The Ballet Automaton: A Formal Model for Human Motion

Amy LaViers and Magnus Egerstedt

Abstract—In this paper we present a discrete event model whose marked language, i.e., sequences of movements, make up canonical warm-up routines in classical ballet. Through composition operations that trim physically infeasible transitions and ensure that the rules of classical ballet are adhered to, a richer model is obtained that supports the generation of free-flowing dance sequences in the style of classical ballet. This type of construction not only allows us to produce a rich set of stylized human motions, but it also allows for variations in the “personalities” of these motions through the potential use of different composition specifications.

I. INTRODUCTION

In a number of emerging applications, the understanding of human motion plays a key role. These applications range from the production of natural gaits for biped humanoid robots to image segmentation and computer animation (e.g., [15], [18], [3]). This paper should be understood in terms of this larger dialogue about how to represent human movement for recognition, imitation, and concise parameterization. A number of human motions and tasks, such as reaching, drawing, and arguably walking, have been successfully encoded using dynamic motion primitives [6], [14]. These primitives, or motemes [4], are designed to produce rich and complex human-like motions through systematic, temporal composition. Traditionally, these primitives are obtained from empirical data, e.g., collected using motion capturing devices, that is segmented (often by hand, [3], but with progress towards automatic segmentation, [7]) into appropriate motion chunks and stored in a motion library [14].

These previous models pick up on very specific, characteristic trajectories involved in short stereotyped movements. A drawback of this representation is that it limits the models’ ability to capture longer, and ultimately produce, composite movement sequences or actual behaviors. Rather than basing the models on empirical data as in [6], [7], and [12], in this paper, we draw inspiration from a highly constrained set of motion patterns, namely, those found in classical ballet. We aim to capture these particular patterns because there are formal rules of movement organization in ballet, and the execution of these rules produces highly sophisticated and complex motions. These highly structured, yet highly expressive, movements provide a convenient initial case study that inspires the structure of our framework.

Dance choreography has been captured using various formalization approaches, e.g., Laban notation [11], but in this paper we model the motion patterns of ballet as a series of event-driven poses\(^1\). Hence, our model takes the form of a finite automaton. The states of the resulting discrete event system represent body poses while transitions between states represent the motion or trajectory between poses.

In particular, we will consider the motions associated with ballet’s traditional warm-up routine. These routines involve a single leg – the working leg, and we will present a 10-state finite state machine whose marked language produces the set of feasible barre routines. By taking the Cartesian composition of two such automata, a system is obtained that now can be used to characterize more free-flowing, two-legged movements – like those found in choreographed performance sequences. However, the resulting system is physically infeasible and, furthermore, recognizes sequences of moves that can no longer be considered valid classical ballet routines. To remedy this, we introduce two operations that remove physically infeasible composite transitions as well as prevents the system from reaching invalid (non-balletic) states.

The reason for this line of inquiry is twofold. First, by producing choreographic control scripts that are not only dynamically feasible (the robot does not fall over or is not asked to execute motions that are in violation of its kinematic constraints) but also that satisfy certain aesthetic constraints, this framework can be extended to endowing and recognizing robotic motion with “personalities” – subtle, yet distinguishing, behavioral quirks. In fact, humans do not behave in tight, stereotyped manners. Rather, we stitch together short movements into greater movement sequences (not only in highly formalized movement schemes like ballet, but also in casual, pedestrian movement modes like hand gestures in conversation, which create patterns that are unique to individuals). This feature will be reflected in the proposed model as well.

Second, by capturing the discrete dynamics associated with pose transitions, the proposed model differs from previous attempts (e.g., [3] and [10]) in that it encodes the actual rules which ballet dancers and choreographers employ into a mathematical construct – thus making a new set of tools which can quantify aspects of choreography, facilitating new debate among scholars and offering unbiased comparison to other styles and genres. In this sense, the proposed model can be considered a follow up to Laban’s early attempts at quantification of aesthetic human behavior [11], [13].

