

# Disturbance attenuation in time-delay systems – a case study on engine air-fuel ratio control

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*Abstract* — In this paper we analyze relative performance of several approaches to disturbance attenuation for systems with time delay including the conventional Proportional-Integral controller, the Smith Predictor, and the Model Reduction controller. The paper proposes a measure of disturbance attenuation capability and computes it analytically for each of the controllers considered. The results are applied to the air-fuel ratio regulation in automotive engines. To meet strict emission regulations, gasoline engines must operate at stoichiometric air-fuel ratio over most of its operating range. A major component towards accomplishing this goal is the closed loop fuel controller. The feedback uses an exhaust-gas oxygen sensor which introduces a long transport delay. This paper discusses the air-fuel ratio regulation problem, explores options in control design for disturbance attenuation in system with time-delay, and shows simulation and experimental, in-vehicle validations.

## I. INTRODUCTION

In practical applications, feedback control is used to stabilize unstable or marginally stable systems, achieve good tracking of reference signals, or attenuate effects of disturbances. These goals may not be completely aligned as fast reference trajectory tracking may not produce good disturbance attenuation and vice-versa. In particular, and this is relevant for the problem considered in this paper, the Smith Predictor delay compensation method is well known for achieving fast reference tracking, but not necessarily good disturbance attenuation (see, for example, [11]).

This paper considers the problem of disturbance attenuation for the air-fuel ratio regulation system in gasoline engines. The three way catalytic converters, employed to remove the three main regulated components (hydrocarbons, oxides of nitrogen, and carbon monoxide) from engine exhaust, achieve very high efficiency only in a very narrow range of air-fuel ratios around stoichiometry (about 14.6 for gasoline). Thanks to their oxygen storage capability, the catalytic converters can operate efficiently for a brief period of time away from stoichiometry. If the oxygen storage gets depleted or saturated, the efficiency drops significantly. Hence, it is very important to keep the air-fuel ratio excursions away from stoichiometry as brief and as shallow as possible. Details on operation of three way catalysts can be found in Section 2.8.2 of [5].

The air-fuel ratio regulation system has two components: feedforward and feedback. While the elaborate feedforward part does its best to estimate and predict how much fuel is needed in the cylinder for combustion, there are still many factors which prevent achieving stoichiometric in-cylinder air-fuel ratio. The remaining error has to be removed by the feedback controller. This feedback loop has a large time delay, which varies with operating conditions (see, for example, [7] and Section 4.2.2 of [5]), that limits the gains which can be used. For this reason, various forms of delay compensation, such as Smith Predictor [1, 10, 16], Internal Model Control [12], and a version of Model Predictive Controller [9], have been employed to tackle this control problem.

In this paper we are specifically interested in the disturbance attenuation aspect for controllers with delay compensation. We establish a measure of disturbance attenuation capability motivated by the observation that, approximately, the oxygen storage in the catalyst behaves as an integrator. For each controller class considered, we establish a generic formula for achievable value of this disturbance attenuation measure. The controllers considered are the conventional PI controller, the Smith Predictor, and the Model Reduction controller. The first two are standard approaches for this and similar types of problems. The Model Reduction controller (see [2, 8]) is considered because it allows simple calculation of controller gains using the “proxy” non-delay system. Its relationship with the “Watanabe-Ito” controller [15] and a version of finite spectrum assignment (FSA) approach described in [14] is briefly discussed in Section IV. Finally, we would like to mention that there is an optimal  $H_\infty$  solution to the disturbance attenuation problem [13], but it is demanding on the control designer as it requires solutions to (a) an algebraic Riccati equation, (b) a differential Riccati equation, and (c) a state transition matrix for a time varying system be computed, possibly repeatedly as one searches for an appropriate gain selection. We emphasize that, for the air-fuel ratio regulation problem, any control design will have to be performed repeatedly over a grid of engine speed-torque points as the system parameters, including the time delay, vary significantly over the operating range.

