Constrained stabilization of a two-input buck-boost DC/DC converter using a set-theoretic method

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Abstract—This paper considers the problem of constrained stabilization of a two-input buck-boost DC/DC converter by linear state-feedback. It is demonstrated that, via an appropriate change of coordinates, a recent synthesis technique for constrained bilinear discrete-time systems can be applied to an averaged nonlinear model of the converter. Moreover, it is proven that the synthesis method yields a polyhedral constrained control invariant set for the converter model in the original coordinate system. The synthesis algorithm requires solving a single linear program off-line. An extensive simulation case study along with a preliminary, successful real-time experiment, demonstrate the effectiveness of the proposed methodology.

Index Terms—DC/DC converters, Bilinear systems, Invariance, Polyhedral Lyapunov functions, Constraints.

I. INTRODUCTION

Buck-boost DC/DC converters are switching devices that have strong nonlinear dynamics and are subject to hard constraints on inputs and states. A very fast switching frequency and small sampling time (ranging from μs to ns) pose a serious challenge to controller synthesis and implementation. That is why simple control solutions, such as PID and Fuzzy controllers, are dominant in PWM controlled low-cost power converters, see, e.g., [1] and [2]. The main issues with this type of controllers are a lack of an a priori stability guarantee and inability to cope with constraints in a non-conservative way. As far as stability is concerned, a direct switching Lyapunov approach was proposed for stabilization of DC/DC converters, see, e.g., [3]. However, this approach can lead to arbitrarily fast switching and does not handle constraints. Recently, model predictive control was proposed as a viable alternative to deal with constraints in control of power converters, see, e.g., [4]–[8] and the references therein. However, due to the bilinear nature of the typical averaged model of a buck-boost converter, these algorithms are computationally intensive and not suitable for low-end converters. Tractable solutions can only be obtained for linear or piecewise affine approximations, which introduce errors and lack an a priori stability guarantee as well. For such classes of systems, an explicit piecewise affine predictive control law can be obtained and stability can be checked a posteriori, see, e.g., [9]. As such, it would be desirable to obtain a tractable synthesis method that is applicable to the full bilinear model of a buck-boost converter, results in a low complexity feedback law and which also offers an a priori guarantee of stability and constraint satisfaction.

This paper indicates that a recent synthesis technique [10] developed for constrained stabilization of general discrete-time bilinear systems with zero as equilibrium can be applied to DC/DC converters. The method of [10] employs invariance conditions [11] for a polyhedral set and yields a stabilizing linear static state-feedback control law that satisfies constraints. The method is computationally advantageous as it requires solving a single linear program off-line. Along with the controller synthesis it yields a polyhedral Lyapunov function for the closed-loop system. Notice that polyhedral Lyapunov functions are preferable to quadratic ones, as they induce polyhedral constrained control invariant (CCI) sets. Moreover, for bilinear systems, quadratic Lyapunov functions lead to fourth order matrix inequalities, which are hardly solvable. However, the results in [10] cannot be applied directly to DC/DC converters, as the corresponding averaged converter model, although bilinear, does not have zero as an equilibrium point. Notice that a simple shift of coordinates does not preserve invariance of a polyhedral set, when the system model is bilinear.

The main contribution of this paper consists of a set of sufficient conditions that render the results of [10] applicable to a standard two-input buck-boost DC/DC converter averaged model. It is shown that if these conditions hold for an auxiliary bilinear model obtained via an appropriate, specific coordinate change, then the resulting control law is stabilizing and satisfies constraints for the original converter model with a non-zero equilibrium. Moreover, it is indicated how a polyhedral CCI set can be obtained for the original model via a suitable Minkowski translation. Such a set, besides providing a region of attraction for the closed-loop system, is very useful for model predictive control algorithms, i.e., it can be employed as a terminal set, see [12] for more details on this topic. The polyhedral CCI set, obtained with the method proposed in this paper, is much larger than the region where the linear approximation of the bilinear model is reasonable and it virtually covers the entire desired range of operation.

