Identification and Controller Tuning of Cascade Control Systems Based on Closed-Loop Step Responses

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Abstract: This paper presents a new automatic tuning method for cascade control systems based on a single closed-loop step test. The proposed method identifies the required process information with the help of B-spline series expansions for the step responses. The two PID controllers are then tuned using an internal model control (IMC) approach. The secondary controller is designed for enhanced disturbance rejection, and the primary controller is designed, without requiring an additional test, based on an identified process model that accurately accounts for the inner loop dynamics. The desired levels of system robustness explicitly guide the selection of the IMC tuning parameters. The proposed method is robust to measurement noises because of the filtering property of the B-splines, and can provide satisfactory control performance. Simulation examples confirm the effectiveness of the proposed method.

Keywords: Process control; Cascade control; Closed-loop identification; B-spline series; PID tuning

1. INTRODUCTION

Cascade control is one of the most successful control structures for enhancing single-loop control performance, which has led to the extensive implementation of cascade control in chemical process industries. The cascade control structure is to nest one feedback loop inside another feedback loop using two controllers, so that the fast dynamics of the inner loop will provide faster disturbance attenuation. Though recent studies have proposed sophisticated cascade control schemes (e.g., Alfaro et al., 2009; Kaya et al., 2007), the standard cascade control scheme still consists of two nested loops with two PID controllers. The design of this cascade control system involves the tuning of two PID controllers, and is therefore more complicated than that of a single-loop control system.

Previous researches have proposed relay-based auto-tuning techniques to facilitate the design of cascade control systems. The method proposed by Hang et al. (1994) needs a sequential application of the conventional relay-based auto-tuning approach, and is therefore still time consuming. The sequential tuning procedure has been improved so that only a single relay experiment is required for auto-tuning (Leva and Donida, 2009; Mehta and Majhi, 2011; Tan et al., 2000). However, an off-line or ad hoc experiment must be performed in these methods. Veronesi and Visioli (2011) recently proposed a simultaneous closed-loop automatic tuning method for cascade controllers from the set-point step response of a cascade control system.

The literature also reports tuning rules for cascade controllers. Lee et al. (1998) proposed IMC-based PID tuning rules that enable simultaneous tuning of primary and secondary controllers. Lee et al. (2002) subsequently improved the tuning rules using a general cascade control structure that requires two additional set-point filters. However, these studies fail to clarify how the procedure can be automated. The main point of simultaneously tuning cascade controllers (Lee et al., 1998; Lee et al., 2002; Mehta and Majhi, 2011; Veronesi and Visioli, 2011) is to approximate the inner loop dynamics with the inner loop design target (e.g., a desired closed-loop transfer function for the inner loop). Such an approximation allows obtaining a process model for the tuning of primary controller. However, this approximation may be inaccurate because the implemented secondary (PID) controller cannot guarantee achieving the inner loop design target. The proposed method addresses this problem.

This study develops an on-line automatic tuning method that simultaneously tunes two PID controllers for cascade control systems based on a single closed-loop step test. By representing the step responses with B-spline series expansions, this method first identifies the analytical expressions of process transfer functions, which can be used to obtain appropriate process models for controller design. The IMC approach is then used to design secondary and primary controllers. The cascade control design here achieves satisfactory servo and regulatory control performance, while the control structure is stuck to the standard cascade control configuration (i.e., it avoids using more complicated control schemes such as two-degree-of-freedom control). This is accomplished by designing the inner loop for enhanced disturbance performance, and also taking into account the actual inner loop dynamics (without an additional experiment) in the primary controller design. Finally, the explicit inclusion of robustness considerations in controller tuning results in a complete automatic tuning procedure.
2. PROCESS IDENTIFICATION

Figure 1 shows the configuration of a typical cascade control system, where \( G_1 \) is the primary process and \( G_2 \) is the secondary process. The primary process variable \( y_1 \) (with set-point \( r_1 \)) is used by the primary controller \( G_{c1} \) to establish the set-point (\( r_2 \)) for the secondary controller \( G_{c2} \). The secondary process variable \( y_2 \) is transmitted to the secondary controller which adjusts the manipulated variable \( u \). The effectiveness of a cascade control scheme is because the disturbance \( d_2 \) affecting the secondary (inner) loop is effectively compensated before it affects the primary process variable \( y_1 \).

