Handling Infeasibilities in Dual Modifier-Adaptation Methodology for Real-Time Optimization

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Abstract: This work shows an extension of dual-modifier adaptation methodology for RTO to reduce the infeasibilities. The main idea is to add a PI controller that is activated only when the measurements shows a violation in the constraints. Since the dual problem is solved to estimate the gradients of the process, an additional controller must be considered in order to increase the inverse of the condition number of the matrix formed with past values. The methodology presented has been applied in a simulated oxygen consumption plant. The results show that, under modelling mismatch, the method finds the real optimum of the process in a feasible path.

Keywords: Real-time optimization, Dual modifier adaptation, Infeasibility, Condition number, PI Controller

1. INTRODUCTION

In the process industry, plants are composed of many complex interconnected units. Finding the optimal operation point of these facilities is not a trivial task, neither by intuition nor with an off-line computation, due to the inherent difficulties of the process itself, as well as the uncertainties that affect the operating conditions. Therefore, it is necessary to use a systematic mechanism to find out the optimal operating point. In highly automated plants, optimal operation is typically addressed by a decision hierarchy involving several levels including scheduling, real-time optimization (RTO) and process control. At the RTO level, decisions are made on a time scale of hours to a few days, considering economic objectives in an explicit way solving a steady state optimization. (Marlin and Hrymak, 1997).

RTO emerged in the late 1970’s as a two-stage algorithm: parameter estimation and economic optimization. In the first step, the uncertainties are taken into account updating the parameters \( \alpha \) of a nonlinear model, solving a parameter estimation problem (1), where \( u \in \mathbb{R}^m \) is the actual value of the manipulated variables, \( y \in \mathbb{R}^p \) represent the model outputs, and the superscript ‘’ indicates that the variable is measured from the process.

\[
\min_{\alpha} \Phi_{\text{id}} := (y - \bar{y})^T R (y - \bar{y}), \quad \text{s.t.:}
\]
\[
f(x, u, \alpha) = 0, \quad y = h(x, u, \alpha)
\]
\[
g(x, u, \alpha) \leq 0
\]  

(1)

In certain cases (1) can be formulated as a data reconciliation problem. After the model update, an economic optimization is performed, and a new stationary point is found (2). The solution of (2) is applied to the process in an iterative scheme until no further improvements in the cost function are observed.

\[
\min_{u, \alpha} \Phi_{\text{eco}} := \phi(u, x, \alpha), \quad \text{s.t.:}
\]
\[
f(x, u, \alpha) = 0
\]
\[
g(x, u, \alpha) \leq 0
\]  

(2)

The intrinsic uncertainty between the model and the process, which can be understood as a modelling mismatch, and the interaction that takes place when parameter estimation and economic optimization are solved separately, implies that the two-step algorithm will not necessarily converge to the optimum of the process (Roberts, 1979). With this in mind, Roberts solved the integrated system optimization and parameter estimation problem: ISOPE. As a result, to compensate for modelling errors, an additional modifier is added to the economic optimization that takes into account the difference between the gradient of the model and the process. Later, Tatjewski showed that the convergence to the true optimum in the ISOPE method does not depend on the parameter estimation problem, but only on the equality between the output of the process and the model in each RTO iteration (Tatjewski, 2002). Following the same idea, Gao and Engell proposed an extension in order to handle process-dependent constraints (Gao and Engell, 2005). Recently, Marchetti and co-workers (Marchetti et al., 2010, Marchetti et al., 2009) have generalized this methodology presenting the Modifier-adaptation method.

Even though modifier adaptation converges to the optimum of the process, due to the modelling mismatch it might happen that the convergence follows an infeasible path. In this work, a method that tries to reduce the infeasibility occurrence is presented with the purpose of contributing to the applicability of the modifier adaptation approach. The work is organized as follows. In chapter 2 the generalities of modifier adaptation are presented. In chapter 3 the method proposed to avoid infeasibilities is described. Chapter 4 shows a
test of the method in a simulated oxygen consumption unit. Finally, section 5 presents some conclusions and future work.

