A Frequency-Tunable LPV Controller for Narrowband Active Noise and Vibration Control

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Abstract—A design method for a discrete-time gain-scheduling controller for the rejection of a harmonic disturbance with known but time-varying frequency for a single-input single-output linear plant is presented. The controller is obtained through the gain scheduling design method for linear parameter-varying (LPV) systems. This results in a controller where the frequency of the disturbance is the gain-scheduling variable of the controller and closed-loop stability is guaranteed for the whole range of frequencies specified in the design. The work is motivated by active noise and vibration control (ANC/AVC), and experimental real-time results obtained with an ANC headset and an AVC test bed are presented. The design method is very straightforward and the experimental results show that the controller is well suited for narrowband ANC and AVC.

I. INTRODUCTION

Active noise and vibration control (ANC/AVC) has attracted considerable interest from the control and signal processing community over the last decades (see, e.g., [1]). One specific application that motivates the work presented in this paper is the rejection of harmonic disturbances of time-varying, known frequencies. This control objective is applicable for environments where rotating machinery operates and the angular velocities of the machines vary, e.g., in automotive applications [2, 3] or aircrafts.

Good cancellation of harmonic (narrowband) or broadband disturbances can be achieved with adaptive feedforward control methods such as the Filtered-x LMS (FxLMS) algorithm [1]. The FxLMS algorithm often works well in practice, but may have critical issues such as convergence speed and tracking performance. Even for constant frequencies, it is difficult to predict the performance offline for such an adaptive feedforward controller, because the characteristics of the resulting controller are not known beforehand. Only approximate stability results for the FxLMS algorithm seem available to date [1, 4].

Another alternative is feedback control. For good disturbance rejection, the feedback controller has to include a model of the disturbance (this is the internal model principle [5]). This can be achieved through an observer-based controller, where the observer not only estimates the plant states but also the states of a disturbance model. The estimated states of the disturbance model can then be used to cancel the disturbance [2, 3, 6]. For harmonic disturbances, this is equivalent to using an adaptive (or rather, scheduled) version of the “spectral observer” [7]. Since the disturbance model is time-varying, also a time-varying observer is required. The time-varying observer gain can either be calculated on-line (e.g. using a Kalman filter) or a set of observer gains can be calculated offline for a set of fixed frequencies [6]. In the latter case, the current observer gain is calculated through interpolation [6] or by switching between different gains [2, 3]. In the first case, stability can be guaranteed but the computational effort is increased (due to the on-line calculation of the covariance matrix). In the second case the computational effort is lower (table lookup and interpolation operations) but stability is not guaranteed. Another alternative is to use observer-based feedback control but to schedule the state feedback gain instead of the observer [8-10].

The work presented here is motivated by [11, 12], where an LPV gain-scheduling approach is proposed for the rejection of a harmonic disturbance. In [11, 12], the controller design is carried out in continuous time and a polytopic LPV description is used. For practical real-time implementation, the controller has to be implemented in discrete time. Also, in active noise and vibration control, the plant model is often obtained through system identification. This usually gives a discrete-time plant model. It is therefore most natural to carry out the whole design in discrete time. If a continuous-time controller is computed, this controller would have to be discretized. Since the controller is time-varying, this discretization would have to be carried out at each sampling instant. Particularly in LPV gain scheduling control, an approximate discretization is proposed [13]. However, this leads to a distortion of the frequency scale that shifts the controller poles to other frequencies. For the rejection of harmonic disturbances, it is required that the frequencies of the controller poles match the disturbance frequencies exactly. Therefore, a frequency distortion cannot be tolerated for controllers that are designed to suppress harmonic disturbances. Discretization methods that maintain the frequencies of the poles such as step invariance (zero-order-hold) or the matched pole-zero method [14] are computationally too expensive (calculation of a matrix exponential, calculations of poles at each sampling instant). It is therefore not surprising that the continuous-time design methods [6, 9, 11, 12] are only tested in simulations (with a very simple two-mass system as a plant).
In this paper, the LPV gain-scheduling controller is calculated directly in discrete time and the parameter variations are described in linear fractional transformation (LFT) form. The resulting controller is evaluated experimentally on an ANC headset and an AVC test bed.

Advantages of the LPV approach are that the resulting controller structure is very simple, the computational load is not too high and that the stability of the closed-loop is guaranteed even for arbitrarily fast changes of the scheduling variable.

The remainder of this paper is organized as follows. In Sec. II, LPV gain scheduling control is briefly reviewed. The design procedure to achieve harmonic disturbance rejection is described in Sec. III. In Sec. IV, the experimental set-ups are described and experimental results are presented. Conclusions are given in Sec. V.