The three controllers considered were experimentally tested in a vehicle in which a rapid prototyping system was used to overwrite the fueling command computed by the standard engine control module. Even though the gain selection in

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each case is aggressive, subject to the capabilities of the underlying control architecture, the experimental testing confirmed the feasibility of their respective tuning as it withstood uncertainties in the system parameters unavoidable in practical applications. On the other hand, simulations did show increased sensitivity for the controllers with higher gains and wider bandwidths, and also more sensitivity for the Model Reduction controller with finite time integrals compared to the Smith Predictor.

## II. AIR-FUEL RATIO REGULATION AND PI CONTROLLER

To keep the air-fuel ratio (or fuel-air ratio) at stoichiometry, an elaborate feedback-feedforward controller is employed that has the following components:

1. Open loop fuel injection metered to match stoichiometric ratio for an estimated air mass entering the cylinders.
2. Wall wetting compensation that accounts for port fuel puddle accumulation and vaporization.
3. The inner loop control, tasked to minimize pre-catalyst fuel-air ratio deviations from the reference, is based on measurements from a wide range universal exhaust gas oxygen (UEGO) sensor.

Even though the measured fuel-air ratio at the sensor is a result of complex physical processes that involve engine air intake, fuel wall-wetting, combustion, gas mixing dynamics in the exhaust, transport delay, and sensor dynamics, for the purpose of designing feedback controls an adequate lumped parameter model of the system is typically chosen as the first order lag response with time delay [1, 7, 10]. Thus, the transfer function from the deviation of injected fuel-air ratio from stoichiometry, considered the control input  $u$ , to the UEGO sensor reading of fuel-air ratio deviation from stoichiometry, considered the output  $y$ , is

$$P(s) = \frac{ae^{-\tau_d s}}{s + a} \quad (2.1)$$

The time delay  $\tau_d$  and the pole  $a$  of the first order filter will vary with engine speed and the engine load (the air charge normalized by the theoretical maximum) [5, 7]. At a given engine speed and load, the delay and filter pole are fixed. In this paper, we consider operation only at one operating point: 700 RPM engine speed and relative load of 0.15 (neutral transmission at idle) in a vehicle with a large V8 engine. For this operating point, the identified time delay is  $\tau_d = 0.45$  seconds and the time constant of the first order lag is 0.59 seconds (that is,  $a = 1.69$  rad/s).

A natural first choice for feedback control is the PI controller with the transfer function

$$C(s) = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} \quad (2.2)$$

Figure 1 shows the model response (the black-dash trace) and experimental traces for 5 trials of normalized fuel-air ratio (that is, the equivalence ratio  $\phi$ ) response to a step

disturbance introduced by changing a setting for the slope parameter relating the fuel mass injected and injector pulse width. In the top plot, the injector pulse width is decreased by 20% at 3.05 seconds and the measured  $\phi$  indicates lean combustion until the closed loop control can return  $\phi$  to 1. In the lower plot, the transfer function is returned to its original setting and the system now responds rich until the disturbance is rejected. The PI gains for this set of experimental tests are selected as  $k_i = 1.36$  and  $k_p = 0.8$ , corresponding to  $k_i = 0.61/\tau_d$ ,  $k_p = k_i/a$ . This aggressive tuning is close to the upper end of the classical tuning range between those proposed by Bryant [3]  $k_i = 0.4/\tau_d$ ,  $k_p = k_i/a$  and Haalman [4]  $k_i = 0.66/\tau_d$ ,  $k_p = k_i/a$  (see also the discussion and simulations in [6]). The PI controller is implemented in discrete time with the sampling time of 0.03 sec. One can observe that the model (the black-dash traces) matches reasonably, but not perfectly, the experimental runs.