![Figure 1. Configuration of a typical cascade control system.](image)

The process identification is accomplished by conducting a simple step test in the set-point during closed-loop operation of a cascade control system. Compared with open-loop test, the closed-loop test has the significant advantage of not excessively disturbing the system. The controller parameters can be chosen arbitrarily (e.g., roughly tuned). The step response data of the process variables, \( y_1(t) \) and \( y_2(t) \), and the manipulated variable, \( u(t) \), are collected until the new steady-state is reached.

The following subsection briefly introduces the B-splines because the proposed identification method uses B-spline series expansions to represent the measured step responses.

2.1 B-spline series

Since the step responses are functions of time for \( t \geq 0 \), we introduce the B-splines on the semi-infinite interval \([0, \infty)\). Let \( \mathbf{x} = \{x_k\} \) be a non-decreasing sequence of real numbers, defined as \( x_0 \leq x_1 \leq \cdots \leq x_k \leq \cdots \leq x_{m-1} \leq \infty \). For each \( k \geq -m+1 \), \( B_{m,k}(t) \) is called the \( k \)th B-spline of order \( m \) with knot sequence \( \mathbf{x} \). Computations with B-splines are facilitated by stable recurrence relations (Chui, 1997)

\[
B_{m,k}(t) = \begin{cases} 
1 & \text{if } x_k \leq t < x_{k+1}, \quad k = 0,1,\ldots, m-1 \\ 0 & \text{otherwise} 
\end{cases}
\]

(2)

The collection of \( m \)-th order B-splines \( B_{m,k}(t) \), \( k \geq -m+1 \) is a basis of the spline space on \([0, \infty)\). In this case, the B-spline series describes a spline \( S(t) \) as a linear combination

\[
S(t) = \sum_{k=-m}^{\infty} c_k B_{m,k}(t)
\]

(3)

which can be used for representing a signal (function).

The representation of a signal using B-spline series has several advantages. The B-splines are a local basis (with finite time duration) which has greater flexibility in signal representation than a global basis, such as Laguerre functions. B-splines with a finer knot sequence will have enhanced capability for representing a signal with sharp changes. A suitable choice of the knot sequence depends on the “shape” of the signal to be represented. The B-spline series works as a low-pass filter that can effectively remove the high-frequency noise when it is used to represent a noisy signal.

The Laplace transforms of \( B_{m,k}(t) \), designated as \( \tilde{B}_{m,k}(s) \), can be obtained by the following recurrence relations.

\[
\tilde{B}_{m,k}(s) = \begin{cases} 
\frac{1}{s} & \text{for } \ t = 0 \\
\frac{1}{s} \left[ 1 - \frac{m-1}{x_k - x_{m-1}} \tilde{B}_{m-1,k}(s) \right] & \text{for } k = -m+1 \\
\frac{1}{s} \left[ 1 - \frac{m-1}{x_k - x_{m-1}} \tilde{B}_{m-1,k}(s) - \frac{m-1}{x_{m-1} - x_k} \tilde{B}_{m-1,k+1}(s) \right] & \text{for } k \geq -m+2
\end{cases}
\]

(4)

The following discussion uses the 3rd-order \((m=3)\) B-splines to develop the closed-loop identification method, and the knot sequence \( \mathbf{x} \) can be inferred from the context. For conciseness, \( B_{3,k}(t) \) and \( \tilde{B}_{3,k}(s) \) are hereafter represented as \( B_k(t) \) and \( \tilde{B}_k(s) \), respectively.

2.2 Identification of process transfer functions

From the scheme of Fig. 1, the transfer functions of the primary and the secondary processes can be expressed as

\[
G_1(s) = \frac{y_1(s)}{y_2(s)} \quad G_2(s) = \frac{y_2(s)}{u(s)}
\]

(5)

To obtain the Laplace transforms of the measured responses, define new variables in the deviation form as

\[
\overline{y}_1(t) = y_1(t) - y_{1s}; \quad \overline{y}_2(t) = y_2(t) - y_{2s}; \quad \overline{u}(t) = u(t) - u_{s}
\]

