A Frequency Response Identification Method for Discrete-time Processes


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Abstract: A new frequency response identification method is presented to estimate frequency responses of discrete-time processes. It provides exact frequency response data sets for any desired frequency. This method incorporates various process signal types such as both initial and final steady state, initial steady state and final cyclic steady state and both initial and final cyclic steady state. Also, the proposed algorithm completely removes the effect of static disturbances and it shows acceptable robustness to measurement noises.

Keywords: Discrete-time systems, z-transform, Frequency responses, Identification

1. INTRODUCTION

The describing function analysis has been used to identify the ultimate frequency information of processes since the original relay feedback identification method for the automatic tuning of proportional-integral-derivative (PID) controllers (Åström and Hägglund, 1984). This method approximates the square relay feedback signal to a sinusoidal one using the Fourier series of the relay feedback signal. The approximation generally provides a fairly accurate ultimate frequency for usual processes. However, the accuracy is not acceptable when the harmonic terms of the relay signal are dominant. To deal with the problem of the describing function analysis, the Fourier analysis was proposed (Sung and Lee, 1997). It can extract accurate frequency response data of the process because no approximations are used for the derivation of the method.

However, the describing function analysis and the Fourier analysis method can estimate only one or two frequency data. It is not sufficient to tune controllers with high performance. Thus, several improved frequency response identification algorithms have been developed to extract multiple frequency response data (Luyben, 1990, Sung and Lee, 2000, Ma and Zhu, 2006, Cheon et al., 2011). Luyben proposed an identification method using the Fourier transform. It can estimate exact frequency information for a wide range of frequencies. But it is valid only for process data with initial and final steady state. Sung and Lee and Ma and Zhu developed an identification algorithm using a modified Fourier transform. The algorithm uses process data ranging from the initial transient region to the final cyclic steady state part. So, it can provide the frequency responses for all desired frequencies. However, it can be applied only to the case where initial part is steady state and final part is cyclic steady state. This algorithm cannot incorporate other cases such as initial/final steady state and initial cyclic steady state. Cheon et al. proposed an improved identification method which can be applied to various cases (initial/final steady state, initial steady state/final cyclic steady state and initial/final cyclic steady state). It also provides exact frequency responses for all desired frequencies. Furthermore, this method provides still exact frequency responses under the circumstances of static disturbances because it can remove the effects of static disturbances completely.

It should be noted that all the above-mentioned nonparametric process identification methods are to obtain frequency responses for continuous-time processes. In modern control theories, however, discrete-time models have been widely accepted as well. Frequency response identification methods for discrete-time processes have shown no remarkable progress compared with the cases of continuous-time processes. Several nonparametric identification methods for discrete-time processes have been proposed so far (Ljung, 1987). But, no nonparametric identification method for discrete-time processes which is applicable to all the types of process data of initial/final steady state, initial steady state/final cyclic steady state and initial/final cyclic steady state has been developed.

In this paper, a new nonparametric identification method to estimate discrete-time frequency responses is proposed. It can incorporate all the types of the process data (both initial and final steady state /initial steady state and final cyclic steady state / both initial and final cyclic steady state). It can estimate exact frequency responses for any desired frequencies and shows acceptable robustness to measurement noises. Also, the accuracy of the model estimated by the proposed method is not affected by static disturbances because it can completely remove the effects of the static disturbances.

2. TYPES OF PROCESS EXCITATION

Consider the following three types of process excitation: (1) Both initial and final parts are steady state (Fig. 1a). (2) The initial part is steady state and the final part is cyclic steady state (Fig. 1b). (3) The initial unsteady state is stabilized to the cyclic steady state and the final part is cyclic steady state, that
is, both initial and final cyclic steady state (Fig. 1c). These types of process data can be obtained by a proportional controller (for type 1) and relay feedback method (for type 2 and 3).

3. PROPOSED IDENTIFICATION METHOD

A new frequency response identification method applicable to discrete-time processes with initial and final cyclic steady state (type 3) will be developed, followed by the extension of the proposed method to the other types of process excitation.

The following assumptions are assumed in this paper in developing the proposed method.

Assumption 1: The initial part of the process data from $t_i$ to $t_i + P$, and the final part from $t_i$, to $t_i + P$, are cyclic steady state as shown in Fig 1c.