2. MODIFIER ADAPTATION METHODOLOGY

The modifier adaptation problem is summarized in (3).

\[
\begin{align*}
\min_{\mathbf{u} \in \mathcal{U}} & \Phi_{\text{mod}} := \phi(\mathbf{u}, \mathbf{x}, \alpha) + \lambda^T \mathbf{u} \\
\text{s. t.:} & \mathbf{f}(\mathbf{x}, \mathbf{u}, \alpha) = 0 \\
& \mathbf{G}_{m}(\mathbf{x}, \mathbf{u}, \alpha, \mathbf{b}_k, \gamma_k) := \mathbf{g}(\mathbf{x}, \mathbf{u}, \alpha) + \mathbf{b}_k + \gamma_k \mathbf{u} - \mathbf{u}_k \leq 0
\end{align*}
\]

(3)

The modifiers directly correct the problem formulation (2) so that the Karush-Kuhn-Tucker (KKT) conditions of the model (Luenberger and Ye, 2008), match with the ones of the process. Equations (4 – 6) show the definition of these modifiers calculated in the k th RTO iteration: \( \lambda_k \) and \( \gamma_k \) are in charge of correcting the gradient of the objective function and the constraints respectively, while \( \mathbf{b}_k \) attempts to modify the value of the process dependent constraints. It can be noted that they depend both on the gap between process and model variables (values and gradients) and their previous value, using a first order filter with a filter constant \( K \) that tries to smooth the path to achieve the optimum. In the entire document the bar over the name of the variables denotes magnitudes referred to the process.

\[
\begin{align*}
\lambda_k &= (I - K^T) \lambda_{k-1} + K^T \left[ \frac{\partial \phi}{\partial \mathbf{u}} \frac{\mathbf{u}}{\partial \mathbf{u}} \phi(\mathbf{u}_k, \mathbf{x}_k, \alpha) \right] \\
\mathbf{b}_k &= (I - K^T) \mathbf{b}_{k-1} + K^T (\hat{\mathbf{g}}(\mathbf{u}_k) - \mathbf{g}(\mathbf{u}_k, \mathbf{x}_k, \alpha)) \\
\gamma_k &= (I - K^T) \gamma_{k-1} + K^T \left[ \frac{\partial \mathbf{g}(\mathbf{u}_k, \mathbf{x}_k, \alpha)}{\partial \mathbf{u}} \right]
\end{align*}
\]

(4 – 6)

2.1 Implementation of the Modifier Adaptation Methodology

Fig. 1 shows the diagram of the implementation of the modifier adaptation methodology.

Fig. 1. Implementation of Modifier-Adaptation Method.

It can be seen that for every iteration of the RTO layer, it is necessary to estimate the gradient of the cost function and the constraints w.r.t. the decision variables \( \mathbf{u} \), for the process and for the model. The bias \( b \) is not difficult to obtain, but the gradient estimation of the process is the key issue of the RTO methods based in modifiers. Mansour and Ellis (Mansour and Ellis, 2003) present a compilation of methods for estimating these process gradient: Finite differences, Dual control optimization, Broyden’s approximations and Dynamic model identification. In this work only the first two will be applied.

Finite Differences: It consists in applying changes of size \( \delta_u \) in the manipulated variables around the actual operation point, then for each change the value of the output variable in steady state is measured. The derivatives are calculated using the finite differences definition (7).

\[
\frac{\partial y(k)}{\partial u} = \frac{\hat{y}(\mathbf{u}_k + \delta_u) - \hat{y}(\mathbf{u}_k - \delta_u)}{2 \delta_u}
\]

(7)

Dual Control Optimization: It is assumed that there are as many past values of the manipulated variables as the number of outputs. Then, by using the definition of directional derivative, the partial derivative of each measured output can be estimated.

Defining the vectors of differences in the manipulated variables \( \mathbf{s}^{k, t} = \mathbf{u}_{k-t} - \mathbf{u}_k \), \( t = 1 \ldots N_u \), and in the measured variable \( j \): \( \Delta \mathbf{y}^j = [\Delta y^{j, k-1} \ldots \Delta y^{j, k-N_u}]^T \), the gradient estimation of the measured variable \( \Delta \mathbf{y}^j \) can be approximated as in (8), assuming that the previous operation points are close enough to estimate a derivative as a finite difference (Brdys and Tatjewski, 2005).

\[
\frac{\partial \Delta \mathbf{y}^j}{\partial \mathbf{u}} = \mathbf{S}^{-1} \Delta \mathbf{y}^j, \\
\mathbf{S}^k = \left[ \begin{array}{c} S^{k,1} \\
\vdots \\
S^{k,N_u} \end{array} \right]^T
\]

(8)