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\(^1\)Ballet’s movement aesthetic is inherently goal-oriented in that the movements often have a climax or final pose that is held for a moment before the next [19]. For example, the goal pose may be an impressive balance on one foot or a graceful breath of the arms that creates a transition into a new movement theme. As such, this is a reasonable assumption.
The work in this paper can thus be regarded as a first attempt at producing a rich, discrete event model for the actual movement patterns of complex behaviors – in particular, those of classical ballet. Even though ballet is an art form characterized by the subjective effect of its flowing, graceful movements, the movements themselves are on some level objective in that they adhere tightly to a specific style. This style is dictated not only by specific teachers whose movement methods have been codified into systems of ballet technique (which allow for efficient execution of difficult jumps and turns), but also and especially by the choreographers who arrange the elements of ballet vocabulary to produce compositions that elucidate a range of emotions from their audience ([1], [19]). For example, a Romantic ballet will paint a sweet, pastoral image of an innocent princess in one scene and in the next provide a bitter, evil picture of a jealous villain using essentially the same set of basic motions. What differentiates these two sequences? This paper aims to provide the beginnings of an answer to this question – a quantitative articulation of “style” – based on tools and techniques from discrete event systems theory.

The outline of this paper is as follows: in Section II, we develop a discrete event model for classical ballet barre. This canonical one-legged routine can be crisply described by a 10-state barre automaton in which the states correspond to poses while the transitions correspond to moves. Moreover, these moves constitute the basic building blocks in ballet as they are the primary tool used to warm and train muscle patterns at the start of every ballet class. The construction of our two-legged system from the one-legged automaton is the main topic of Section III. A direct, Cartesian composition of two one-legged automata (one for each leg) will result in a system with physically impossible configurations and motions. As such, we propose a composition which trims physically infeasible transitions by checking the outputs associated with the joint transitions. We then further constrain this physically consistent system with a supervisor that prevents the system from reaching states that are incongruous with the aesthetics of classical ballet. Eventually, different such supervisors can specify different dance styles and even stylistic variations on ballet, e.g., the Cecchetti and Vaganova methods. To demonstrate the viability of our construction, we consider random sample paths through the final system which are viable dance sequences, in the style of classical ballet. Finally, in Section IV we provide philosophical motivation for our approach.

II. THE BARRE AUTOMATON

In order to understand and implement the rules of classical ballet, we draw inspiration from established warm-up exercises, the barre. A concept central to ballet’s doctrine is that the barre trains and safely warms the muscle groups critical to the correct execution of the freestanding, full-fledged movements that comprise the second portion of class and performances. Hence, these canonical exercises contain the poses and allowable trajectories through them that construct the more rich and expressive remaining vocabulary of ballet.

The term “barre” refers to the physical hand-railing, or bar, that dancers hold onto in order to balance during the warm-up. Exercises typically focus on one side of the body and are repeated twice in order to work both sides of the body. The working leg is the leg that is away from the bar and is more active than the other (standing or supporting) leg during a given iteration of the exercise. For now, we limit our focus to positions with the working leg in the body’s coronal plane.

We define 10 states which correspond to poses constructed from a triplet of joint angles, as seen in Fig. 1. These poses represent shapes critical to the experience of ballet. They are chosen from goal positions at the barre and, as such, are highly recognizable snapshots from the vocabulary of ballet that are found in more complex movements used for choreography. The state transitions are given by events modeled as the movements from the barre exercises. These movements are listed in the table below, together with the transition labels (assigned according to the first four letters of the name of the movement from which the event was derived). Additionally, we distinguish two transitions for each movement listed in the table using a subscript to indicate an in and out variant. The variants stem from the fact that each movement has a goal end pose; during a movement sequence, the dancer system is either on its way out to the goal pose or on its way back in, to a previous state. [19]

<table>
<thead>
<tr>
<th>Movement</th>
<th>Transition Label</th>
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<tbody>
<tr>
<td>plié</td>
<td>plie</td>
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<tr>
<td>relevé</td>
<td>rele</td>
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<tr>
<td>battement tendu</td>
<td>tend</td>
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<tr>
<td>degajé</td>
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<tr>
<td>coupé</td>
<td>coup</td>
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<tr>
<td>frappé</td>
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<tr>
<td>grand battement</td>
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<td>possé</td>
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<td>battement</td>
<td>batt</td>
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<tr>
<td>développé</td>
<td>deve</td>
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Formally speaking, we will model our system as the finite state machine
\[ G = (X, E, O, f, \Gamma, o, x_0, X_m, \epsilon, \omega), \] (1)
where \( X \) is the finite state space, \( E \) is the event set, \( O \) is an output set, \( f : X \times E \rightarrow X \) is the state transition function, \( \Gamma : X \rightarrow 2^E \) is the set of feasible events (at a given state), \( o : X \times E \rightarrow O \) is the output map, \( x_0 \in X \) is the initial condition, and \( X_m \subseteq X \) is a set of marked states. In order to allow for both event and asynchronous transitions once we take the Cartesian composition of two such systems, we also explicitly need to use “empty” transitions, which are defined by the symbol \( \epsilon \). The interpretation is that for our finite state machine, we insist on \( \epsilon \in E \), with the result that \( \epsilon \in \Gamma(x) \) as well as \( f(x, \epsilon) = x, \forall x \in X \). Moreover, we associate \( \omega \in O \) with the outputs from “empty” events, i.e., \( o(x, \epsilon) = \omega, \forall x \in X \).