Assumption 2: The dynamics of the discrete-time process is described by the following linear time-invariant transfer function:

$$G(z) = \frac{y(z)}{u(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{k_m} z + b_k}{z^r + a_1 z^{-1} + \cdots + a_{k_m} z + a_k}$$

(1)

Data Pre-processing: The pre-processing procedure is as follows: (1) Obtain the input and output signal in Fig. 2 by repeating the initial part (one period) from $t_i$ to $t_i + P$ in Fig. 1c. Let us denote the input and output signal in Fig. 2 by $u_m(k)$ and $y_m(k)$, respectively. (2) Define deviation variables of $\tilde{u}(k) = u(k) - u_{m}(k)$ and $\tilde{y}(k) = y(k) - y_{m}(k)$, where $u(k)$ and $y(k)$ are the process input and the process output in Fig 1c. Then, $\tilde{u}(k)$ and $\tilde{y}(k)$ from $t_i$ to $t_i + P$ is zero steady state and the final part after $t_i$ is cyclic steady state of which period is the common multiple of $P_i$ and $P_f$, that is $P = m P_i = n P_f$, where $m$ and $n$ are integers.

3.1 Proposed Algorithms

All the discrete-time frequency responses of the process for any frequencies can be estimated by the proposed algorithm 1 in the case that the initial and final part of the process input and output are cyclic steady state as shown in Fig 1c.

Algorithm 1 for initial and final cyclic steady state: For any nonzero frequency, $\omega \neq 0$:

$$G(e^{j \omega}) = \frac{(1 - e^{j \omega} \xi_{n+i}) \xi_{n} - (1 - e^{-j \omega} \xi_{n}) \xi_{n+i} - (1 - e^{-j \omega} \xi_{n}) \xi_{n+i}}{(1 - e^{-j \omega} \xi_{n+i}) \xi_{n+i+i} - (1 - e^{j \omega} \xi_{n+i}) \xi_{n+i+i} - (1 - e^{j \omega} \xi_{n+i}) \xi_{n+i+i}}$$

(2)

Where
Substituting $e^{j\omega t}$ for $z$ gives the proposed algorithm 1 (Eq. 2) and the zero frequency response estimator (Eq. 3) is obtained by applying L’Hospital’s rule to Eq. 9. Q.E.D.

The proposed algorithm has several advantages. First, it can estimate any desired frequency responses. Second, it gives exact frequency response data because any approximation is not used in developing the algorithm. Third, the algorithm can be extended to the other process signal types, which is discussed in the next section. Fourth, it can still estimate exact frequency responses under the circumstances of static disturbances. Note that $\tilde{u}(k) = u(k) + \delta - (u_{\text{ref}}(k) + \delta)$ is valid in the case that the static disturbance $\delta$ is added to the process input of $u(k)$. This means that the deviation variables $u(k)$ and $\tilde{y}(k)$ (equivalently, the proposed method) are not affected by the static disturbance. It is notable that the static disturbances are equivalent to wrong deviation variables. It means that the proposed algorithm estimates exact frequency responses even in the presence of wrong deviation variables.

3.2 Extension to the other cases

The case that the period $P_1$ or $P_2$ in the proposed algorithm 1 (Eq. 2 and 3) is 1 is equivalent to the case that the initial or final part of the input and output signals is steady state. So, the algorithm 1 can be easily extended to the other cases of Fig. 1 just by substituting 1 for $P_1$ or $P_2$.

Algorithm 2 for initial steady state and final cyclic steady state:

The algorithm 2 of Eq. 10 applicable to the initial steady state and final cyclic steady state (Fig 1b) is derived by choosing the initial period $P_1$ of the algorithm 1 as 1.

For any nonzero frequency, $\omega \neq 0$ :

$$G(e^{j\omega}) = \frac{\tilde{y}(z)}{u(z)} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} / \sum_{i=0}^{\infty} u_{\text{ref}}(z)z^{-i} - \frac{1}{1-z^{-1}}$$

$$= \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} / \left(1 - \sum_{i=0}^{\infty} u_{\text{ref}}(z)z^{-i} \right)$$

Without loss of generality, assume that $P$, $m$, and $n$ are infinite. Then, Eq. 8 is obtained from Eq. 7.

$$G(e^{j\omega}) = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} / \left(1 - \sum_{i=0}^{\infty} u_{\text{ref}}(z)z^{-i} \right)$$

Eq. 7 is also valid for $u(k)$ and $u_{\text{ref}}(k)$.