II. LPV CONTROL DESIGN

An LPV system in LFT form is shown in Fig. 1. It consists of a generalized plant \( G(z) \) that includes input and output weighting functions and a parametric uncertainty block \( \theta \). For this general system, a gain-scheduling controller can be calculated following the method presented in [15]. In this method, two sets of linear matrix inequalities (LMIs) are solved. The first set of LMIs determines the feasibility of the problem which means that a bound on the control system performance in the sense of the \( H_\infty \) norm can be satisfied. With the second set of LMIs, the controller matrices are calculated from the solution of the first set of LMIs.

![Fig. 1. General LPV system.](image)

A harmonic disturbance with frequency \( f \) can be modeled as the output \( y_d \) of the state space model

\[
\begin{align*}
x_{d,k+1} &= A_d(f)x_{d,k} + B_{d,w}w_{d,k}, \\
y_{d,k} &= C_{d,y}x_{d,k},
\end{align*}
\]

with

\[
A_d(f) = \begin{bmatrix} 0 & 1 \\ -1 & a(f) \end{bmatrix}, \quad B_{d,w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_{d,y} = \begin{bmatrix} 1 & 0 \end{bmatrix},
\]

and

\[ a(f) = 2\cos(2\pi f T). \]

The frequency varies between \( f_{\text{min}} \) and \( f_{\text{max}} \), therefore, \( a \) will vary between \( a_{\text{min}} \) and \( a_{\text{max}} \) and can be written as

\[ a = a_0 + a_1 \theta, \]

where \( a_0 \) and \( a_1 \) are real constants and \( \theta \in [-1, 1] \). This model can be expressed in LFT-LPV form as

\[
\begin{align*}
x_{d,k+1} &= A_{d,0}x_{d,k} + B_{d,\theta}w_{d,k} + B_{d,w}w_{d,k}, \\
q_{\theta,k} &= C_{d,\theta}x_{d,k}, \\
y_{d,k} &= C_{d,y}x_{d,k},
\end{align*}
\]

with

\[
A_{d,0} = \begin{bmatrix} 0 & 1 \\ -1 & a_0 \end{bmatrix}, \quad B_{d,\theta} = \begin{bmatrix} 0 \\ a_1 \end{bmatrix}, \quad C_{d,\theta} = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\]

In the examples considered below, the frequency varies between 80 Hz and 130 Hz and the sampling time is 1 ms. For this case, the matrices for the disturbance model are

\[
\begin{align*}
A_{d,0} &= \begin{bmatrix} 0 & 1 \\ -1 & 1.5607 \end{bmatrix}, \quad B_{d,\theta} = \begin{bmatrix} 0 \\ 0.2130 \end{bmatrix}, \\
B_{d,w} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_{d,\theta} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad C_{d,y} = \begin{bmatrix} 1 & 0 \end{bmatrix}.
\end{align*}
\]

It is assumed that the disturbance enters at the plant input. The state-space model of the plant is

\[
\begin{align*}
x_{p,k+1} &= A_p x_{p,k} + B_p u_{p,k} + B_p y_{d,k},
\end{align*}
\]
The plant and the disturbance model can then be combined as shown in Fig. 3, where $G_p(z)$ and $G_d(z)$ are the transfer functions of the plant and the time-invariant part of the disturbance model, respectively. The state space model of this combined system is

$$\dot{x}_{k+1} = Ax_k + Bu_{p,k} + B_p w_{d,k} + B_w w_{d,k},$$

$$y_k = Cx_k,$$

with

$$A = \begin{bmatrix} A_p & B_p C_{d,y} \\ 0 & A_{d,0} \end{bmatrix},$$

$$B_c = \begin{bmatrix} B_p \\ 0 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0 \\ B_{d,\theta} \end{bmatrix},$$

$$C = \begin{bmatrix} C_p & 0 \end{bmatrix}.$$


### IV. EXPERIMENTAL RESULTS

The controller obtained from the design procedure outlined in Sec. II and Sec. III is experimentally validated on a Sennheiser PXC 300 headset and on an AVC test bed.

#### A. Active Noise Control

The ANC headset has two microphones placed in the ear cups of the headset (Fig. 5). The aim of the ANC system is to cancel a harmonic disturbance generated by an external loudspeaker. An anti-aliasing filter is used for the output signal and a reconstruction filter for the control input. The control algorithm is implemented on a rapid prototyping unit (dSpace MicroAutoBox).

The transfer function between the output of the control unit and the input to the control unit is the plant $G_p(z)$ (usually called secondary path in the AVC/ANC literature [1]). To design the control algorithm, this transfer function is required. To obtain $G_p(z)$, the system is excited with a multisine test signal and the output is recorded. The transfer function can be estimated using a standard black-box technique (oe). All usual methods (arx, oe, n4sid) resulted in models that were suitable for control design. If a transfer function model is identified, this is converted to a state-space description for the controller design. The identified transfer functions for the left and the right sides of the headset were almost identical, therefore, the same control algorithm was implemented on both sides. Both control algorithms work independently from each other. The experimental results shown are for the right side.