In particular, for the barre automaton that we will use to generate ballet-like motions, \( X_{\text{barre}}, E_{\text{barre}}, f_{\text{barre}}, \Gamma_{\text{barre}} \), and \( \Omega_{\text{barre}} \) are defined in Fig. 2. (Note that, for clarity, we neglected to draw the self-loops that are needed at each state to capture the effect of \( \epsilon \)). Moreover, the rest position is given by state 2, and as we always aim at returning to that pose, we let \( x_0 = 2 \) and \( X_m = \{2\} \). The resulting marked language, i.e., the set of event strings that start at \( x_0 \) and end in \( X_m \), produce feasible barre routines. Some strings might be somewhat unusual, but they will certainly be recognizable as classical ballet.

In this paper, we will use the output map as a means of preventing physically infeasible motions from happening when composing two barre automata in order to emulate full-fledged two-legged dance routines rather than one-legged warm-up exercises with our system. For this, we consider the continuous set \( \mathcal{X} \) which spans the entire physical space of configurations that the leg may take, defined in terms of the three angles in Fig. 1. From the figure and general kinematic constraints of humanoid geometry, we see that the range of \( \mathcal{X} \) is limited, e.g.,

\[ \mathcal{X} = \left\{ (\theta_1, \theta_2, \theta_3) \mid \theta_1, \theta_2 \in [0, \pi], \theta_3 \in \left[ \frac{3\pi}{2}, \pi \right] \right\}. \] (2)

We can now think of transitions between the discrete states of the automaton as tracing paths through \( \mathcal{X}^2 \). And, as our formulation associates every event with a unique state, we can associate the output map with the path that the corresponding pose transition sweeps in \( \mathcal{X} \). Hence, for \( x \in X_{\text{barre}} \), we let \( o_{\text{barre}}(x, \epsilon) \in 2^X = O_{\text{barre}} \), and we denote by \( g_{\text{barre}} \) the particular finite state machine used to encode the barre exercises in classic ballet.

In order to demonstrate how a sample path through the system works, consider, for example, a développé; this movement is found both in barre exercises and more complex ballet movement phrases. A développé is the action when the working leg’s foot is moved to the ankle, then the knee, then extends from the body so that the leg is parallel to the floor. However, lifting the foot to the ankle or knee (without, for example, any extension to follow) are allowable movements called coupé and possé, respectively. Thus, to keep our transitions (and trajectories) uniquely defined, we model this as three separate events for the working leg: coup_\text{a}, poss_\text{a}, deve_\text{a}. Next, the dancer performs a closing movement where the foot remains extended from the body and the leg is lowered till the foot is returned to the starting stance. This is modeled as the event batt_\text{i} – the transition from pose 8 directly to pose 2 with a label that corresponds to the trajectory of a battement (a simpler movement that looks like a high straight-legged kick). The events coup_\text{i}, poss_\text{i}, and deve_\text{i} are also defined, that is, the reverse pose transitions are allowed and used for more complex movements. Of course, for the stationary standing leg, the movement is simply a repeated \( \epsilon \) event; the next section will illustrate event strings for two legs.

\(^2\)A relaxed version of this may take into account that the angles corresponding to the states in the automaton are, in practice, approximate. They may even vary slightly dancer to dancer and, for a given dancer, execution to execution. In this case we may think of these paths as “tubes.”
III. A COMPOSITIONAL SYSTEM

Given two finite state machines of the form of Eq. 1, we want to be able to take the Cartesian composition of these two systems while transitions that are deemed impossible or incorrect are removed from the composed system. The reason for this is that we want to be able to go from a one-legged barre automaton to a system involving two legs without violating the laws of physics or the rules of ballet. For this, we need to introduce the following compositional operations.

