Simulation and Experiment Validation of Simultaneous Vibration Control and Energy Harvesting from Buildings using Tuned Mass Dampers

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Abstract—For the protection of the structure safety and occupant comfort, the vibrations of the tall buildings are serious concerns for both engineers and architects. In order to mitigate the vibration, different approaches have been proposed, among which Tuned Mass Dampers (TMDs) are one of the most preferable and have been widely used in practice. Instead of dissipating the vibration energy into heat waste via the viscous damping element, this paper presents an approach to harvest the vibration energy from tall buildings with TMDs, by replacing the damping element with electromagnetic harvester. This paper demonstrates that vibration control and energy harvesting can be obtained simultaneously. Based on the proposed switch based energy harvesting circuit with both power regulation and force control functions, two control strategies are investigated in this paper. One is the clipped Linear Quadratic Gaussian (LQG) control, where the TMD is controlled in a semi-active way with practical force constraints while electricity is harvested. The other one is an implemented energy-harvesting TMD with the duty cycle of the energy harvesting circuit controlled by Pulse Width Modulation (PWM). In this case, the vibration performance is quite close to the traditional TMD while the vibration energy is converted into electricity. An experiment using the second control strategy is conducted based on a three-stories building prototype with electricity generating TMD composed of a rotational motor and rack-pinion mechanism.

I. INTRODUCTION

VIBRATION has been a serious concern since the early days when the tall buildings and long-span bridges were built. These structures are subjected to huge dynamic loadings from the winds, earthquakes, water waves, traffics, and human motions. The large vibration amplitude can damage the structures or the secondary components, or cause discomfort to its human occupants [1]. Extensive research has been conducted to mitigate this harmful vibration, by, for example, structure design [2], vibration isolation systems [3], and auxiliary damping systems. Among these methods, the TMD has been proved to be a very simple and effective vibration suppression device, with many practical implementations on tall buildings, such as Taipei 101 in Taipei, Citi Group in New York, and many others. The original TMD, which was invented by Frahm in 1911 [4], only consists of an auxiliary mass and a spring connected to the primary system to mitigate vibration in a very narrow frequency range near the resonance. Ormondroyd and Den Hartog [5] increased the working frequency range by introducing an additional damper and optimizing the parameters.

Although TMD composes great percentage of the supplemental damping systems currently in use, it has its inherent drawbacks. When the parameters of the primary systems change a small amount, the performance of TMDs will be greatly defected. It is so called off-tuning problem, which has been investigated by researchers, for example, Bergman [6] and Setareh [7]. Hence, active TMD is developed to handle this problem. It has been proved that the active TMD can provide better vibration mitigation performance than the passive one, at the cost of large amount energy consumption [8]. However, the active TMD system is more complex and costly, which limits its practical implementations. Besides, active TMD suffers from the inherent disadvantage that it relies on external energy, which is not countable in the hazardous situations like earthquakes or hurricanes. Hybrid active TMD, which is composed of passive TMD and one additional actuator in parallel with the damping element, can reduce the power consumption and increase the robustness in the event of power failure [9], [10].

Realizing both the limitations of passive and active TMDs, researchers proposed semi-active TMDs to provide better vibration mitigation performance than the passive one without the drawbacks of active TMDs. Semi-active control using Magneto Rheological (MR) or variable orifice dampers have been investigated by many researchers [11]-[15], especially in vibration control of buildings with TMDs [14] and vehicle suspension systems [15]. However, considering that the semi-active control force is essentially a passive force, which means that the vibration energy is being dissipated through the transducers. Instead of being dissipated into heat waste, the energy can be converted into electricity through electromagnetic transducers. The vibration control with energy harvesting is also called regenerative vibration control, as can be seen in regenerative vehicle suspension [16] and regenerative building vibration control [17]-[20].