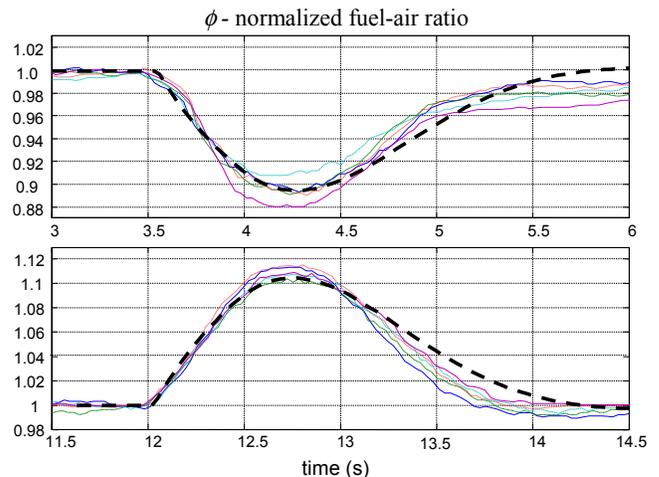


Fig. 1. Model (bold-dash curves) and repeated test responses to a 20% fuel-air ratio disturbance induced by changing injector slope.

Recall that, due to the catalyst oxygen storage capability, rich or lean air-fuel ratio excursions of short duration can be effectively absorbed by the catalyst. The simplest model of the catalyst oxygen storage is that of an oxygen bucket which has to be prevented from overflowing or depleting (see equation (2.193) in [5]). Hence, from the point of view of emission reduction, our control goal is to minimize the amount of oxygen added or removed by a fuel-air ratio disturbance, which correlates to the integral under the curve for the rich or lean traces in Figure 1.

To put a quantitative measure to the above deliberation, we consider the maximal value of the integral of the system output  $y(t)$ , caused by a step disturbance. As this integral is monotonically increasing (or almost monotonically increasing) for over-damped and slightly under-damped systems, the maximal value will occur at infinity. Hence, our performance objective is to minimize the cost function  $J$  defined by

$$J = \left| \lim_{t \rightarrow \infty} \int_0^t y(t) dt \right| \quad (2.3)$$

where  $y(t)$  is the response to the unit step disturbance at time 0. If we denote by  $G_{dy}(s)$  the transfer function from the disturbance input  $d$  to the output  $y$ , the optimization function  $J$  in the Laplace domain is

$$J = \left| \lim_{s \rightarrow 0} \frac{G_{dy}(s)}{s} \right| \quad (2.4)$$

In deriving (2.4) from (2.3) we have used a few standard properties of the Laplace transform  $F(s)$  of a time signal  $f(t)$  ( $L\{\cdot\}$  denotes the Laplace transform of a signal):

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s); \quad L\left\{ \int_0^t f(\theta) d\theta \right\} = \frac{1}{s} F(s), \text{ and}$$

$L\{d(t)\} = 1/s$  for  $d(t)$  the unit step function. To simplify notation, we'll assume that the limit in (2.4) is positive and subsequently suppress the absolute value in computing  $J$ .

It is obvious that the disturbance attenuation measure  $J$  may be a function of plant parameters, controller architecture, and controller parameters. To achieve a finite value of  $J$ , the controller has to have an integral action. Hence, we pick the controller structure as

$$C(s) = C_p(s) + \frac{k_i}{s}$$

where  $C_p(s)$  could denote just a P-gain of the PI controller or a more general frequency shaped controller. Either way, the value of the measure  $J$  is simply

$$J = \lim_{s \rightarrow 0} \frac{G_{dy}(s)}{s} = \lim_{s \rightarrow 0} \frac{P(s)}{s + [k_i + sC_p(s)]P(s)} = \frac{1}{k_i} \quad (2.5)$$

Note that the value is independent of the plant transfer function  $P(s)$ . In particular, this formula applies to the plant (2.1) and the PI controller (2.2). Hence, an increase in the integral gain produces a reciprocal reduction in the optimization function. Unfortunately, due to the delay, there is a limit to the integral gain that can be used before the system response deteriorates. For the classical PI controller tuning of Bryant and Haalman, the resulting  $J$  is between  $1.5\tau_d$  and  $2.5\tau_d$ . With the tuning of the PI gains for the tests shown in Figure 1, the value of  $J$ , computed from (2.5), is  $J = 1.64\tau_d$  or, with  $\tau_d = 0.45$ ,  $J = 0.74$ .