Given two finite automata
\[ G_i = (X_i, E_i, O_i, f_i, \Gamma_i, o_i, x_{i,0}, X_{i,m}, e_i, \omega_i), \quad i = 1, 2, \]
we let the Cartesian composition of these two systems be given by
\[ G = G_1 \times G_2 = (X, E, O, f, \Gamma, o, x_0, X_m, e, \omega), \]
where
\[ X = X_1 \times X_2 \]
\[ E = E_1 \times E_2 \]
\[ O = O_1 \times O_2 \]
\[ f((x_1, x_2), (e_1, e_2)) = (f_1(x_1, e_1), f_2(x_2, e_2)) \]
\[ \Gamma((x_1, x_2)) = \Gamma_1(x_1) \times \Gamma_2(x_2) \]
\[ o((x_1, x_2), (e_1, e_2)) = (o_1(x_1, e_1), o_2(x_2, e_2)) \]
\[ x_0 = (x_{1,0}, x_{2,0}) \]
\[ X_m = X_{1,m} \times X_{2,m} \]
\[ e = (e_1, e_2) \]
\[ \omega = (\omega_1, \omega_2) \]

Note that this is a synchronous composition in the sense that events have to happen to both of the two systems in order for a transition to happen. However, through the introduction of the empty word, we can produce asynchronous transitions directly through the use of the events \((e_1, e_2)\) or \((e_1, e_2)\) in a straightforward manner.

If we perform this operation on two barre automata as \( G = G_{\text{barre}1} \times G_{\text{barre}2} \), we obtain a system that no longer is a one-legged warm-up routine, but rather a two-legged dance model. Even limited to movements taking place in the coronal plane, \( G \) can demonstrate how simple barre exercises define the grammar for more varied and larger ballet movements. For example, \textit{pas de chat\(^3\)} is a jump in which the dancer picks up each leg in sequence causing him or her to move side to side. It is modeled for either leg in this automat as the event sequence
\[ coup_o, poss_o, poss_i, plie_o, plie_i \in E_{\text{barre}}^* \]
where \(^*\) denotes the Kleene closure. The corresponding event string for the composite system would be
\[ (e_1, coup_o), (coup_o, poss_o), (poss_o, poss_i), ..., \]
\[ ..., (poss_i, plie_o), (plie_o, plie_i), (plie_i, e_2). \]

Note that both of these joint event strings belong to the composite set \((E_{\text{barre}} \times E_{\text{barre}})^*\).

\(^3\)Literal translation from French: “step of the cat.”

As the barre automaton does not tell us anything about the forces required for jumping but accepts the correct sequence of leg positions during the jump, it is entirely possible that the Cartesian composition contains events that are not physically possible to execute. The characterization of the physically feasible joint events is encoded in the output values associated with the events. We want to prevent any corresponding, physically infeasible events from occurring in the composite system, and thus, we introduce the operation \(\text{infeas} \). Let
\[ G = (X, E, O, f, \Gamma, o, x_0, X_m, e, \omega) \]
be a finite state machine as per Eq. 1 and let \(O_{\text{infeas}} \subseteq O\) be a subset of the output set that does not contain \(\omega\). The \(\text{infeas} \) operation will take \( G \) and \(O_{\text{infeas}}\) as arguments and return a new finite state machine,
\[ \text{infeas}(G, O_{\text{infeas}}) = (X, E, O, f, \hat{\Gamma}, o, x_0, X_m, e, \omega), \]
where we only have changed the definition of \(\Gamma\) i.e., the set of events that are allowed to happen at a given state. The new such set is given by
\[ e \in \hat{\Gamma}(x) \iff e \in \Gamma(x) \text{ and } o(x, e) \notin O_{\text{infeas}}. \]
be the supervised system $G \backslash S_{\text{aesth}}$, where $S_{\text{aesth}}$ is the maximally permissive, non-blocking supervisor that ensures that the set $X_{\text{unaesth}}$ is never reached. Note that such supervisors can be automatically generated, e.g., [5].