Due to the fact that this technique involves the calculus of \( S^{-1} \), the value of its condition number (\( \kappa \)) is crucial in order to ensure appropriate estimations of the gradient in each RTO iteration. Previous discussion implies that the additional constraint (9) must be added to the general formulation of the modifier adaptation methodology (3), which transforms the original problem in the so called “Dual” problem (Tatjewski, 2002, Marchetti et al., 2010).

\[
d(\mathbf{s}^{k+1}) \geq d^{LO}, d(\cdot) = \kappa(\cdot)^{-1}, \\
\mathbf{s}^{k+1} = \left[ \begin{array}{c} \mathbf{s}^{k+1,0} \\
\vdots \\
\mathbf{s}^{k+1,N_u-1} \end{array} \right]^T
\]

(9)

Being \( d^{LO} \) a lower bound for the inverse of \( \kappa \).

Even though both methods presented give a suitable estimation of the gradient, finite differences has the disadvantage that many disturbances on the process are required, which can be difficult to apply for problems with a large number of decision variables.

3. HANDLING INFEASIBILITIES IN DUAL MODIFIER ADAPTATION METHODOLOGY

Because of the modelling mismatch, the path followed by the process to reach the optimum given by the modifier adaptation RTO scheme can include infeasible points, even if the starting and the end points are feasible. According to Marchetti and co-workers, these infeasibilities can be reduced decreasing the value of the filter constant presented in (4 – 6) (Marchetti et al., 2009).

The option proposed in this work is to reduce the occurrence of infeasibilities between the \( k \) and the \( k+1 \) RTO iteration, performing continuous corrections to the value given by the real time optimization during the transient. If the infeasibilities are detected by the control layer, these corrections can be done at every sampling time of a controller, for instance, a PI controller with an anti-windup system.
It will be assumed that the process constraints can be directly measured or estimated, which is a valid assumption considering that a modifier must be calculated for these variables. Then, the infeasibility error of the process constraint $g$ in the $n$th sampling time $e^g_n$ can be defined as the maximum among zero and the difference between the measured constrained variable $\bar{g}_n$ and its upper bound $g^{UP}(10)$:

$$e^g_n = \max(0, g_n - g^{UP})$$

(10)

It is possible to state a correction action of the manipulated variable using a PI controller, in order to calculate its new value to be applied in the $n+1$ sampling time of the controller using:

$$u_{n+1} = u_n + K_a^d e^g_n \quad u_1 = u^{RTO}_k$$

(11)

Being $u^{RTO}_i$ the solution of the RTO layer in its $k$th iteration and $K_a^d$ the gain of the controller.

If dual control optimization has been used as a method to estimate the process gradients, corrections in the modified variables can affect the dual constraint (9). This can produce inaccurate estimations of the process gradients in the next RTO iteration. Therefore, an additional controller must be added to ensure compliance with this constraint. Defining the infeasibility error of the inverse of the condition number in the $n$th sampling time $e^{d^{-1}}_n$, as (12) presents:

$$e^{d^{-1}}_n = \max(0, d^{IO} - d_n)$$

(12)

Where the inverse of the condition number $d_n$, is calculated using previous corrected values given by the RTO and the corrected value from (11):

$$d_n = \kappa(S^{n+1})^{-1}$$

$$S^{n+1} = [s_1^{n+1} \; s_2^{n+1} \; \ldots \; s_{n+1,n}^{n+1}]^T$$

$$S^{n+1} = u_k^{RTO} - u_k^{-1}$$

(13)

Variable $u_k^{RTO}$ represents the previous value given by the RTO layer in the $i$th iteration, corrected with the infeasibility controller proposed until steady state, i.e., the last corrected value during the transient between the $i$ and the $i+1$ RTO iteration. Finally, the value to be applied to the process is calculated from a PI controller:

$$u_{n+1} = u_{n+1}^{RTO} + K_d^d e^{d^{-1}}_n$$

(14)

Figure 2 summarizes the application of the method proposed. It can be noted that if no infeasibilities are detected during the transient, the value applied in each sampling time of the controller is equal to the one calculated by the RTO layer.

Because persistent corrections are performed when infeasibilities are detected, steady state is not reached until the process becomes feasible (assuming no saturation in the manipulated variable), hence the Modifier-adaptation methodology with the infeasibility controller proposed will always converge to the optimum in a feasible path.