### III. DELAY COMPENSATION WITH THE SMITH PREDICTOR

To improve the closed loop performance, one could use a method to compensate for time delay. For stable systems, such as the one considered here, the standard choice is the Smith Predictor. Indeed, its application to the problem of engine air-fuel ratio regulation has already been reported in the literature [1, 10, 16]. Figure 2 shows a typical configuration of a Smith Predictor for the engine model given by (2.1). In this case, the standard PI controller is augmented with the Smith Predictor structure (blue blocks in Figure 2). The transfer function  $P_0(s)$  denotes the delay free part of the plant:  $P(s) = P_0(s) e^{-s\tau_d}$ .

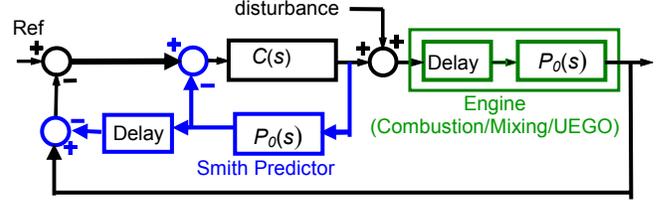


Fig. 2. The closed loop system with Smith Predictor

For the system in Figure 2, assuming no parametric uncertainty, the plant output  $y$  and the controller output  $u$  are given by

$$\begin{aligned} y &= P_0(s)e^{-\tau_d s}(u + d) \\ u &= C(s)[r - P_0(s)(1 - e^{-\tau_d s})u - y] \end{aligned} \quad (3.1)$$

By solving for  $u$ , we obtain the closed loop transfer functions, from the reference input  $r$  and the disturbance  $d$  to the output  $y$ , given by

$$\begin{aligned} y &= \frac{C(s)P_0(s)e^{-\tau_d s}}{1 + C(s)P_0(s)} r + \frac{[1 + C(s)P_0(s)(1 - e^{-\tau_d s})]P_0(s)e^{-\tau_d s}}{1 + C(s)P_0(s)} d \\ &= G_{ry}(s)r + G_{dy}(s)d \end{aligned} \quad (3.2)$$

Note that the denominator of the transfer functions does not depend on the delay. Hence, in contrast to the PI controller, the Smith Predictor gains are not limited by the delay itself, but rather by the uncertainty in the delay. As the gains are increased, the system becomes more sensitive to delay and time constant mismatches between the plant and its model used inside the controller. Nevertheless, if the model is known accurately, we could push the gains and the closed loop bandwidth much higher. It is well known, however, that the Smith Predictor's capability to attenuate disturbances may be limited [11]. Indeed, our measure of disturbance attenuation capability  $J$  reveals such a limit.

First, to reject a constant disturbance and produce finite value of the disturbance measure  $J$ , the controller has to have an integral action – hence,  $C(s) = C_p(s) + k_i/s$ . As before,  $C_p(s)$  may just be a simple proportional gain or a frequency shaping transfer function. The value of the measure  $J$  is independent of this choice:

$$\begin{aligned} J &= \lim_{s \rightarrow 0} \frac{G_{dy}(s)}{s} = \\ &= \lim_{s \rightarrow 0} \frac{[1 + (k_i + sC_p(s))P_0(s)\frac{1 - e^{-\tau_d s}}{s}]P_0(s)e^{-\tau_d s}}{s + (k_i + sC_p(s))P_0(s)} = \\ &= \frac{1}{k_i} + \tau_d P_0(0) \end{aligned} \quad (3.3)$$

Note that this formula for the disturbance attenuation measure with Smith Predictor applies for any (stable) plant  $P(s)$  and any stabilizing controller with integral action  $C(s)$ . It establishes a lower limit for the "integral under the curve" of the system response to a step disturbance input equal to  $\tau_d P_0(0)$  (if  $C_p(s)$  is stable,  $k_i$  and  $P_0(0)$  have the same sign).

