Neural Network Control of Quadrotor UAV Formations

Travis Dierks and S. Jagannathan

Abstract—In this paper, a novel framework for leader-follower formation control is developed for the control of multiple quadrotor unmanned aerial vehicles (UAVs) based on spherical coordinates. The control objective for the follower UAV is to track its leader at a desired separation, angle of incidence, and bearing by using an auxiliary velocity control. Then, a novel neural network (NN) control law for the dynamical system is introduced to learn the complete dynamics of the UAV including unmodeled dynamics like aerodynamic friction. Additionally, the interconnection dynamic errors between the leader and its followers are explicitly considered, and the stability of the entire formation is demonstrated using Lyapunov theory. Numerical results verify the theoretical conjectures.

Index Terms—Formation Control, Quadrotor UAV, Neural Networks, Lyapunov Stability

I. INTRODUCTION

In recent years, quadrotor helicopters have become a popular unmanned aerial vehicle (UAV) platform, and their control has been undertaken [1] by many researchers. However, a team of UAVs working together is often more effective than a single UAV in scenarios like surveillance, search and rescue, and perimeter security. Therefore, the formation control of UAVs [2-6] has been proposed.

The authors in [2] present a modified leader-follower framework with a model predictive nonlinear control algorithm without proof of convergence and stability. In [3], a kinematic-based formation control law is proposed while ignoring the individual dynamics and the formation dynamics of UAVs; proof of stability is not provided. The work [4] offers an algorithm for perimeter security using UAVs without considering the UAV and formation dynamics.

On the other hand in [5], cylindrical coordinates and contributions from wheeled mobile robot formation control [8] are considered for leader-follower based formation control scheme by assuming dynamics are known. In [6], experimental results are provided by using a dynamic model and assuming that measured position and velocity of the leader to its followers are communicated. In [7], a robust formation controller is proposed based on higher order sliding mode controllers in the presence of disturbances.

By contrast, in this work, a new leader-follower formation control framework for quadrotor UAVs based on spherical coordinates is introduced where the desired position of a follower UAV is specified using a desired separation, $s_d$, a desired angle of incidence, $\alpha_d$, and bearing, $\beta_d$. Then, a new control law is derived using neural networks (NN) to learn the complete dynamics of the UAV online, including unmodeled dynamics like aerodynamic friction and in the presence of bounded disturbances. Although a quadrotor UAV is underactuated, a novel virtual NN control input scheme is proposed which allows all six degrees of freedom of the UAV to be controlled using only four control inputs.

II. BACKGROUND

A. Quadrotor UAV Dynamics

Consider a quadrotor UAV with six DOF defined in the inertial coordinate frame, $E^b$, as $[x, y, z, \phi, \theta, \psi]^T \in \mathbb{R}^6$ where $\rho = [x, y, z]^T \in \mathbb{R}^3$ are the position coordinates of the UAV and $\Theta = [\phi, \theta, \psi]^T \in \mathbb{R}^3$ describe its orientation referred to as roll, pitch, and yaw, respectively. The translational and angular velocities are expressed in the body fixed frame attached to the center of mass of the UAV, $E^b$, and the dynamics of the UAV in the body fixed frame can be written as [1]