II. PRELIMINARIES

A. Mathematical notation and definitions

Let \( \mathbb{R} \), \( \mathbb{R}_+ \), \( \mathbb{Z} \), \( \mathbb{Z}_+ \) denote the set of real numbers, the set of non-negative reals, the set of integer numbers and non-negative integers, respectively. Given two sets \( P \) and \( S \), \( P \cap S := \{ x \in P \mid x \in S \} \). For a \( \lambda \in \mathbb{R} \) and a set \( P \subset \mathbb{R}^n \), let \( \lambda P := \{ \lambda x \mid x \in P \} \).
\[ \mathbb{R}^{n \times m} \text{ denotes the set of real } n \times m \text{ matrices. For a matrix } \\
Z \in \mathbb{R}^{n \times m} , [Z]_{ij} \in \mathbb{R} \text{ denotes the element on the i-th row} \\
\text{ and the j-th column of } Z, [Z]_{i} \in \mathbb{R}^{1 \times m} \text{ denotes the i-th row of } Z \\
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\text{ absolute value. } \]

Theorem II.3 \[ 10 \text{ Suppose there exist matrices } D_j , H \in \mathbb{R}^{p \times p} , j \in \mathbb{Z}_{[1,p]} , K \in \mathbb{R}^{n \times n} , \text{ a non-negative matrix } L \in \mathbb{R}^{2m \times 2p} \text{ and } \varepsilon \in \mathbb{R}_{[0,1]} \text{ that satisfy} \]

\[ G(A + BK) = HG , \]

\[ \sum_{i=1}^{n} [G]_{ij} C_i K = G^T D_j G , j \in \mathbb{Z}_{[1,p]} \]

\[ \begin{bmatrix} H^+ & H^- \\ H^- & H^+ \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \]

\[ \begin{bmatrix} D_1^+ \circ W^M + D_1^- \circ W^M + D_1^- \circ W^M \\ \vdots \\ D_p^+ \circ W^M + D_p^- \circ W^M + D_p^- \circ W^M \end{bmatrix} \leq \varepsilon \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} , \]

\[ L \begin{bmatrix} G_{-} \\ -K \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \leq \begin{bmatrix} u^M \\ u^m \end{bmatrix} , \]

where \( C_i \in \mathbb{R}^{n \times m} \text{ for all } i \in \mathbb{Z}_{[1,n]} , A \in \mathbb{R}^{n \times n} , B \in \mathbb{R}^{n \times m} \text{ and } K \in \mathbb{R}^{n \times n} \). Let a subset of initial conditions be given, i.e., \( Q := \Psi(G, w_1, w_2) \subseteq X \), for some \( G, w_1, w_2 \) and let the input constraints set be defined as \( U := \Psi(I_m, u^M, u^M) \) for some \( u^M, u^m \in \mathbb{R}^m \).

Corollary II.5 \[ 10 \text{ Suppose that Problem II.4 has a feasible solution with } \varepsilon \in \mathbb{R}_{[0,1]} \text{. Then the set-induced function} \]

\[ \hat{V}(x) := \max_{j \in \mathbb{Z}_{[1,p]}} \left\{ \frac{[G]_{ij} \circ \varepsilon}{[w_1]_{ij}}, \frac{[-G]_{ij} \circ \varepsilon}{[w_2]_{ij}} \right\} \]

is a Lyapunov function for the closed-loop system (2).

For a formal definition of a Lyapunov function for discrete-time systems, the interested reader is referred to \[ 11 \].
To summarize, in order to synthesize a stabilizing state-feedback law, one has to impose a candidate \( \varepsilon \)-contractive set \( Q \), which satisfies the state constraints and has the origin in its interior, and solve the Problem II.4. If \( \varepsilon < 1 \) then the resulting state-feedback is stabilizing and the set \( Q \) is indeed \( \varepsilon \)-contractive, otherwise one has to chose another candidate set and repeat the procedure.

III. PLANT DESCRIPTION AND PROBLEM FORMULATION

The non-inverting buck-boost converter is essentially one buck and one boost converter connected in series. This type of converter can produce lower as well as higher output voltages than the supplied one. For more information on the subject of power conversion, see [1], [13], [14]. The converter topology employed in this paper has a separate control input for each stage. The control signal is a PWM waveform with a constant frequency and controlled duty-cycle. The schematic representation of such a converter is shown in Fig. 1.