(6)

where \( y_{1s}, y_{2s}, \) and \( u_s \) designate the new steady-state values of \( y_1(t), y_2(t), \) and \( u(t) \), respectively, after the step change in the set-point. Then, each of the responses in (6) is approximated as a spline using the 3rd-order B-spline series expansions as

\[
\overline{y}_1(t) \approx \tilde{y}_1(t) = \sum_{k=0}^{\infty} c_{y,k} B_k(t); \quad \overline{y}_2(t) \approx \tilde{y}_2(t) = \sum_{k=0}^{\infty} c_{y,k} B_k(t)
\]

(7)

\[
\overline{u}(t) \approx \tilde{u}(t) = \sum_{k=0}^{\infty} c_{u,k} B_k(t)
\]

Each response in the deviation form has a zero steady-state value, which allows it to be approximated with finite terms of B-splines \((k = -2, -1, 0, \ldots, N_0)\). After applying the Laplace transform to (6) and (7), the approximated original responses can be expressed in the Laplace domain as

\[
\tilde{y}_1(s) = \sum_{k=0}^{\infty} c_{y,k} \tilde{B}_k(s) + \frac{y_{1s}}{s}; \quad \tilde{y}_2(s) = \sum_{k=0}^{\infty} c_{y,k} \tilde{B}_k(s) + \frac{y_{2s}}{s}
\]

(8)

The B-spline coefficients \( c_{y,k}, \quad k = -2, -1, 0, \ldots, N_0 \) can be obtained by minimizing the following cost function

\[
\min_{t_a} \sum_{m=1}^{M} \left[ \tilde{y}_1(t_m) - \tilde{u}(t_a) \right]^2
\]

(9)

where \( M \) is the number of measurements and \( t_m \) is the time of the \( m \)th measurement. The above optimization problem can be easily solved by a least-squares estimation method. The coefficients \( c_{y,k} \) and \( c_{u,k} \) can be obtained in a similar way.
By choosing the knot sequence, the use of B-splines is better able to represent the closed-loop responses. A suitable knot sequence can be chosen based on the characteristics of the measured responses. Because the closed-loop step responses usually have time delay and sharp transient dynamics (especially for the manipulated variable $u$), a finer knot sequence can be chosen for the initial period of response, and a coarser knot sequence can be chosen when the response approaches the steady-state. This method of choosing a non-uniform knot sequence provides a favorable trade-off between approximation accuracy and computational effort.

From (5), the analytical expressions of process transfer functions now can be calculated by substituting (8) as

$$
\hat{G}_1(s) = \sum_{k=1}^{N} c_{1,k} \hat{B}_k(s) + \frac{y_{1,k}}{s},
\hat{G}_2(s) = \sum_{k=1}^{N} c_{2,k} \hat{B}_k(s) + \frac{y_{2,k}}{s}
$$

(10)

The process transfer functions obtained from (10) are high-order with complex forms. These transfer functions can be directly used to calculate the process frequency responses, $\hat{G}_1(j\omega)$ and $\hat{G}_2(j\omega)$, by substituting $s = j\omega$ for arbitrary frequencies. The transfer functions can also be reduced to low-order plus time delay models, which are suitable for controller design, by a simple model reduction technique based on the calculated frequency response data. First, the steady-state gain of the model, $K$, is determined as that of the approximated process transfer function by

$$
K = \lim_{s \to 0} \hat{G}(s)
$$

(11)

The other model parameters then can be determined by minimizing the difference between the estimated process frequency response and the frequency response of the reduced model. To avoid an iterative nonlinear optimization procedure in the parameter estimation, a two-step least-squares technique is suggested. Because the amplitude ratio is independent of time delay, the first step estimates the model parameters, excluding time delay $\theta$, by minimizing the difference in the amplitude ratio. The estimated model parameters are then used in the second step to estimate the time delay of the model by minimizing the difference in the phase angle. Both minimization problems can be solved by a standard linear least-squares technique. The computation of the model parameters can be repeated for various types of low-order models, and the model that minimizes the residuals of the least-squares problem is chosen.

3. PID CONTROLLER TUNING

This study proposes to use an IMC approach (Morari and Zafiriou, 1989) for the controller design.