$$
\dot{v} = \tilde{S}(\omega) v + \left[ N_1(v) + G(R) \right] + u + \tau_d,
$$

where $u = \begin{bmatrix} 0 & 0 & u_z \end{bmatrix}^T \in \mathbb{R}^3$, $M = \text{diag}(m l_1, J) \in \mathbb{R}^{3\times 3}$ represents the positive definite inertia matrix, $v(t) = [v_{x,b}, v_{y,b}, v_{z,b}]^T \in \mathbb{R}^3$ represents the translational velocity, $\omega(t) = [\omega_{x,b}, \omega_{y,b}, \omega_{z,b}]^T \in \mathbb{R}^3$ represents the angular velocity, $N_i(\bullet) \in \mathbb{R}^{3\times 1}, i = 1, 2, 3$, are the nonlinear aerodynamic effects, $\tilde{S}(\omega) = \text{diag}(-m S(\omega), S(I\omega)) \in \mathbb{R}^{3\times 6}$, $u_z \in \mathbb{R}^1$ provides the thrust along the z-direction, $u_z \in \mathbb{R}^1$ provides the rotational torques, $\tau_d = [\tau_{d_1}, \tau_{d_2}]^T \in \mathbb{R}^6$ and $\tau_d \in \mathbb{R}^1$, $i = 1, 2$ represents unknown, but bounded disturbances such that $\parallel \tau_d \parallel < \tau_M$ for all time $t$, with $\tau_M$ being an unknown positive constant, $I_{nxn} \in \mathbb{R}^{nxn}$ is an $nxn$ identity matrix, and $0_{nx1} \in \mathbb{R}^{nx1}$ represents an $n \times 1$ matrix of all zeros. Furthermore, $G(R) \in \mathbb{R}^{3\times 3}$ is the general form of a skew-symmetric matrix defined in [1].

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The matrix \( R(\Theta) \in \mathbb{R}^{3 \times 3} \) is the translational rotation matrix which is used to relate a vector in the body fixed frame to the inertial coordinate frame defined as [1]

\[
R(\Theta) = R = \begin{bmatrix}
c_\psi & -s_\psi & 0 \\
s_\psi & c_\psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(2)

where the abbreviations \( s(\bullet) \) and \( c(\bullet) \) have been used for \( \sin(\bullet) \) and \( \cos(\bullet) \), respectively. It is important to note that \( \|R\| = R_{max} \) for a known constant \( R_{max} \), \( R^{-1} = R^T \), \( \hat{R} = RS(\omega) \) and \( R^T = -S(\omega)R^T \). It is also necessary to define a rotational transformation matrix from the fixed body to the inertial coordinate frame as in [1]

\[
T(\Theta) = T = \begin{bmatrix}
1 & s_\theta & c_\theta \\
0 & c_\theta & -s_\theta \\
0 & s_\theta/c_\theta & c_\theta/c_\theta
\end{bmatrix}, \quad T^{-1} = \begin{bmatrix}
1 & 0 & -s_\phi \\
0 & c_\phi & s_\phi \\
0 & -s_\phi/c_\phi & c_\phi/c_\phi
\end{bmatrix}
\]  

(3)

where the abbreviation \( t(\bullet) \) has been used for \( \tan(\bullet) \). The transformation matrix \( T \) is bounded as long as \(- (\pi/2) < \phi < (\pi/2), \quad - (\pi/2) < \theta < (\pi/2) \) and \(- \pi \leq \psi \leq \pi \). These regions will be referred to as the stable operation regions of the UAV, and under these flight conditions, it is observed that \( \|T\| < T_{max} \) for a known constant \( T_{max} \).

Finally, the kinematics of the UAV can be written as

\[
\dot{\rho} = Rv, \quad \Theta = T\omega
\]  

(4)

B. Neural Networks

In this work, two-layer NNs are considered consisting of one layer of randomly assigned constant weights \( V_\gamma \in \mathbb{R}^{ac} \) in the first layer and one layer of tunable weights \( W_\delta \in \mathbb{R}^{Lb} \) in the second with \( a \) inputs, \( b \) outputs, and \( L \) hidden neurons. The sigmoid activation function is considered here. Furthermore, on any compact subset of \( \mathbb{R}^{n} \), the target NN weights are bounded by a known positive value \( W_{min} \) such that \( \|W_\delta\| \leq W_{min} \) [9]. For complete details of the NN and its properties, see [9].