A. Nonlinear averaged model

No dead-time nor other nonlinear behavior of circuitry components were considered during the mathematical modeling of the converter. The only parasitic elements taken into account are the resistances of power transistors, output capacitor and inductor, which are lumped into \( R_L \) and \( R_C \). Thus, the resulting average discrete-time model of the system is bilinear in input and states, i.e.,

\[
x(k+1) = \phi(x(k), u(k)) \quad (9)
\]

\[
:= Ax(k) + Bu(k) + \begin{bmatrix} x(k)^T C_1 \\ x(k)^T C_2 \end{bmatrix} u(k) + w
\]

where \( x(k) := \begin{bmatrix} v_C(k) \\ i_L(k) \end{bmatrix}^T \in \mathbb{R}^2 \) is the system state (i.e., the capacitor voltage and the inductor current) and \( u(k) := \begin{bmatrix} d_1(k) \\ d_2(k) \end{bmatrix} \) is the system input (i.e., the duty cycles) at the time instant \( k \in \mathbb{Z}_4 \), respectively. In this paper, a constant current source is considered as load. Notice that other load types can be accommodated in a similar fashion. The matrix coefficients from (9) are specific to the circuitry implementation and they are described in terms of system parameters such as inductance, capacitance and resistance, i.e.,

\[
A = I_2 + T_s \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R_L}{L} \end{bmatrix}, B = T_s \begin{bmatrix} 0 & 0 \\ \frac{v_s}{L} & -\frac{I_{load}}{C} \end{bmatrix}, \quad (10)
\]

\[
w = T_s \begin{bmatrix} -\frac{I_{load}}{0} \\ 0 \end{bmatrix}, C_1 = T_s \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, C_2 = T_s \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{L} \end{bmatrix},
\]

where \( v_s \) is the supply voltage, \( i_{load} \) is the load current and \( T_s \) is the sampling time. The sampling time and the PWM period are assumed to be equal throughout this paper. The system is subject to hard constraints, i.e., \( u(k) \in U := \Psi(I_2, u^M, -u^m) \) and \( x(k) \in X := \Psi(I_2, x^M, -x^m) \), for all \( k \in \mathbb{Z}_4 \), where \( u^m, u^M, x^m, x^M \in \mathbb{R}^2 \) are suitable vectors. The constraints on the states can be softened within certain limits in most situations.

Remark III.1 In (10), it can be observed that some of the system matrices are functions of the supply voltage \( v_s \) and the load current \( i_{load} \). In this paper their values are considered constant and known a priori. In practice, they are either measured or estimated. Further work deals with parametrization of the control law with respect to \( v_s \) and \( i_{load} \).

B. Control problem formulation

The goal of the controller is to maintain the output voltage of the converter at a prescribed value while maintaining the system state and input within specified limits. The stationary value of the inductor current can vary in certain limits without affecting the value of the output voltage. Generally, a low value of the stationary inductor current is preferred to minimize the power dissipation of the converter.

Throughout the paper it is assumed that a specific reference is provided for both the output voltage \( V_{ref} \) and the inductor current \( I_{ref} \). The notation \( x^s := \begin{bmatrix} V_{ref} & I_{ref} \end{bmatrix}^T \) will be employed throughout the rest of the paper.

In conclusion, the controller synthesis problem has the following formulation.

Problem III.2 Given the system (9), sets \( U, X \) and \( P \subseteq X \), and desired equilibrium state \( x^s \) (along with corresponding control input \( u^s \)), construct an affine state-feedback control law such that \( P \) is \( \varepsilon \)-contractive for the resulting closed-loop system.

IV. MAIN RESULTS

As mentioned in Section II-B, the algorithm described in [10] is applicable only to systems with zero as equilibrium. Thus, system (9) must be transformed in order to render the results from [10] applicable. Moreover, this transformation must be such that its corresponding reverse transformation preserves stability and invariance and thus, constraint satisfaction.

To this end, let us begin with the analysis of a coordinate change problem for affine systems. Consider an affine system

\[
x(k+1) = Ax(k) + Bu(k) + w \quad (11)
\]
and the coordinate transformation
\begin{align}
z(k) &= x(k) - x^*, \quad (12a) \\
s(k) &= u(k) - u^*, \quad (12b)
\end{align}
where \( x^* \) is the desired equilibrium point for the closed-loop system and \( u^* \) is selected such that \( Bu^* = x^* - w - Ax^* \).

Then, one obtains
\[ z(k + 1) = Az(k) + Bs(k), \quad (13) \]
which has exactly the same form as the linear part of (11).

Next, a separate linear feedback law is considered for each of the systems (11) and (13), i.e.,
\begin{align}
u(k) &= K(x(k) - x^*) + u^*, \quad (14a) \\
s(k) &= Kz(k), \quad (14b)
\end{align}
for all \( k \in \mathbb{Z}_+ \).