This paper focuses on the realization and implementation of the regenerative TMD in both passive and semi-active ways, as well as the experimental demonstration of the feasibility of simultaneous vibration control and energy harvesting. It should be noted that there are some relevant experimental work on self-powered MR dampers using electromagnetic induction [21],[22]. However, these self-powered MR dampers are essentially two devices: MR damper and electromagnetic harvester connected in parallel. Besides, most of energy is still dissipated by the MR damper.
rather than harvested. This paper uses single electromagnetic harvester. An electromagnetic transducer consisting of a rotational motor with gears and rack-pinion mechanism is developed to replace the conventional viscous damper of TMD. The rotational motor is chosen because of its high energy density. A switch based circuit is proposed, where the damping force can be controlled by the duty cycle of the PWM signal and the low voltage generated by the rotation motor is stepped up to charge the battery by taking advantage of the motor inductance and high-frequency switching. Numerical simulation is carried out to show the effectiveness of the regenerative semi-active TMD, where the controller is designed based on clipped optimal control with practical force constraints. Experiments on the regenerative electromagnetic TMD are conducted to demonstrate the feasibility of simultaneous vibration control and energy harvesting.

This paper is organized as follows: First, the modeling of the building with TMD and circuit for the simultaneous vibration control and energy harvesting are presented. Then the two control strategies will be investigated. Finally, simulation and experiment results are presented.

II. MODELING OF THE SYSTEM

A. Modeling of the Building

![Fig. 1. Modeling of building with regenerative TMD.](image)

Fig. 1 shows the modeling of building with TMD, where only the first vibration mode is considered and the damper of the TMD is replaced with an electromagnetic motor. The primary structure is subjected to the wind load disturbance $F$. The force $u$ is exerted by the electromagnetic motor. Then, the dynamics equations of this building with TMD can be written as:

$$\begin{align*}
    m_s \ddot{x}_s + k_s x_s + c_s \dot{x}_s &= k_1 (x_1 - x_s) + F - u \\
    m_1 \ddot{x}_1 + k_1 (x_1 - x_s) &= u
\end{align*}$$

which can be further expressed in the following form:

$$\begin{align*}
    M \ddot{x}(t) + K x(t) + C \dot{x}(t) &= G F(t) + H u(t) \\
    (2)
\end{align*}$$

where $M = \begin{bmatrix} m_s & 0 \\ 0 & m_1 \end{bmatrix}$, $K = \begin{bmatrix} k_s + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}$, $C = \begin{bmatrix} c_s & 0 \\ 0 & 0 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x(t) = [x_s \ x_1]^T$.

Defining the state space as $q = [q_1, q_2, q_3, q_4]^T = [x_s, \dot{x}_s, \dot{x}_s, \dot{x}_1]^T$, Equation (2) can be further written in the state space form as:

$$\dot{q}(t) = Aq(t) + B_p F(t) + B_u u(t)$$

where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$, $B_p = \begin{bmatrix} 0 \\ M^{-1}G \end{bmatrix}$, $B_u = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}$.

B. Switch-based Circuit

![Fig. 2. Energy harvesting circuit with both vibration damping force control and power regulation capacities.](image)

Fig. 2 shows the proposed circuit for the simultaneous vibration control and energy harvesting implementation. The electromagnetic motor is modeled as a voltage source $e_m$ with the inherent inductor $L_m$ and resistor $R_m$ connected in series. The relative motion between the two masses can induce a back electromotive voltage $e_m$ in the coils, which is proportional to the relative velocity $v_r = \dot{x}_1 - \dot{x}_s$. The rack-pinion mechanism and gears transfer the linear motor into rotation motion with a reduction ratio $M$. Therefore, the relative speed between the stator and mover is $M$ times of the relative motion between the TMD and the structure.

$$e_m = M k_e v_r$$

where $k_e$ is the back electromotive force (EMF) coefficient of the electromagnetic motor. Meanwhile, the current flows inside of the motor coil will induce a back electromotive force proportional to the current:

$$f_b = M k_l i$$

where $k_l$ is thrust constant of electromagnetic motor.

The circuit shown in Fig. 2 has both vibration damping force control and power regulation capacities. The switch $S$ is controlled using PWM in a frequency that is several orders higher than the first natural frequency of the building. Hence, the mechanical system can be considered as static from the electrical circuit point of view. When the switch is ON, the electromagnetic motor tends to provide the maximum damping coefficient which is equal to

$$e_{m_{\text{max}}} = \frac{h^2 k_t k_c}{R_m}$$

Meanwhile, the current flow inside the motor will increase to its maximum. When the switch is turned OFF, due to the effect of the inductance of the motor the current continues flowing even when the voltage generated by the motor is less than the battery voltage $E_b$. However, the current will decrease. Such high frequency switching with different duty cycle will result different mean current in the
motor and damping force. Since the switch frequency is orders higher than the mechanical system, the mean force and current can be controlled in real-time. It should be noted that this circuit topology has been used in [20] for the purpose of eliminating the dead zone when the voltage generated by the motor is lower than the battery voltage and in [23] for boosting the electricity from the low voltage to higher voltage. In this paper, we will extend its potential to the semi-active force control.