The control algorithm is designed for a harmonic disturbance in the frequency range of 80 Hz to 130 Hz. A sampling frequency of 1 kHz was chosen such that the Nyquist frequency of 500 Hz is well above the highest disturbance frequency. The identified plant model is of 12th order and the controller is of 15th order.

Design and experimental results are shown in Figs. 6-10 for the ANC system. Figs. 6 and 7 show the amplitude frequency responses of the open-loop and the closed-loop disturbance transfer functions for fixed frequencies of 80 Hz and 120 Hz for the ANC system. The resulting closed loop transfer functions represent notch filters with a zero at the disturbance frequency. Due to Bode’s sensitivity integral, the disturbance attenuation at the disturbance frequency leads to some disturbance amplification for frequencies below and above the disturbance frequency (the “waterbed” or “spillover” effect [16]). Whether this is
tolerable in a practical application depends on the spectral content of the background noise. Real-time results for the rejection of constant disturbances of 80 Hz and 120 Hz for the ANC system are shown in Figs. 8 and 9. As expected from the frequency responses, excellent disturbance rejection is achieved.

Fig. 6. Open-loop and closed-loop amplitude frequency responses for the ANC system; the closed-loop transfer function is calculated for a fixed disturbance frequency of 80 Hz.

Fig. 7. Open-loop and closed-loop amplitude frequency responses for the ANC system; the closed-loop transfer function is calculated for a fixed disturbance frequency of 120 Hz.

Fig. 8. Pressure measured for a fixed disturbance frequency of 80 Hz; the control sequence is off/on/off.

In Fig. 10 the behavior for a disturbance with a time-varying frequency is demonstrated. The disturbance is a sine sweep with a linearly increasing frequency from 80 Hz to 120 Hz in 10 seconds. The control systems also performed well for a sweep of 5 s duration. For even faster sweeps, the system remained stable but did not achieve satisfactory disturbance attenuation.

B. Active Vibration Control

The AVC test bed is schematically shown in Fig. 11. Two shakers (inertia mass actuators) are attached to a steel cantilever beam. One shaker acts as the disturbance source and the other shaker is used to counteract this disturbance. An accelerometer is used to measure the output signal.

A multisine test signal is used to identify the transfer function of the system. A sampling frequency of 1 kHz was chosen. The identified system is of 12th order and the controller of 15th order.
Figs. 12 and 13 show the amplitude frequency responses of the open-loop and the closed-loop disturbance transfer functions for fixed frequencies of 90 Hz and 130 Hz for the AVC system. Real-time results for the rejection of constant disturbances of 90 Hz and 130 Hz are shown in Figs. 14 and 15.

In Fig. 16 the behavior for a disturbance with a time-varying frequency is demonstrated. The disturbance is a sine sweep with a linearly increasing frequency from 90 Hz to 130 Hz, respectively, in 10 seconds. The controller achieves an excellent disturbance rejection for constant and for varying disturbance frequencies.
V. CONCLUSIONS

A discrete-time LPV gain-scheduling controller is presented as an approach to cancel a harmonic disturbance with time-varying, but known (measured) frequency based on the LFT-LPV gain scheduling method of [15]. To the best of the authors’ knowledge, this design method has not been used for this control problem before.

The controller is designed by augmenting the plant with a weighting function that represents a harmonic disturbance and using the frequency as the gain scheduling variable. In this paper, only a single-frequency disturbance is considered. The method is used in [17] and [18] for the rejection of two and six frequencies, respectively.

The design in discrete-time avoids problems with controller implementation (discretization at each sampling instant or frequency distortion resulting from approximate discretization) and results in a controller that can be readily implemented. Experimental results demonstrate that excellent disturbance rejection is achieved even when the disturbance frequency changes fairly rapidly.

A major advantage of the algorithm seems to be that stability of the closed-loop system is guaranteed for all values of the scheduling variable (frequency) in the range specified in the design process. This is a favorable result compared to the usual adaptive filtering methods. Compared to approaches in which an observer-based controller is used and the observer gain is calculated online to guarantee stability [6], the computational effort of the LPV controller is much lower. The computational complexity of the LPV controller is about the same as for an observer-based controller where the observer gain is switched [2, 3] or interpolated [6], but these approaches do not guarantee stability.

The fact that the closed-loop system is stable would also allow for an application without measurements of the frequency. For example, an ANC headset is possible where the user to the dominant frequency that is present in a noisy environment.

Future work will focus on the rejection of multiple harmonics, alternative LPV controller structures and multi-input multi-output systems.

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REFERENCES