Theorem IV.1 The set \( \mathbb{P} \) is \( \varepsilon \)-contractive for the equilibrium state \( x^* \) of the system (11) in closed-loop with the state-feedback (14a) and input constraints set \( \mathbb{U} \) if and only if the set \( \tilde{\mathbb{P}} \) is \( \varepsilon \)-contractive for the zero equilibrium of the system (13) in closed-loop with the state-feedback (14b) and input constraints set \( \tilde{\mathbb{U}} \).

Proof: The proof of this theorem follows straightforward from the equivalence of the two closed-loop systems. Let \( k \in \mathbb{Z}_+ \) be arbitrary. More precisely, by construction, for any \( x(k) \in \mathbb{P} \), \( z(k) = x(k) - x^* \) is in \( \tilde{\mathbb{P}} \) it holds that \( z(k + 1) \in \tilde{\mathbb{P}} = \varepsilon(\mathbb{P} + \{-x^*\}) \) and thus, \( x(k + 1) - x^* = z(k + 1) \in \varepsilon(\mathbb{P} + \{-x^*\}) \) with \( s(k) \in \tilde{\mathbb{U}} \) and \( u(k) \in \mathbb{U} \), respectively. The same reasoning applies when starting with any \( z(k) \in \tilde{\mathbb{P}} \), which yields \( z(k + 1) \in \varepsilon \mathbb{P} \) with \( s(k) \in \mathbb{U} \).

Corollary IV.3 Suppose that the set \( \tilde{\mathbb{P}} \) is \( \varepsilon \)-contractive with \( \varepsilon \in \mathbb{P}(0,1) \) for the zero equilibrium of the auxiliary system (17) in closed-loop with the state-feedback (14b) and input constraints set \( \tilde{\mathbb{U}} \). Then the set-induced function
\[ V(x) := \max_{j \in [1,m]} \left\{ \frac{[G]_{j*}(x - x^*)^T}{[u_1]_{j}}, -[G]_{j*}(x - x^*) \right\} \]
is a Lyapunov function for the system (15) in closed-loop with the state-feedback (14a) and input constraints set \( \mathbb{U} \).

The proof follows directly from the equivalence of the two closed-loop systems established in Theorem IV.2 and Corollary II.5.

Using the methodology described above, an auxiliary bilinear system can be constructed for the converter model (9). The auxiliary system has zero as equilibrium. Thus, a solution to the constrained stabilization problem can be obtained by solving Problem II.4. Based on the resulting control law and \( \varepsilon \)-contractive set, a suitable, stabilizing control law and \( \varepsilon \)-contractive set can be calculated for the original converter model (9) with a non-zero equilibrium.
<table>
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<td>$V_{load}$</td>
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<td>220µH</td>
<td>$T_s$</td>
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V. ILLUSTRATIVE CASE STUDY

In this section, we consider the buck-boost converter model (9)-(10) with the parameter values as summarized in Table I. The constraint sets on states and inputs are $X := \Psi(I_2, [\begin{bmatrix}22\end{bmatrix}], [\begin{bmatrix}9\end{bmatrix}])$ and $U := \Psi(I_2, [\begin{bmatrix}1\end{bmatrix}], [\begin{bmatrix}9\end{bmatrix}])$. Two different linear state-feedback controllers were constructed for a pre-specified, candidate $\varepsilon$-contractive set using Problem II.4. One of the controllers was synthesized using the average nonlinear model (9) of the system and the coordinate transformation proposed in Theorem IV.2. The other controller was obtained using a linearized model around the desired equilibrium state $x^*$ and the coordinate transformation proposed in Theorem IV.1, i.e.,

$$z_i(k+1) := A_i z_i(k) + B_i s_i(k), \quad (22)$$

$$A_i = \begin{bmatrix} 1.0000 & 0.1818 \\ -0.0182 & 0.9855 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0.4545 \\ -0.9098 \end{bmatrix},$$

where the index $l$ denotes the fact that the matrices $A_l, B_l$ and vectors $z_l, s_l$ correspond to the linearization of (9). Notice that matrices $A_l$ and $B_l$ in (22) are different from $A$ and $B$ in (9).