In general, Fig. 3 shows the feasible force region when the damping force is controlled by the circuit in Fig. 2. A maximum force limitation is applied in order to maintain the mechanical safety of the motor. When the amplitude of the relative velocity \( v_r \) is smaller than \( \frac{E_p}{MK_e} \), the minimum force is the 0, when the switch is always OFF. The maximum damping coefficient will be \( \frac{M^2k_e k_e}{R_m} \), when the switch is always ON. On the other hand, when the amplitude of \( v_r \) is larger than \( \frac{E_p}{MK_e} \), the minimum force will be \( \frac{M^2k_e k_e}{R_m} - \frac{MK_e E_p}{R_m} \), when the switch is always OFF. Fig. 3 will be the force constraints applied for the regenerative semi-active control design in Section III.

III. CONTROL STRATEGIES

A. Clipped LQG control

The clipped optimal control has been investigated by many researchers with its application on the building vibration control [12, 17, 24]-[25]. It is also used in the paper to design the semi-active controller. Because we need the information of relative velocity to determine whether the force is essentially passive or active force, the full state feedback control is assumed.

First, the desired active force \( u_{des} \) is obtained by LQG active controller design. Then \( u_{des} \) is clipped due to constraints of semi-active control. To get the desired force, the root-mean-square (RMS) value of the structure’s displacement is used as the performance index. Hence the performance index in chosen as:

\[
J = \int_0^\infty (q^T Q q + ru_{des}^2) dt
\]

where \( Q \) and \( r \) are time invariant weights of the state and control force. By minimizing the quadratic cost, we can minimize the response to Gaussian white noise input.

The optimal control law is expressed:

\[
u_{des} = -r^{-1} B^T S q(t) = -K q(t)
\]

where \( S \) is the solution of the algebraic Riccati Equation:

\[
A^T S + SA - SB_r r^{-1} B_r^T S + Q = 0
\]

The optimal control force \( u_{des} \) is then clipped using the feasible force regions (Fig. 3) constrained by the proposed switch circuit (Fig. 2). The final control law of the whole system with considering the force limitation or damping coefficient limitation is realized by the block diagram shown in Fig. 4.

B. Regenerative Electromagnetic TMD

The effect of passive TMD can be achieved by regenerative electromagnetic TMD with the charging circuit in Fig. 2 controlled by the duty cycle of the PWM. Following the methodology in [26], the equivalent resistance of the circuit connected to the motor/generator at discontinuous mode can be obtained as:

\[
R_{eq} = \frac{2L_m (\alpha - 1)}{TD^2 \alpha}, \quad \text{for} \quad D \leq 1 - \frac{1}{\alpha}
\]

where \( T \) and \( D \) are the period and duty cycle of the PWM, respectively, and \( \alpha \) is the ratio of the battery voltage and motor back EMF voltage. Hence, the regenerative EMF force can be controlled to achieve a viscous damping coefficient \( \frac{M^2k_e k_e}{R_m + \epsilon R_{eq}} \).

As we know, there is an optimal damping ratio for the traditional passive TMD. The optimal parameters of traditional TMD can be found using the equations summarized in [27]. By adjusting the duty cycle of the proposed circuit, the TMD can be tuned to have the desired damping coefficient to minimize the vibration while harvesting the vibration energy.

It should be noted that if the two electrodes of the motor are shunt directly with a resister \( R_e \) and the motor inductance is neglectable, the electromagnetic motor will also appear to be a viscous damper. Desired damping can then be achieved by adjusting the value of \( R_e \), although the regenerated electricity is not really harvested.