If we consider the plant given by (2.1) and the  $C(s)$  in the Smith Predictor given by (2.2), we obtain the characteristic polynomial given by  $s^2 + a(1+k_p)s + ak_i$ . Now, one could use much higher gains and attain much wider bandwidth for the closed loop system than with the PI controller. Under the same operating conditions described in Section 2, we pick the Smith Predictor PI gains to be  $k_i = 6$  and  $k_p = 1.6$ , producing two closed loop poles at  $-2.2 \pm j2.3$  rad/s. Figure 3 shows the performance of the Smith Predictor in response to a 20% step disturbance in fuel-air ratio and the comparison to the response of the conventional PI controller. The traces in this and subsequent figures represent averages of five tests to remove some randomness in responses (see the traces in Figure 1 for illustration of the run-to-run variability). Small misalignment in the starting points is the result of traces being manually time-aligned.

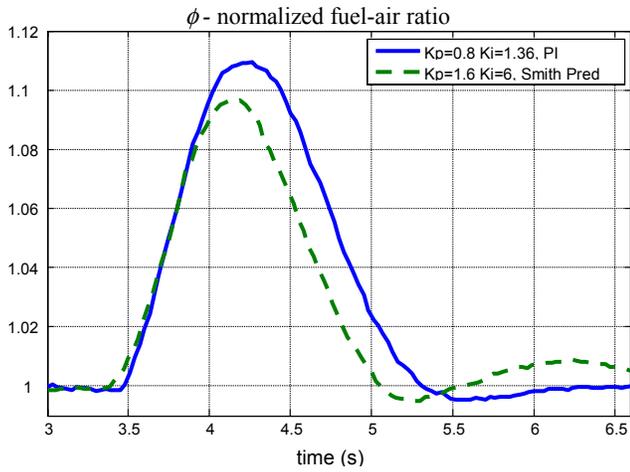


Fig. 3. Comparison of the Smith Predictor (green-dash) and the PI controllers (blue-solid) responses to a 20% FAR step disturbance.

With the value of  $P_0(0) = 1$ , from (3.3) the value of the disturbance attenuation measure  $J$  is

$$J = \frac{1}{k_i} + \tau_d \quad (3.4)$$

Hence, no matter how high PI gains are employed, the value of  $J$  is limited from below by  $\tau_d$  (if  $C(s)$  has no right half plane poles). This point is illustrated in Figure 4 which shows experimental vehicle traces with two additional tunings of the Smith Predictor controller: (a) high gain SP with  $k_i = 12.2$  and  $k_p = 4.4$  resulting in the poles at  $-4$  and  $-5.2$  rad/s and (b) very high gain SP with  $k_i = 27.4$  and  $k_p = 7$  resulting in the poles at  $-6.8 \pm j0.8$  rad/s. It is clear from Figure 4 that there is a limit to improving disturbance rejection capability of the Smith Predictor.

#### IV. MODEL REDUCTION CONTROLLER

To overcome Smith Predictor's limitations for disturbance attenuation, controllers with finite time integrals have been considered (see [11, 14]). In this paper we use the approach based on system augmentation with an integral of the output tracking error and the Model Reduction (MR) method (see,

for example, [2, 8]). Even though MR and the Watanabe-Ito method [15] will turn out to have the same disturbance rejection capability, they are not closely related. For MR, the delay compensation must account for the added critically stable mode (the added integrator) forcing the implementation to use a discrete approximation of the finite time integral. In return for an increase in complexity (and higher sensitivity observed in simulations), the MR method provides a straightforward selection of feedback gains and would have worked for multi-input, multi-delay systems, as well as unstable plants. Compared to the version of finite spectrum assignment proposed in [14], the MR method considered here employs a full state feedback design through the "proxy" system, rather than an output feedback design by solving a Diophantine equation for the system augmented by the integrator (as proposed in Section 2.4 of [14]). To use the MR design for output feedback, one can design a conventional finite dimensional observer (delays at the plant input don't affect the observer error equation).