The first step in controller design is to define the coordinate transformations. Note that the $u^*$ for both the linear and bilinear system is not uniquely defined in Section IV. For the particular case of the considered buck-boost converter, the matrices $B_l$ and $B_i$ are invertible. Thus, a unique $u^*$ can be calculated for the chosen $x^*$. Next, the candidate $\varepsilon$-contractive set $\mathcal{P}$ for the zero equilibrium auxiliary bilinear system is imposed and Problem II.4 is solved. For this particular case study, the elements of the matrices $H$ and $D_j$ were restricted to take only positive values. The corresponding Problem II.4 has 65 optimization variables, 26 equality and 70 inequality constraints, and yields the solution $K = \begin{bmatrix} 0.0037 & -0.2965 \\ 0 & 0 \end{bmatrix}$ and $\varepsilon = 0.9875$. This solution was obtained for the candidate $\varepsilon$-contractive set $\mathcal{P} := \Psi(G, w_1, w_2)$ with

$$G = \begin{bmatrix} 0 & 0.8 \\ 1 & 1.16 \end{bmatrix}, \quad w_1 = \begin{bmatrix} 0.5 \\ 1.8 \\ 2.9 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 2.5 \\ 14 \\ 20 \end{bmatrix}.$$  \quad (23a)

The steady state input $u^* = \begin{bmatrix} 0.8157 \\ 0.4 \end{bmatrix}$ was computed using (16). The control input is computed at each time instant using (14a). The trajectories of closed-loop system starting from the vertices of the $\varepsilon$-contractive set $\mathcal{P}$ are shown in Fig. 2.

A similar technique, as described in Section IV, was applied to obtain a linear state-feedback controller for the linearized system model around $x^*$. The same candidate $\varepsilon$-contractive set was imposed as for the auxiliary bilinear system. Note that the uniqueness of the steady-state input for a specified $x^*$ requires the same $u^*$ for both linearized and bilinear models. Problem II.4 in this setup has 38 optimization variables, 14 equality and 43 inequality constraints, and yields the solution $K_l = \begin{bmatrix} -0.0001 & -0.0635 \\ -0.0125 & 0.1324 \end{bmatrix}$ and $\varepsilon_l = 0.9823$. Note that in this case the feasibility of the controller synthesis methodology does not guarantee the stability of the closed-loop system due to differences between linearized and bilinear models. For example, to illustrate the significance of these differences, consider the set $\mathcal{S} := \Psi(I_2, [\begin{bmatrix}0.5\end{bmatrix}], [\begin{bmatrix}0.3\end{bmatrix}])$. The average one-step prediction error of a linearized system model, in comparison with the bilinear model, turned out to be higher than 12% within $\mathcal{S}$. In Fig. 3, the yellow rectangle represents the set $\mathcal{S} \oplus \{x^*\}$.

The trajectories of the closed-loop system with the controller designed using a linear model, for all vertices of $\mathcal{P}$, are plotted in Fig. 3. These trajectories clearly violate the state constraints. The results shown in Fig. 2 and Fig. 3 clearly illustrate the advantages of a synthesis method that is applicable to the full bilinear averaged model.

As it can be seen from Fig. 2, the $\varepsilon$-contractive set $\mathcal{P}$ does not contain the origin, which is a regular starting point for the converter. One possible solution is to apply a constant input $u(k) = u_{ct}$ until $x(k) \in \mathcal{P}$, $k \in \mathbb{Z}_+$. After the system state reaches the $\varepsilon$-contractive set the affine state-feedback control law (14a) can be applied. In this specific case it is also sufficient to clamp the control input, which is illustrated by the simulation result in Fig. 4. In any case, the satisfaction
of the constraint $v_C \geq 0$ is not guaranteed for the startup from the origin and under a constant load current.

The proposed design method of an affine state-feedback control law was tested in real-time on a real-life hardware platform. A circuit with the same parameters as the ones employed in the previous simulations was used for the experiments. A hardware implementation of the controller was obtained and executed on the Virtex 5 FPGA device on board of the NI PXI-7852R multifunction DAQ from National Instruments. The evolution of output voltage of the converter at startup is shown in Fig. 5.

VI. CONCLUSIONS

This paper proposed a novel, set-theoretic method for constrained stabilization of DC/DC power converters. The developed method makes use of a recent result [10] on the stabilization of bilinear discrete-time systems with zero as equilibrium. The main contribution was to design a coordinate transformation that renders this result applicable to buck-boost DC/DC power converters, which typically have a non-zero equilibrium. The resulting synthesis has several advantages, which include low computational complexity, an a priori guarantee of stability and constraints satisfaction and applicability to the full bilinear averaged model of the converter.

Future research deals with further enlarging the region of attraction via alternative synthesis methods for constrained discrete-time bilinear systems and optimizing the speed of convergence.

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