IV. SIMULATIONS AND EXPERIMENTS RESULTS

A. Simulation results of TMD with clipped LQG

The simulation is carried for case when the regenerative semi-active control is designed based clipped LQG. The steady state responses of the semi-active system to a
harmonic excitation are periodic with the same period as the
excitation which means the system is piecewise linear and
the frequency response of the system can be plotted [11].
However, the force constraints like Fig.3 will introduce
nonlinearity to the system [24], [25]. Yet, the frequency
response can still be used to reasonably illustrate the
effectiveness semi-active TMD, though not strictly. The
parameters used in this simulation are based on the prototype
built for experiments as shown in Fig. 6, the detailed
parameters of which are listed in Table I.

![Fig. 5. Frequency response of passive, electricity-generating and active
TMDs](image)

It can be seen from Fig. 5, regenerative semi-active TMD
can provide much better vibration mitigation performance
than the passive TMD optimized using the formula in [27].
With the practical constraints shown by Fig.3, the
performance is not better than the active TMDs. However, it
can harvest energy rather than consuming energy. It should
be noted that in the LQG controller design, $Q$ is defined
such that the displacement of the primary system as the
performance index $Q=\text{diag}([1,0,0,0])$, $r$ is chosen $5e-7$ to
limit the maximum control force.

<table>
<thead>
<tr>
<th>TABLE I  PARAMETERS USED FOR SIMULATION</th>
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<tbody>
<tr>
<td>Parameters</td>
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<tr>
<td>EMF coefficient $k_e$</td>
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<tr>
<td>Thrust constant $k_t$</td>
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<tr>
<td>Resistance of the motor $R_m$</td>
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<tr>
<td>Inductance of motor $L_m$</td>
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<tr>
<td>Gear ratio of motor $M$</td>
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<td>Battery voltage $E_b$</td>
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<td>Modal mass of the building $m_r$</td>
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<td>Mass ratio $\mu$</td>
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<tr>
<td>Magnitude of forced force $F_f$</td>
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<tr>
<td>First natural frequency of building</td>
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</tbody>
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**B. Experiments results of electricity-generating TMD**

The second control strategy which is electromagnetic
TMD with energy harvesting function is demonstrated
experimentally here, where the passive force of (10) is
realized by the circuit in Fig.2. A three-stories building
prototype with regenerative TMD is built for the purpose, as
shown in Fig.6. The building prototype is 1.9m tall. It is
supported by 4 beams with mass blocks located on the
second, third and roof level respectively. It is built with
aluminum, which totally weights 24kg.

The frequency response of the building is shown in Fig. 7,
which is measured using sweep sine excitations. It indicates
the first three natural frequencies are 3.32 Hz, 10.66Hz and
16.29Hz, respectively.

![Fig. 6. A 3-story building prototype with regenerative TMD: a), the whole
view of the building; b), the 3-D drawing of the regenerative TMD, c), a
close-up view of the TMD prototype.](image)

![Fig. 7. Measured frequency response of the 3-story building prototype
without TMD](image)

The electromagnetic TMD system is shown in Fig.6 (b)
and (c). The TMD mass is supported by a two flexible beam
structures, which can provide large stiffness in vertical
direction to support the gravity load and smooth motion in
the horizontal direction with very little friction. A geared
motor with resistance load acts as the damping element of
the TMD system. A rack pinion mechanism is used to
transfer the linear motion to the motor rotation.

In this experiment, the building is modeled as a single
DOF system in the mode space, because usually the TMD is
used for the first vibration mode reduction. The tuning of
TMD requires the mass ratio , which is the mass of the
TMD over the first modal mass of the building. However,
the modal mass is not known exactly. We proposed a simple
and yet effective engineering method to obtain the modal
mass by experiment, as in the appendix. Hence, three are
totally three steps to tuning the TMD: 1). Obtaining the mass
ratio; 2). Tuning the frequency ratio; 3). Tuning the damping
ratio. In the third step, the damping ratio can be tuned via
adjusting the duty cycle of the proposed circuit for
simultaneous energy harvesting.

The mass of the TMD is 0.68 kg. The mass ratio is calculated to be 5.2% and hence the actual modal mass of the building is 13.08 kg.

The frequency response of the building prototype with TMD tuning by the switch circuit in Fig. 2 is shown in Fig. 8 with a fixed duty cycle 10%. The vibration is significantly reduced compared with the one without TMD.