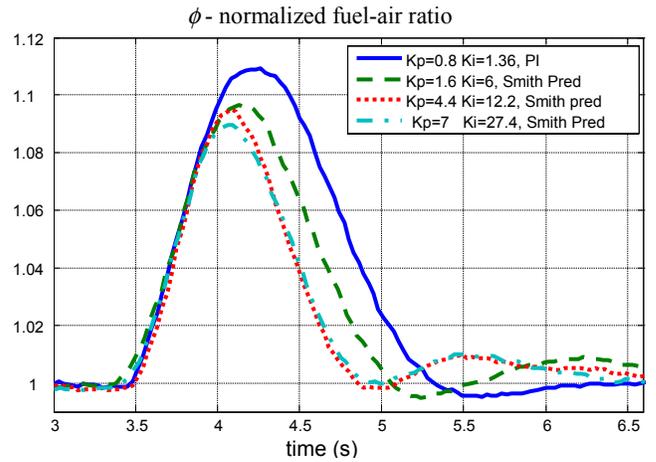


Fig. 4. Disturbance rejection with increasing gains: PI (blue-solid), moderate gain SP (green-dash), high gain SP (red-dot), very high gain SP (cyan-dash-dot).

The Model Reduction is the method of control design for systems with input delays that relies on a transformation into a "proxy" system without delays. Given the system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^l B_i u(t - \tau_i) \quad (4.1)$$

one can design the closed loop controller with the help of the non-delay system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^l e^{-A\tau_i} B_i u(t) \quad (4.2)$$

A feedback law

$$u(t) = -Kx(t) \quad (4.3)$$

which stabilizes (4.2), provides gain matrix  $K$  for control

$$u(t) = -Kx(t) - K \sum_{i=0}^l \int_0^{\tau_i} e^{-A\theta} B_i u(t + \theta - \tau_i) d\theta \quad (4.4)$$

that stabilizes (4.1). Moreover, the poles of the proxy closed loop system (4.2), (4.3) are the same as the closed loop system with delays (4.1), (4.4). Hence, the delay system is stabilized and has a finite spectrum.

Given a state space representation

$$\begin{aligned}\dot{z}(t) &= A_0 z(t) + b_0 [u(t - \tau_d) + d(t - \tau_d)] \\ y &= c_0 z\end{aligned}\quad (4.5)$$

of a single-input, single-output plant  $y = P_0(s)e^{-s\tau_d}(u + d)$ , we augment the state vector with  $z_0 = 1/s(y - r)$ . The dynamics for the augmented state  $x^T = [z_0, z^T]$  is

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B[u(t - \tau_d) + d(t - \tau_d)] \\ y &= Cx\end{aligned}\quad (4.6)$$

$$C = [0 \ c_0], \quad A = \begin{bmatrix} 0 & c_0 \\ 0 & A_0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b_0 \end{bmatrix}$$

The transfer function from the disturbance  $d$  to the output  $y$  for the closed loop system can be computed from the following set of equations obtained by taking the Laplace transforms of (4.4) and (4.6):

$$\begin{aligned}x &= (sI - A)^{-1} B e^{-s\tau_d} (u + d) \\ u &= -Kx - K(sI - A)^{-1} (e^{-A\tau_d} - e^{-s\tau_d} I) B u \\ y &= Cx\end{aligned}\quad (4.7)$$

This transfer functions is given by

$$G_{dy}(s) = P_0(s) e^{-s\tau_d} \frac{1 + K(sI - A)^{-1} (e^{-A\tau_d} - e^{-s\tau_d} I) B}{1 + K(sI - A)^{-1} e^{-A\tau_d} B} \quad (4.8)$$

where  $P_0(s) = c_0(sI - A_0)^{-1} b_0$ . Note that the resulting transfer function has the finite number of closed loop poles determined by the proxy system feedback design.

Using the special structure of the matrices in (4.6) we obtain

$$(sI - A)^{-1} = \frac{1}{s} \begin{bmatrix} 1 & c_0(sI - A_0)^{-1} \\ 0 & (sI - A)^{-1} \end{bmatrix}, \quad e^{-A\tau_d} = \begin{bmatrix} 1 & A_0^{-1}(e^{-A_0\tau_d} - I) \\ 0 & e^{-A_0\tau_d} \end{bmatrix}$$

Partitioning the feedback gain  $K = [k_i \ K_p]$ , with the scalar  $k_i$  being the feedback gain multiplying the integral state  $z_0$ , we can rewrite (4.8) as

$$\begin{aligned}G_{dy}(s) &= P_0(s) e^{-s\tau_d} \times \\ &\times \frac{s + \beta + (k_i c_0 + sK_p)(sI - A_0)^{-1} (e^{-A_0\tau_d} - e^{-s\tau_d} I) b_0}{s + \beta + (k_i c_0 + sK_p)(sI - A_0)^{-1} e^{-A_0\tau_d} b_0}\end{aligned}$$

where  $\beta = k_i c_0 A_0^{-1} (e^{-A_0\tau_d} - I) b_0$ .