4. SIMULATION EXAMPLE

To test how the proposed method works, it has been applied to a simulation example shown in Fig.3: a reactor that uses oxygen to perform a chemical reaction.

![Diagram of one consumption unit in the ODN](image)

In this unit, two streams with a mixture of nitrogen and oxygen ($G_1$ and $G_2$, each of them with different concentrations), are fed to a continuous stirred reactor ($G_{in}$). Together with the gas mixture, an aqueous solution containing sodium sulphite (Na$_2$SO$_3$) and cobalt chloride (CoCl$_2$) is sent to the reactor ($G_{in}$) to be treated. The oxygen that comes from the gas mixture can diffuse into the solution and react with the Na$_2$SO$_3$ producing sodium sulphate (Na$_2$SO$_4$) with the help of CoCl$_2$ that acts as a catalyst, just like (15) states.

$$Na_2SO_3 + \frac{1}{2}O_2 \rightarrow Na_2SO_4$$

(15)

The aqueous product is removed from the reactor ($G_{out}$). Because of the solubility limits of oxygen in water, only one fraction of the fed oxygen can be used in the reactor, while the unused gas leaves the unit ($G_{out}$).

4.1 Model of the process

To model the process, a first principle approach has been used with the following assumptions:

- Ideal gas behaviour (pressure is 1bar)
- The reactor can be modelled as a CSTR
- Absence of Na$_2$SO$_4$ and O$_2$ in the liquid influent
- The Nitrogen cannot diffuse in the solution
- There are not lateral reactions
- The equilibrium produced in the gas – liquid interface, can be described using the Henry’s Law (Treybal, 1980).

Mole balance in liquid phase for each compound.

$$V \frac{d}{dt} (c_{Na_2SO_3}^{out}) = F^{in} \varepsilon^{in}_N (c_{Na_2SO_3}^{in} - c_{Na_2SO_3}^{out}) - V \cdot r_{Na_2SO_3}$$

(16)

$$V \frac{d}{dt} (c_{Na_2SO_4}^{out}) = -F^{in} (c_{Na_2SO_4}^{out}) + V \cdot r_{Na_2SO_4}$$

(17)
\[ \frac{d}{dt}(C_{O_2}) = N_A \cdot V - F^{in} \cdot (C_{O_2}^{out}) - V \cdot r_{O_2} \]  
(18)

Where \( r_{O_2} \) is the rate of conversion of the solubilized oxygen due to the chemical reaction, which can be calculated as:

\[ r_{O_2} = K \cdot \frac{C_{Na,SO_2}}{C_{Na}} \cdot \frac{C_{Ca}}{C_{Ca}^B} \cdot C_{out} \]  
(19)

The relationship between the compounds involved in the chemical reaction can be obtained from the stoichiometric coefficients from (15), as (20) presents.

\[ r_{Na,SO_2} = r_{Na,SO_4} = \frac{1}{2} \cdot r_{O_2} \]  
(20)

Mole balance in gas phase:

\[ \frac{d}{dt}(M_{O_2}) = \rho_{O_2}(G^{in} \cdot Y_{O_2}^{in} - G^{out} \cdot Y_{O_2}^{out}) - V \cdot N_A \]  
(21)

The variable \( N_A \) corresponds to the oxygen flux transferred between each phase. Generally, in gas – liquid reactors, this term is a function of the gradient of the concentrations between the interface and the liquid bulk (Treybal, 1980). Assuming thermodynamic equilibrium in the interface, the oxygen concentration in the liquid that is in contact with the gas bubble, is equal to maximum concentration that can be reached for a given temperature: the saturation concentration obtained from the Henry’s Law (23,24).

\[ N_A = K_{LA} \cdot (C_{O_2}^{SAT} - C_{O_2}^{out}) \]  
(23)

\[ C_{O_2}^{SAT} = \left( \frac{m}{n} \right) \cdot Y_{O_2}^{out} \]  
(24)

The nomenclature used in the model is listed in Table 1 and the superscripts “in” and “out” can be followed from Fig. 3.