![Figure 8](image)

Fig. 8. Measured frequency response. — Frequency response of building without TMD. — Frequency response of building with TMD tuned using resistor of 360Ω. — Frequency response of building with TMD tuned using the circuit shown in Fig. 2, where the duty cycle of the switch is 10%)

Fig. 9 shows transient current flow in diode D1 with a duty cycle of 10% and the excitation is 15 N harmonic force of 3.3 Hz, where the effect of switch can be obviously seen. The mean value of the current over one vibration period is about 0.02 A. The battery we used has 3 V voltage. Hence the mean power harvested is about 60 mW.

![Figure 9](image)

Fig. 9. Measured instant electrical current in diode D1

As a comparison, the frequency response of building is also plotted in Fig. 8 for the TMD tuned by a resistor, where the optimal tuning resistance $R_t$ is chosen as 360 Ω by trial and error. As we can see from Fig. 8, the vibration control effect of the TMD with electrical charging circuit is close to the one tuned by a resistor. There is still some difference, and the reason is because the switch circuit is essentially nonlinear and there is unmodeled friction of the rack-pinion. Ideally the duty cycle should be controlled with feedback instead of being kept as a constant value.

In addition to the steady frequency response, we also experimentally studied the transient response of building with electricity-generating TMD. We pull the building to an initial position and release it. The transient vibrations are shown in Fig. 10, from which we can see that the vibration is reduced very quickly in the case when TMD is tuned with resistor and duty cycle control respectively. After 1.5 second the higher mode vibration continues and lasts for longer time because TMD is designed for the first vibration mode only.

![Figure 10](image)

Fig. 10. The transient acceleration response of the building. (Without TMD, — with TMD tuned with optimal resistor, — with TMD tuned by duty cycle)

Fig. 11 shows the voltage of the tuning resistor which is 360 Ω. The electricity of up to 2 V voltage is generated while mitigating the vibration.

![Figure 11](image)

Fig. 11. Voltage on the 360 Ω resistor generated by the electromagnetic TMD

Figure 12 shows the current that charges the two 3 V batteries. Since the instant current charges the battery has very high frequency due to the effect of switching (as in Fig. 9), in Fig. 12 we use the average current of time period 0.01 s. About 5.85 mJ energy charges the battery in this free vibration. Also from Fig. 11 and Fig. 12, we can find that almost no power is dissipated by the resistor or harvested after 1.5 second. This suggests that the relative motion between the TMD and building is stopped due to the static friction of the rack-pinion at low vibration amplitude. The vibration is then mitigated slowly by the inherent damping of the building prototype.

![Figure 12](image)

Fig. 12. Transient average current $i$ that charges the battery of the energy-generating TMD controlled by duty cycle

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V. CONCLUSIONS

This paper for the first time demonstrated the feasibility of simultaneous vibration control and energy harvesting from the building structure using electromagnetic harvester. Two control strategies are investigated, namely regenerative semi-active control and regenerative electromagnetic TMD with energy harvesting. The effectiveness of regenerative semi-active control is shown by simulation where the controller is designed based on the clipped LQG control with practical force constraints. The regenerative semi-active control can provide better vibration mitigation than the passive one. The simultaneous vibration control and energy harvesting is experimentally demonstrated using regenerative electromagnetic TMD, where the desired damping is controlled using the proposed circuit with dual functions of force control and power regulation. 60mW energy is harvested when the prototype building is excited by a harmonic force with amplitude of 15N at 3.3Hz.

APPENDIX

We propose a simple and effective method to experimentally determine the mass ratio. First the mass of TMD is removed from the building, and the systems natural frequency is measured \( \omega_1 \) (3.32Hz for the prototype building). Then, by adding the TMD mass to the building and locking the TMD together with the building, we can obtain a new natural frequency of the building with additional TMD mass \( \omega_12 \) (3.237Hz for the prototype building). By using the two measured frequencies, the tuning ration can be calculated.

\[
\frac{\omega_1^2}{\omega_12} = \frac{k_1}{m_1}\frac{1}{1 + \mu} = 1 + \mu \quad \text{(A.1)}
\]

ACKNOWLEDGMENT

The authors would like to thank Min Chen, Zhujian Liu, Ethan Li for helping us build the prototype and Richard LaGrandier for his assistance in testing the circuit. The authors gratefully acknowledge the funding support from National Science Foundation (NSF) CMMI#1031038.

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