Finally, we arrive at the generic formula for the disturbance attenuation measure  $J$  with the MR method for delay compensation:

$$\begin{aligned}J &= \lim_{s \rightarrow 0} \frac{G_{dy}(s)}{s} = \\ &= \frac{1}{k_i} - \left( \frac{K_p}{k_i} + c_0 A_0^{-1} \right) A_0^{-1} (e^{-A_0\tau_d} - I) b_0 + \tau_d P_0(0)\end{aligned}\quad (4.9)$$

Returning back to the model of the fuel-air ratio regulation we have  $A_0 = -a$ ,  $b_0 = a$ ,  $c_0 = 1$  and the disturbance attenuation measure is

$$J = \frac{1}{k_i} + \left( \frac{k_p a}{k_i} - 1 \right) \frac{e^{a\tau_d} - 1}{a} + \tau_d \quad (4.10)$$

The characteristic equation, given by

$$\chi(s) = s^2 + (a + k_i(1 - e^{a\tau_d}) + ak_p e^{a\tau_d})s + ak_i \quad (4.11)$$

produces two closed loop poles determined by  $k_i$  and  $k_p$ . Note that by selecting  $k_i = k_p a$  in (4.9) as was recommended by the conventional PI tuning rules, the disturbance rejection performance of the Smith Predictor is recovered. On the other hand, if  $k_i$  is increased relative to  $k_p$ , the system closed loop response will get oscillatory and eventually unstable as one can see by analyzing the roots of (4.11). Interestingly, the Model Reduction, the Watanabe-Ito method [15], and the FSA version of [14] achieve the same value of  $J$  for the same set (pair) of the closed loop poles.

To select feedback gains, we use the fact that the closed loop poles of (4.1), (4.4) are the same as those of the proxy system without delay (4.2), (4.3). For the latter we use the conventional LQR method and tune the matrix  $Q$  to obtain  $k_i = 12.2$  and  $k_p = 5.9$ , placing the closed loop poles at -4 and -5.2 rad/s. As mentioned above, the finite time integral cannot be implemented in the "Smith Predictor" form and has to be approximated as a discrete sum. We employed the standard trapezoidal rule with 0.03 sec. sampling time.

We have compared the performance of the MR controller with the conventional, aggressively tuned PI controller and the high-gain Smith Predictor from Section 3. The high-gain Smith Predictor has gains  $k_i = 12.2$  and  $k_p = 4.4$ , producing the same closed loop poles as the MR controller. Figure 5 compares the responses of the three controllers to the 20% increase in fuel producing a rich air-fuel ratio disturbance.

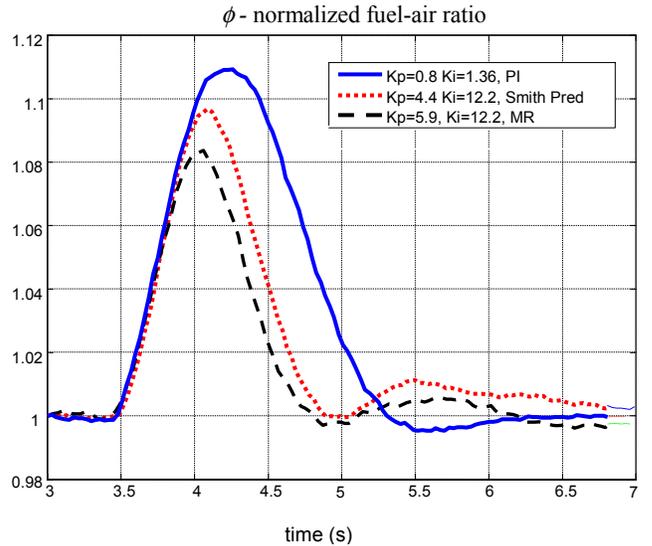


Fig. 5. Average of 5 trials for PI controller (blue-solid), high-gain Smith Predictor (red-dot) and MR controller (black-dash).

