the steps of the control input computation and sensitivity matrix calculation is needed. Local linearization is justifiable only when the computed inputs \(u_k, \ldots, u_{k+m-1}\) do not deviate from \(u_{k-1}\) by much.

The computed control moves are implemented in receding horizon fashion; first, optimization problem is calculated at time \(k\) over the predictor horizon of \(P\) time steps. Then, only first move \(\Delta u\) is implemented for the calculation of next output. In the next step, the whole optimization procedures are repeated at the next sampling time.

### 5.3 Results of model predictive control

The MPC based on successive linearization is implemented on the microalgal bioreactor systems to control the amount of mass fraction of lipid. Since the slow dynamics of the microalgal bioreactor systems, the sampling time is chosen as 24 hr and the results of MPC with selected conditions are shown in Figure 3. As shown in Figure 3, the mass fraction of algal oil is controlled by manipulating the flow rate of nitrogen and carbon feed. As the mass fraction of oil is increased, the flow rate of carbon feed also increased. The oil production is optimized in nitrogen starvation conditions, and replenishment of carbon is necessary to maintain the oil production.

**Fig. 3. Results of the model predictive control**

### 6. CONCLUSION

Microalgal bioreactor systems are investigated for optimal model-based control of microalgae. A first principles ODE model with 6 states and 12 parameters was used. Design of optimal input signal was performed using D-optimality criterion to maximize the information (effect of parameter values on the system outputs) for the sensitivity analysis. Then, global sensitivity analysis was performed using the input signals computed from the D-optimality criterion and found five dominant system parameters. For the control and optimization of mass fraction of oil, model predictive control based on successive linearization is implemented.

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**REFERENCES**