3.1 Design of the secondary controller

According to the method presented in the previous section, a secondary process model, $G_{n2}$, can be obtained from the frequency response $\hat{G}_2(j\omega)$. The IMC controller $Q_2$ is designed as

$$
Q_2(s) = G_{n2}(s)^{-1} F_2(s)
$$

(12)

where $G_{n2}$ is the invertible part of the model $G_{n2}$, and $F_2$ is the IMC filter to deal with the robustness issue of the inner loop. For stable processes, this IMC filter is typically chosen as lag element without lead dynamics, which results in sluggish response to disturbance $d_2$ if $G_{n2}$ has slow poles. Because the performance of cascade control in the presence of disturbance $d_2$ is usually the principal concern, the inner loop must respond quickly to effectively reduce the effects of disturbance $d_2$. To this end, this study proposes that an IMC filter $F_2$ designed so that $(1-G_{n2}, F_2)$, where $G_{n2}$ is the non-invertible part of the model $G_{n2}$, cancels the slow poles in $G_{n2}$. As it is typical in the industrial context, the secondary process is modeled as a first-order plus time delay (FOPTD) dynamics:

$$
G_{n2}(s) = \frac{K_2}{\tau_2 s + 1} e^{-\delta_2 s}
$$

(13)

In this case, the IMC filter is chosen as

$$
F_2(s) = \frac{\alpha s + 1}{(\lambda_{2, s} + 1)}; \quad \alpha = \frac{\tau_2}{1 - \left(1 - \frac{\lambda_{2, s}}{\tau_2}\right)^2 e^{-\delta_2}}
$$

(14)

where the parameter $\lambda_2$ is the adjustable parameter to make trade-off between performance and robustness of the inner loop. The design target for the complementary sensitivity function of the inner loop, $F_2$, then becomes

$$
T_2(s) = G_{n2}(s) F_2(s) = \frac{\alpha s + 1}{(\lambda_{2, s} + 1)} e^{-\delta_2 s}
$$

(15)

The lead term in (15) may cause an excessive overshoot in the set-point response of the inner loop. Therefore, a set-point filter is usually used to eliminate the overshoot, resulting in a more complicated control system (Lee et al., 2002). Nevertheless, an additional set-point filter for the inner loop of cascade control system is not required because the response of the primary process variable $y_1$ is the major concern. This is because the primary process attenuates the lead dynamics of the inner loop. Furthermore, the primary controller can be designed to compensate the lead dynamics of the inner loop so that the response of the primary process variable remains satisfactory.

The conventional feedback controller $G_{c2}$ is related to the IMC controller $Q_2$ as

$$
G_{c2}(s) = \frac{Q_2(s)}{1 - G_{n2}(s) Q_2(s)} = \frac{(\tau_{2, s} + 1)(\alpha s + 1)}{K_2 \left(\lambda_{2, s} + 1\right) - e^{-\delta_2 s} (\alpha s + 1)}
$$

(16)

The above equivalent controller is not in the form of PID controller. Therefore, the Maclaurin series expansion formula is applied to obtain a PID controller which approximates $G_{c2}$ given in (16) as

$$
G_{c2}(s) = \frac{s}{\tau_{c2}} f_2(s) + f_1(0) + \frac{f_0(0)}{2} s^2 + \cdots
= K_{c2} \left(1 + \frac{1}{\tau_{c2}} + \tau_{c2} s\right)
$$

(17)

where $f_s(s) = s G_{c2}(s)$. The PID parameters are obtained as

$$
K_{c2} = f_2(0); \quad \tau_{c2} = f_1(0); \quad \tau_{c2} = f_0(0)
$$

(18)

Notice that a PI controller or a P controller can be approximated using only the first two terms or using only the second term, respectively, in (17).
3.2 Design of the primary controller

With the designed secondary controller, the primary controller is designed based on an apparent process \( G_1^* \) seen by the primary controller as

\[
T_1^r(s) = T_1(s)G_1(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)} G_1(s) \tag{19}
\]