In the process, the liquid influent and effluent, and also the system pressure can be considered as known disturbances, whereas the influent gas streams are manipulated variables that can be modified to fulfill certain operational goals.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>Molar concentration in liquid phase</td>
<td>( mol \ \cdot m^{-3} )</td>
</tr>
<tr>
<td>( F )</td>
<td>Volumetric liquid flow</td>
<td>( m^3 \cdot s^{-1} )</td>
</tr>
<tr>
<td>( G )</td>
<td>Volumetric gas flow</td>
<td>( m^3 \cdot s^{-1} )</td>
</tr>
<tr>
<td>( H )</td>
<td>Henry’s constant</td>
<td>( Pa )</td>
</tr>
<tr>
<td>( K_{LA} )</td>
<td>Volumetric mass transfer coefficient</td>
<td>( m^3 \cdot s^{-1} )</td>
</tr>
<tr>
<td>( K, A, B, C )</td>
<td>Kinetic parameters</td>
<td>-</td>
</tr>
<tr>
<td>( N_A )</td>
<td>Molar flux in the interface</td>
<td>( mol \ \cdot m^{-3} \cdot s^{-1} )</td>
</tr>
<tr>
<td>( P )</td>
<td>Pressure</td>
<td>( Pa )</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature</td>
<td>( K )</td>
</tr>
<tr>
<td>( V )</td>
<td>Volume of the liquid in the reactor</td>
<td>( m^3 )</td>
</tr>
<tr>
<td>( V_g )</td>
<td>Volume of the gas chamber</td>
<td>( m^3 )</td>
</tr>
<tr>
<td>( Y )</td>
<td>Mole fraction in gas phase</td>
<td>-</td>
</tr>
<tr>
<td>( r )</td>
<td>Conversion rate</td>
<td>( mol \ \cdot m^{-3} \cdot s^{-1} )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Molar density</td>
<td>( mol \ \cdot m^{-3} )</td>
</tr>
</tbody>
</table>

### 4.2 RTO of the unit

The purpose of the unit is to transform the entire incoming sulphite in sulphate in the cheapest way. This means that enough oxygen must be fed in the reactor in order to ensure this transformation. Owing to the fact that the kinetic of the reaction under the presence of the catalyst is very fast (Zhao et al., 2005), an excess of oxygen must be taken into account in order to reject disturbances in the sulphite concentration of the influent, which means that the oxygen concentration in the liquid bulk of the reactor \( \left( C_{O_2}^{out} \right) \) must be greater than a lower bound \( \left( C_{O_2}^{LO} \right) \). Therefore, the economic optimization problem to be solved in the RTO layer can be expressed as minimizing the cost of oxygen as (25) states:

\[ \min_{(g_1, g_2)} \Phi = C_{g_1} g_1^2 + C_{g_2} g_2^2 \]

s.t.: Steady state model of the process \( (16) - (24) \) setting in zero the time derivatives. \( G^{LO} \) and \( G^{UP} \) represents the lower and upper bounds of the manipulated variables.

In bubbled reactors, as the one used in the example, mass transfer is the limiting phenomenon (Quijano et al., 2010). Several factors can affect this effect: bubble diameter, power of the stirrer, temperature, pressure and gas flow among others. In general the mass transfer coefficient can be estimated as the product of the contribution of this effects using non-dimensional numbers (Treybal, 1980). For a given system, where only the gas flow can change while the rest of the factors that can affect the mass transfer can be considered constants, the value of \( K_{LA} \) can be expressed as:

\[ K_{LA} = m (G_{in})^n \]  
(26)

Where \( m \) and \( n \) are parameters that can be estimated experimentally or calculated from available correlations.

About the chemical reaction that occurs at the bulk of the liquid phase, it can be said that the order of this reaction can change with the concentration of the reactants (Zhao et al., 2005). Therefore, kinetic parameters \( A, B \) and \( C \) from (19) are uncertain.

Previous discussions about the physical phenomena inside the reactor, give insights about the main sources of uncertainty of the system. Also, they provide the sources of more important modelling mismatches that can be expected. Table 2 shows the value of the parameters that will be used in the process and in the model for the RTO, summarizing the simulated modelling mismatch.

<table>
<thead>
<tr>
<th>n</th>
<th>m</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.3342</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>3.7e9</td>
</tr>
<tr>
<td>1</td>
<td>0.062</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.029</td>
</tr>
</tbody>
</table>

### 4.3 Results

Modifier-adaptation method with the infeasibilities controller added was applied to the problem presented in 4.2, with the purpose of comparing its performance with previous methods. 2-Stage RTO and modifier adaptation with gradient
estimation using finite differences and dual control optimization were also tested starting from three different initial points (A, B and C) that corresponds to feasible modes of operation. The evolution of the system until no more appreciable changes were observed in the decision variables between each RTO iteration are presented in Figs. 4 to 7. In all these Figures the real optimum of the simulated process, obtained by means of simulations over the entire feasible region, is represented with R.