Then, Eq. 9 is obtained straightforwardly.

$$G(z) = \frac{\tilde{y}(z)z^{-i} + (1-z^{-i})\tilde{y}_{\text{ref}}(z)z^{-i}}{1-z^{-1}}$$

$$= \tilde{y}(z)z^{-i} / \left(1 - \sum_{i=0}^{\infty} u_{\text{ref}}(z)z^{-i} \right)$$

Where

$$S_{i,n} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i}$$
$$S_{j,n} = \sum_{i=0}^{\infty} \tilde{y}_{\text{ref}}(z)z^{-i}$$
$$S_{k,n} = \sum_{i=0}^{\infty} u(k)z^{-i}$$

For zero frequency, $\omega = 0$ :

$$G(1) = \frac{\sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} - \sum_{i=0}^{\infty} u_{\text{ref}}(z)z^{-i}}{\sum_{i=0}^{\infty} u(k)z^{-i}}$$

Proof

Consider the transfer function of a discrete-time process.

$$G(z) = \frac{\tilde{y}(z)}{u(z)} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} / \sum_{i=0}^{\infty} u(z)z^{-i}$$

(4)

Where, the numerator can be rewritten as follows:

$$\sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i + \ldots}$$

(5)

Because $\tilde{y}(k)$ for $k \geq t_j$ is a periodic function of which the period is $P$, $\tilde{y}(k) = \tilde{y}(k - P)$ for $t_j + P \leq k \leq t_j + 2P - 1$ ,

$$\tilde{y}(k) = \tilde{y}(k - 2P)$$

for $t_j + 2P \leq k \leq t_j + 3P - 1$ , are valid. So, Eq. 5 is equivalent to Eq. 6.

$$\sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i + \ldots}$$

(6)

Since $\tilde{y}(k) = y(k) - u_{\text{ref}}(k)$ and $P = mP_2 = nP_1$ , Eq. 7 can be straightforwardly derived from Eq. 6:

$$\sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} = \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}(z)z^{-i}$$

(7)

Without loss of generality, assume that $P$, $m$, and $n$ are infinite. Then, Eq. 8 is obtained from Eq. 7.

$$\tilde{y}(z) = \sum_{i=0}^{\infty} \tilde{y}(k)z^{-i} + \sum_{i=0}^{\infty} \tilde{y}_{\text{ref}}(k)z^{-i}$$

(8)

Eq. 7 is also valid for $u(k)$ and $u_{\text{ref}}(k)$.

Then, Eq. 9 is obtained straightforwardly.

$$G(z) = \frac{\tilde{y}(z)z^{-i} + (1-z^{-i})\tilde{y}_{\text{ref}}(z)z^{-i}}{1-z^{-1}}$$

Where

$$S_{i,n} = \sum_{i=0}^{\infty} \tilde{x}(k)z^{-i}$$
$$S_{j,n} = \sum_{i=0}^{\infty} \tilde{y}_{\text{ref}}(k)z^{-i}$$
$$S_{k,n} = \sum_{i=0}^{\infty} u(k)z^{-i}$$

Substituting $e^{j\omega t}$ for $z$ gives the proposed algorithm 1 (Eq. 2) and the zero frequency response estimator (Eq. 3) is obtained by applying L’Hospital’s rule to Eq. 9. Q.E.D.
Algorithm 3 for both initial and final steady state:
The algorithm 3 of Eq. 12 applicable to both initial and final steady state case (Fig. 1a) is derived by substituting $P_i$ and $P_f$ of the algorithm 1.

\[
G(e^{-j\omega}) = \frac{\sum_{i=0}^{\infty} y(k) - P_i \mu_i (t)}{\sum_{i=0}^{\infty} u(k) - P_i \mu_i (t)}
\]

(11)

4. SIMULATION STUDY

In this section, several simulations are conducted to confirm the performance of the proposed method. Consider the following third order transfer function of the discrete-time process.

\[
G(z) = \frac{0.0000461 z^2 + 0.00005763 z + 0.00001418}{z^3 - 2.714245 z^2 + 2.4557004 z - 0.7405967}
\]

(13)

The process is activated by the P controller and relay feedback method (Figure 3a, 4a and 5a) and the frequency response data is estimated by the proposed algorithms in each case (Figure 3b, 4b and 5b). The results show that the proposed method provides exact frequency response estimation and covers all the desired frequencies. And it can also be applied to various signal types.
Fig. 5. (a) Process signal (both initial and final cyclic steady state) (b) Identified frequency responses

Fig. 6a shows the process input/output data excited by the relay feedback method in the presence of a static input disturbance of 0.1. As expected, Fig. 6b confirms that the proposed method provides exact frequency responses under the circumstances of static disturbances.

Consider the process output signal of Fig. 7a contaminated by random measurement noises distributed uniformly between -0.03 and 0.03. The frequency responses of Fig. 7b estimated by the proposed method confirms that it shows acceptable robustness to measurement noises.

5. CONCLUSIONS

In this study, a new frequency response identification method for discrete-time processes is proposed. The proposed methods can incorporate all the three cases of both initial and final steady state, initial steady state and final steady state, both initial and final cyclic steady state process data. It guarantees exact frequency response models for all desired frequencies even under the circumstances of static disturbances. Also, it shows acceptable robustness to measurement noises.

REFERENCES