Most existing works for simultaneous cascade controller design (Lee et al., 1998; Lee et al., 2002; Mehta and Majhi, 2011; Veronesi and Visioli, 2011) approximate this apparent process using the design target of the complementary sensitivity function of the inner loop, i.e., \( G_1^* = T_1^r G_1^* \) for primary controller design. Because the ideal secondary controller is approximated as a classical PID-type controller for implementation, the actual secondary complementary sensitivity function may deviate from its design target \( T_1^r \), depending on the accuracy of PID approximation. This deviation may be especially significant when the ideal controller is approximated as a P or PI controller, which is usually the controller type of the secondary controller. To account for the non-ideality of the inner loop, this study identifies a model of \( G_1^* \) according to the actually implemented secondary controller. After the frequency responses of the secondary and primary processes have been identified, (19) permits the calculation of the frequency response of \( G_1^*(j\omega) \) in terms of frequency responses of the designed secondary controller, the secondary process, and the primary process. Therefore, a model of the apparent process, \( G_{1a}^* \), can be identified from \( G_1^*(j\omega) \) using the previously presented least-squares method. Note that the identification of \( G_{1a}^* \) involves only simple linear least-squares computation and does not require conducting an additional experiment.

With the model \( G_{1a}^* \), the IMC design is applied again to design the primary controller \( G_{1i} \). The IMC controller \( Q_i \) is designed as

\[
Q_i(s) = G_{1a}^*(s) F_i(s) \tag{20}
\]

where \( F_i \) is the IMC filter dealing with the robustness issue of the outer loop. If the primary controller is designed for enhanced disturbance performance, a set-point filter for the outer loop must be incorporated to reduce the excessive overshoot in the set-point response, which complicates the control system. To maintain the simplicity of the control system, the typical IMC filter design is adopted for \( F_i \), i.e.,

\[
F_i(s) = \frac{1}{\lambda \omega + 1} \tag{21}
\]

where \( \lambda \) is the adjustable parameter to make trade-off between performance and robustness of the outer loop.

The apparent process for the primary controller design is generally modeled using second-order plus time delay and zero (SOPTDZ) dynamics:

\[
G_{1a}^*(s) = \frac{K_s(z^*s+1)}{a_s^2 s^2 + b_s^2 s + 1}, \quad z^* \geq 0 \tag{22}
\]

Therefore, the design target for the complementary sensitivity function of the outer loop, \( T_1 \), now becomes

\[
T_1^r(s) = G_{1a}^*(s) F_i(s) = \frac{1}{\lambda \omega + 1} e^{-\theta_i} \tag{23}
\]

For the SOPTDZ model, the conventional feedback controller \( G_{1i} \) is related to the IMC controller \( Q_i \) as

\[
G_{1i}(s) = \frac{Q_i(s)}{1 - G_{1i}(s) Q_i(s)} = \frac{a_i \omega^2 s^2 + b_i \omega s + 1}{K_s(z^*s+1)(\lambda \omega + 1)e^{-\theta_i}} \tag{24}
\]

The Maclaurin series expansion formula is applied to obtain a PID controller which approximates \( G_{1i} \) given in (24). Defining \( f_i(s) = s G_{1i}(s) \), the PID parameters are obtained as

\[
K_i = f_i(0); \quad \tau_i = f_i'(0); \quad \tau_i = f_i'(0) \tag{25}
\]

3.3 Guidelines for the selection of tuning parameters

In the proposed approach, \( \lambda_i \) and \( \lambda_s \) are the tuning parameters to handle the trade-off between the speed of response and system robustness. This study includes robustness considerations in the controller design and develops explicit guidelines for the automated selection of tuning parameters to achieve the desired level for the robustness. This aim is accomplished by using the maximum sensitivity as a robustness measure. The maximum sensitivity \( (M_S) \) is defined as

\[
M_S = \max_s S(j\omega) = \max_s \| -T(j\omega) \| \tag{26}
\]

where \( S \) denotes the sensitivity function with \( S + T = 1 \). As \( M_S \) decreases, the dynamic response of closed-loop system becomes more robust. The recommended values for \( M_S \) are typically within the range \( 1.2 < M_s < 2.0 \).