Fig. 4. Evolution of the 2-Stage RTO

The 2-Stage RTO was applied, estimating model parameters m and K (from Table 2) in each iteration. Fig. 4 shows the evolution of the outcomes of this strategy for each starting point. As it was told previously, under presence of modelling mismatch the method does not reaches the real optimum of the process.

The evolution of the manipulated variables with Modifier-adaptation method using finite differences and dual control optimization is shown in Fig 5. Only the constraint of C_{O_2}^{out} is modified using the gradient and value correctors, since the objective function only depends on the manipulated variables. Equation (27) presents the modified constraint \( \dot{G}_{mod} \) that must be employed instead of \( \dot{G} \) in equation (25).

\[
\begin{align*}
G_{mod} &:= \dot{C}_{O_2} - \dot{C}_{O_2}^{out} + b_k + \gamma_k (G - G_k) \\
b_k &:= \dot{C}_{O_2}^{out}(k) - \dot{C}_{O_2}(k) \\
\gamma_k &:= \left[ \frac{\partial \dot{C}_{O_2}^{out}(k)}{\partial \dot{\theta}} - \frac{\partial \dot{G}_{O_2}(k)}{\partial \dot{\theta}} \right] \\
G &= [G_1, G_2]^T, \quad \{G_1, G_2\} \in [G^{LO}, G^{UP}].
\end{align*}
\]

Where the variables with the bar are referred to the process and the ones that present the indicator \( k \) are constants in the optimization problem, estimated from the previous solution of the RTO.

As it was expected, for the two methods of gradient estimation, the modified problem converges to the real optimum from the three starting points. Also, it can be noted that the estimation of process gradients with finite differences needs more iterations to reach the optimum because additional perturbations are required (represented as single points in Fig. 5), in fact, the necessary RTO iterations to reach the optimum of the process for dual control optimization was reduced by a factor of three with respect to the finite differences. Due to the fact that the optimum of the process is over the inequality constraint, and the starting points A and B are close to the restriction, the evolution of the system goes near to the constraint until it reaches the real optimum. Fig.6 shows a close up to the constraint for the evolution of the Modifier-adaptation method with dual control optimization.

Fig. 5. Evolution of the Modifier-adaptation method

It can be noted that infeasibilities appear during the evolution of the RTO method (marked in red circles in Fig. 6a), that seems to be more frequent as the method approaches the optimal point (Fig. 6b).

Fig. 6. Close up for the Modifier-adaptation method

Figure 7 shows the evolution of the Modifier-adaptation method with the infeasibility controller applied in the constraints of C_{O_2}^{out} and in the inverse of the condition number. In Figure 7a, the optimal solution given by the Modifier - adaptation RTO and the real value applied to the process after being corrected by the infeasibility controller is plotted (marked as RTO and REAL respectively). It can be noted how the system reaches the optimum of the process while infeasibilities in the oxygen concentration are avoided during the whole evolution of the process, just as it was expected.

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mentioned previously. Figure 7b magnifies the region close to the optimum of the process, where it can be noted again how the infeasibility controller avoid infeasible solutions.

Fig. 7. Evolution of the Modifier-adaptation method with the infeasibility controller proposed.

Referring to the constraint over the inverse of the condition number, it can be mentioned that the number of RTO iterations has remained unchanged with respect to the Modifier-adaptation method without the infeasibility controller. This means that in both cases (with and without the controller), the quality of the gradient estimator is the same. Therefore, the constraint over the inverse of the condition number is fulfilled.

As a final remark, it can be said that for the intermediate points that are placed in the feasible region of the problem, the behaviour of the method proposed is the same than the one observed in original modifier-adaptation method.

5. CONCLUSIONS

To avoid infeasible points that can be observed when the modifier-adaptation method is applied, an infeasibility PI controller has been proposed and tested in a simulated example. Assuming that manipulated variables can be adjusted to cope with constrained variables, the system might evolve until the optimum of the process in a feasible path.

As a future work, modifier adaptation method must be extended for dynamic problems (Brdys and Tatjewski, 2005), in order to join control and RTO layers into a single problem (De Prada et al., 2007, Engell, 2007). Then, all available degrees of freedom can be used to operate economically a plant and fulfilling operational constraints.

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