The definition of $X_{\text{ballet}} \backslash S_{\text{unaesth}}$ in the ballet model is a straightforward list of two-legged states which are perhaps considered ugly as judged by the metric of ballet; often, these are asymmetrical poses or poses which cannot be seen from the audience’s distant perspective. Finally, with all of these components in place, the final model $G_{\text{ballet}}$ for free-flowing classic ballet dance is given by

$$aesth(\text{infeas}(G_{\text{barre1}} \times G_{\text{barre2}}, O_{\text{infeas}}, X_{\text{ballet}})).$$

In order to test the effectiveness of our chosen rules (as these behaviors which we define may be very subjective and require a human eye to validate), we generate random allowable sample paths through two systems: 1) the original Cartesian composition between two single-leg automata with no control scheme and 2) a system which adds the restrictions contained in $O_{\text{ballet}}$ and $X_{\text{unaesth}}$ and enforced by the operations outlined in this section as in Eq. 6. The results are animated using a MATLAB script; snapshots of an illustrative sample case are provided in Fig. 3. These animations have been evaluated by a trained eye and found to be a reasonable initial model of ballet technique. Clearly, even to the untrained eye, significant changes take place between the distinct cases of systems that we animated.

IV. TOWARDS A METHOD FOR STYLE SPECIFICATION AND COMPARISON

An emerging philosophy in several disciplines is that dynamical equations in terms of external parameters such as joint angles are an inherently poor choice of coordinates for parameterizing human motion. Although these external quantities are easy to measure, we present three examples where they have been demonstrated to break down. (1) Recent results in neuroscience ([9]) indicate that the motor cortex (the part of the brain that controls animal movements) is organized in terms of behavioral actions, not body parts and joint angles. (2) It is an emerging practice in dance education to give students corrections in terms of actions, not body part placement [2]. The philosophy is that, in general, humans do not have control of individual body parts. As a result, corrections that identify misplaced body parts result in weird, undesirable movement patterns that the dancer essentially invents in attempt to align something which he or she cannot control directly. (3) Biophysicists studying the movements of C. Elegans, a microscopic worm that is one of the most well-studied model organisms in biology, provided the first quantitative explanation of its movement patterns by phrasing their analysis in terms of four body poses. These body poses, and their linear combinations, were shown to account for 95% of the worm’s behavior [16]. Viewing the worm’s movement in this so-called shape space ([17]) allowed the researchers to, for the first time, accurately predict the motion of the worm due to external stimuli. Thus, it is perhaps an internal parameterization, such as body position, that will provide a more simple, useful model for human movement.

The extension of this philosophy into a system with well-defined inputs and outputs presents interesting questions for systems theory, and as has been presented in this paper, systems theory allows a new articulation of the creative process involved in choreographing human movement. Namely, using body position parameterization, we have specified a grammar for body positions in ballet movements restricted to the coronal plane. This specification may lead to systems (humanoid or otherwise) which behave in a way that is natural for its given context. A deterministic program can limit the flexibility of a system’s ability to cope and adapt to its environment. When implemented on a robotic system, a behavior, as outlined here, provides one or more states that the system may enter given its current state. By affixing further outputs to the system’s transitions and assigning...
preferential weighting to available states, an automatic decision making process may select one of the available states outlined by the behavioral framework.

Encapsulated in this framework is the facility to produce different styles of movement adjusting only the contents of our barre automaton and two sets: $O_{\text{infeas}}$ and $X_{\text{unaesth}}$. These sets imply rules that arise from physical and aesthetic constraints, respectively. Instantiations which encode a different style, perhaps with the addition of more poses (than in this initial presentation) may be compared via a quantitative metric as they each have simple mathematical representations. This venue for movement and style analysis is the second contribution of this model. Such a quantitative survey of specific dance styles (between different genres and choreographers) would bolster and perhaps corroborate years of qualitative dance study which hold that specific movement patterns evoke very different aesthetic and emotive effects in dance choreography.

In summary, ballet is a highly ordered behavior of a truly complex biological system whose attributes have important analogs in systems theory that warrant quantitative study. By formulating aesthetic style from a systems theoretic perspective and, thus, resolving the attributes of human movement which typify and comprise stylized movement, we are beginning to define a metric for a previously abstract concept. Furthermore, the structure of the aesthetic movement explored here provides an interesting challenge for control theory, namely that of discrete event systems and their composition.

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