The complementary sensitivity function for the inner loop (15) can be normalized by \( \tau_i \) as

\[
\tilde{T}_i(s) = \frac{\tilde{B} \omega + 1}{(\tilde{\lambda} \omega + 1)} e^{-\tilde{\theta}_i} \tag{27}
\]

where \( \tilde{\theta} = \alpha / \tau_i = 1 - (1 - \tilde{\chi}) \tilde{\theta}_i \), \( \tilde{\chi} = \lambda / \tau_i \), and \( \tilde{\theta} = \theta / \tau_i \). Equation (27) indicates that the maximum sensitivity of the inner loop, \( M_{S2} \), depends on \( \tilde{\chi} \) and \( \tilde{\theta} \). Therefore, this study calculates the maximum sensitivity \( M_{S2} \) for the combinations of various values of \( \tilde{\chi} \) and \( \tilde{\theta} \). The resulting relations among the normalized design parameter \( \tilde{\chi} \), the inner loop robustness \( M_{S2} \), and the normalized time delay \( \tilde{\tau}_i \) are correlated as the following inner loop robust design criterion:

\[
\tilde{\tau}_i = p_1 M_{S2} + p_2 \tag{28}
\]

where

\[
p_1 = \frac{-3.998 \tilde{\theta}_i^2 + 4.279 \tilde{\theta}_i - 0.0397}{\tilde{\theta}_i^2 - 3.242 \tilde{\theta}_i^2 + 2.133 \tilde{\theta}_i + 0.139} \tag{29}
\]

This criterion provides the required value of \( \tilde{\chi} \), for a given \( \tilde{\theta}_i \), to achieve a desired value of \( M_{S2} \). Notice that (28) is applicable to \( 1.2 \leq M_{S2} \leq 1.8 \) and \( 0.05 \leq \tilde{\theta}_i \leq 1 \).
The complementary sensitivity function for the outer loop (23) can be normalized by $\theta^*$ as

$$T_\theta(s) = \frac{1}{\lambda s + 1} e^{-s}$$  \hspace{1cm} (30)

where $\lambda = \lambda_1 / \theta^*$. The maximum sensitivity of the outer loop, $M_{S1}$, depends only on $\lambda_1$. Therefore, this study calculates $M_{S1}$ for various values of $\lambda_1$. The resulting relationship between the normalized design parameter $\lambda_1$ and the outer loop robustness $M_{S1}$ is correlated as the following outer loop robust design criterion, which provides the required value of $\lambda_1$ to achieve a desired value of $M_{S1}$.

$$\lambda_1 = -0.7289 M_{S1} + 1.555 \frac{M_{S1}}{1.006}$$  \hspace{1cm} (31)

Notice that (31) is applicable to $1.2 \leq M_{S1} \leq 1.8$.

4. SIMULATION EXAMPLE

Two simulation examples are provided to demonstrate the effectiveness of the proposed automatic tuning method.

4.1 Example 1

Consider the high-order process given by the following transfer functions (Lee et al., 1998; Mehta and Majhi, 2011)

$$G_1(s) = \frac{10(-5s+1)}{(30s+1)(10s+1)} e^{-s}, \quad G_2(s) = \frac{3}{13s+1} e^{-s}$$  \hspace{1cm} (32)

The cascade control system with PID/PI control mode tuned by Mehta and Majhi (2011) is adopted for closed-loop test. Gaussian random noises with standard deviations 0.02, 0.006, and 0.004 were added to the original unit step responses of $y_1$, $y_2$, and $u$, respectively, to represent real measurements. Figure 2 shows the noisy responses and the approximated responses by B-spline series. The B-spline series expansions provide smooth approximated responses, effectively reducing the effect of noise. Figure 3 shows the identified process frequency responses and comparisons with the actual frequency responses for $G_1$ and $G_2$. An excellent agreement exists between the estimated and actual frequency responses.

To tune the secondary controller, an FOPTD model for $G_2$ is first identified from $\hat{G}_2(j\omega)$ as

$$\hat{G}_2(j\omega) = \frac{9.979}{1922.3s^2 + 81.1s + 1} e^{-3.77s}$$  \hspace{1cm} (34)

Choosing $M_{S2} = 1.5$, $\lambda_2 = 40.905$ is obtained and the resulting PID parameters are $K_2 = 0.109$, $\tau_1 = 9.024$, and $\tau_2 = 30.33$. Figure 4(a) shows the closed-loop response to a unit step set-point change at $t = 0$ followed by step disturbance inputs $d_1 = 0.5$ at $t = 700$ and $d_2 = 0.2$ at $t = 1500$, and presents the results obtained by Lee et al. (1998) for comparison.

Fig. 2. Noisy and approximated closed-loop step responses for example 1.