It is clear that the Model Reduction is able to improve the disturbance rejection further over the Smith Predictor even though the two systems have the same closed loop poles and the same  $k_i$  gain. Note that we were unable to do this by increasing the gains and the bandwidth of the Smith

Predictor (see Figure 4). The predicted value of the disturbance attenuation measure  $J$  is reduced from 0.74 for the PI controller, to 0.53 (equation (3.4)) for the high gain Smith Predictor, to 0.41 (equation (4.10)) for the Model Reference controller. Further reduction in  $J$  with the MR controller would have been possible by reducing the ratio of  $k_p$  over  $k_i$ , (see equation (4.10)), that is by decreasing the damping of the closed loop system. On the other hand, reduced damping leads to more oscillatory response and tend to reduce robustness to parametric uncertainties.

Finally, we show simulations that confirm some of the findings in this paper. The top plot shows the simulation responses to a 20% disturbance input for PI (blue-solid), SP (red-dot), MR (black-dash), and Watanabe-Ito (WI) controller (orange-dash-dot). The tuning of the first three is the same as for the experimental runs in Fig. 5, while the WI gains are selected to place the closed loop poles at -4 and -5.2 rad/s (the same as SP and MR). The bottom plot confirms the calculation of  $J$  in (2.5), (3.4), and (4.10) as the values at time  $T = 20$  sec. are equal to  $0.2J$  (0.2 being the magnitude of the disturbance input). Note that the WI value for  $J$  is the same as that for MR.

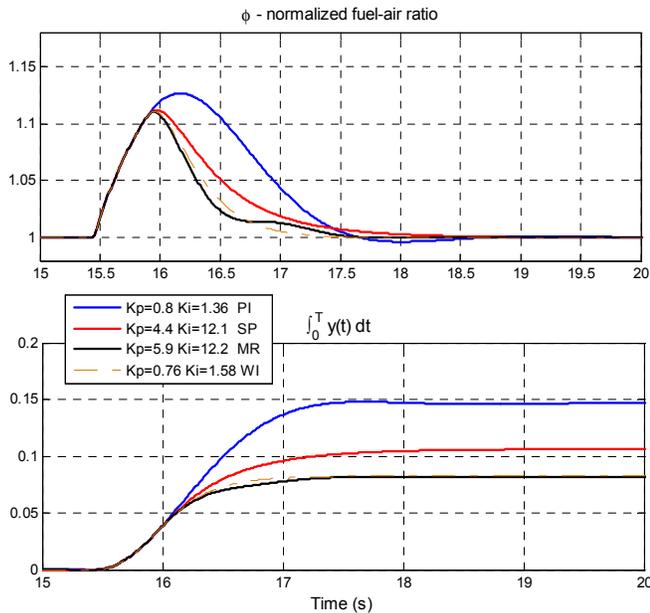


Fig. 6: Response to a 0.2 magnitude step disturbance (top plot) and its time integral (bottom plot) for PI, SP, and MR controllers plus the Watanabe-Ito controller (orange-dash dot).

## V. SUMMARY

In this paper we consider the problem of disturbance attenuation for the air-fuel ratio regulation system. First a measure of disturbance attenuation, relevant for emission reduction aspect for this problem, is proposed. Then, the three approaches – PI controller, Smith Predictor, and Model Reduction – are compared analytically and experimentally by implementing them in a vehicle. As one may expect, it has been observed in simulations (not

included in the paper) that as the gains and the bandwidth are increased, the sensitivity to parametric uncertainties also increases. The MR controllers with finite time integral appeared more sensitive than the Smith Predictor ones (with a similar bandwidth), but offer an opportunity to overcome SP limitation in disturbance attenuation as observed in vehicle test results.

## VI. REFERENCES

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