Fig. 3. Actual and identified process frequency responses (Nquist plots) for example 1.

$$G_2(s) = \frac{2.972}{12.970s + 1} e^{-3.05s}$$  \hspace{1cm} (33)

Choosing $M_{S2} = 1.5$, $\lambda_2 = 40.905$ is obtained and the resulting PID parameters are $K_2 = 0.109$, $\tau_1 = 9.024$, and $\tau_2 = 30.33$. Figure 4(a) shows the closed-loop response to a unit step set-point change at $t = 0$ followed by step disturbance inputs $d_1 = 0.5$ at $t = 700$ and $d_2 = 0.2$ at $t = 1500$, and presents the results obtained by Lee et al. (1998) for comparison.

These results illustrate that the proposed automatic tuning method outperforms the method of Lee et al. (1998). To illustrate the robustness to parameter variations, the control systems are evaluated on a perturbed plant where +20% deviations in gain and time delay from the nominal values for both primary and secondary processes are considered. Figure 4(b) shows the responses of perturbed systems. The response achieved with the proposed method is still better, indicating that the proposed cascade control system not only provides a fast response, but also maintains a reasonable robustness.
4.2 Example 2

As mentioned previously, most existing works design the primary controller based on an apparent process model that is obtained using the design target of the inner loop. To illustrate the drawback in previous works, the cascade control system with the PID/P control mode and the following processes is considered.

\[
G_s = \frac{1}{5s+1}e^{-2s}, \quad G_p = \frac{1}{s+1}
\]  
(35)

This study compares the proposed tuning method with that of Lee et al. (1998). Perfect models are assumed for controller design. By choosing \( M_2 = 1.5 \), the proposed method suggests the secondary P controller with \( K_2 = 0.565 \). The tuning parameter in the method of Lee et al. (1998) is chosen as \( \lambda = 1.18 \) so that the same \( K_2 \) is resulted.

To tune the primary controller, the proposed method identifies a SOPDTZ model for the apparent process as

\[
G_{ap}(s) = \frac{0.361(0.634s+1)}{1.698s^2+5.34s+1}e^{-3.78s}
\]  
(36)

Choosing \( M_3 = 1.7 \), the resulting PID parameters are obtained as \( K_3 = 3.157 \), \( \tau_3 = 5.936 \), and \( \tau_{i3} = 0.766 \). On the other hand, the method of Lee et al. (1998) uses the following approximated model for the primary controller design.

\[
G_{ap}^*(s) = \frac{1}{\lambda s+1}e^{-\theta \lambda} \quad G(s) = \frac{1}{(1.18s+1)(5s+1)}e^{-3s}
\]  
(37)

The tuning parameter \( \lambda \) in Lee et al. (1998) is selected so that the resulting system has the same \( M_3 \) with the proposed system, leading to \( K_3 = 2.371 \), \( \tau_3 = 7.586 \), and \( \tau_{i3} = 1.999 \).

Figure 5 compares the closed-loop responses to a unit step set-point change at \( t = 0 \) followed by unit step disturbance inputs \( d_2 \) and \( d_1 \) at \( t = 60 \) and \( t = 120 \), respectively. These results illustrate the superior performance of the proposed method. Figure 6 shows the frequency responses of the actual apparent process and the models considered in the proposed method and in Lee et al. (1998). The model considered in Lee et al. (1998) (i.e., (37)) significantly deviates from the actual apparent process because the secondary controller is a P controller. This figure reveals that the superior performance by the proposed method is the result of accurately modeling the apparent process in the primary controller design.

5. CONCLUSIONS

A novel automatic tuning technique for cascade control systems has been developed to tune the two PID controllers simultaneously. Based on a single closed-loop step test, the process information is identified with the help of B-spline series representation for the step responses. Distinctive features of the presented tuning method include: (1) the secondary controller is designed for enhanced disturbance performance; (2) the primary controller is designed based on a process model that accurately accounts for the inner loop dynamics; (3) the desired robustness levels explicitly guide the selection of the IMC tuning parameters. The effectiveness of the proposed method is apparent from the satisfactory performance shown in the simulation examples.

Fig. 5. Closed-loop responses for example 2.

Fig. 6. Comparison of frequency responses (Nyquist plots) of the apparent process model for example 2.

REFERENCES