C. Three Dimensional UAV Formation Framework

Throughout the development, the follower UAVs will be denoted with a subscript \( j' \) while the formation leader will be denoted by the subscript \( j \). To begin, an alternate reference frame is defined by rotating the inertial coordinate frame about the z-axis by the yaw angle, \( \psi_j \), and denoted by \( E_{j'} \). In order to relate a vector in \( E_{j'} \) to \( E_j \), the transformation matrix is given by

\[
R_{ij} = \begin{bmatrix}
\cos(\psi_j) & \sin(\psi_j) & 0 \\
-\sin(\psi_j) & \cos(\psi_j) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(5)

where \( R_{ij}^{-1} = R_{ij} \). The objective of the proposed leader-follower formation control is for the follower UAV to maintain a desired separation, \( s_{id} \in \mathbb{R} \), at a desired angle of incidence, \( \alpha_{id} \in E_{j'} \), and bearing, \( \beta_{id} \in E_{j'} \), with respect to its leader. The incidence angle is measured from the \( x_{aj}-y_{aj} \) plane of follower \( j \) while the bearing angle is measured from the positive \( x_{aj} \)-axis as shown in Fig. 1. It is important to observe that each quantity is defined relative to the follower \( j \) instead of the leader \( j \) [5],[8]. To specify a unique configuration of follower \( j \) with respect to its leader, the desired yaw of follower \( j \) is selected to be the yaw angle of leader \( i \), \( \psi_i \in E^{a} \) as in [2]. Then, the measured separation between follower \( j \) and leader \( i \) is written as

\[
\rho_j - \rho_i = R_{ij}^{-1} s_{id} \Xi_j
\]  

(6)

where

\[
\Xi_j = [\cos(\alpha_{id}) \cos(\beta_{id}) \cos(\alpha_{id}) \sin(\beta_{id}) \sin(\alpha_{id})]^T
\]  

(7)

Thus, to solve the leader-follower formation control problem in the proposed framework, a control velocity must be derived to ensure

\[
\begin{align*}
\lim_{t \to \infty}(s_{id} - s_{id}) &= 0, \\
\lim_{t \to \infty}(\beta_{id} - \beta_{id}) &= 0, \\
\lim_{t \to \infty}(\alpha_{id} - \alpha_{id}) &= 0, \\
\lim_{t \to \infty}(\psi_{id} - \psi_{id}) &= 0
\end{align*}
\]  

(8)

Throughout the development, the desired separation, angle of incidence and bearing \( s_{id}, \alpha_{id} \) and \( \beta_{id} \), respectively, will be taken as constants, while it is assumed that each UAV has knowledge of its own constant total mass, \( m_\bullet \), where \( \bullet \) is \( i \) for the leader and \( j \) for the follower. Additionally, it will be assumed that reliable communication between the leader and its followers is available [4],[6], and the leader communicates its measured orientation and angular rate vector, \( \Theta \) and \( \omega_j \), respectively, and its desired states, \( \psi_{id}, \psi_{id}, \psi_{id}, \psi_{id}, \psi_{id}, \psi_{id} \). Future work will relax this assumption.

It is worth noting that communicating the desired states as opposed to the measured states does not necessarily reduce the amount of communication overhead. The benefit of considering the desired states of the leader in the design of the follower UAVs’ control laws becomes significant when compensating for the formation dynamics which become
incorporated in the follower UAVs dynamic controller design. Further, considering the desired states reduces the reliance on noisy sensor measurements.

III. LEADER-FOLLOWER FORMATION TRACKING CONTROL

A. FOLLOWER UAV CONTROL LAW

Given a leader \( i \) subject to the dynamics and kinematics (1), and (4), respectively, define a reference trajectory at a desired separation \( s_{jad} \), at a desired angle of incidence, \( \alpha_{jad} \), and bearing, \( \beta_{jad} \), for follower \( j \) given by

\[
\rho_{jad} = \rho_i - s_{jad} R_{adj}^T \Xi_{jad} \tag{9}
\]

where \( R_{adj} \) is defined as in (5) but written in terms of \( \psi_{jad} \), and \( \Xi_{jad} \) is defined as in (7) but written in terms of the desired angle of incidence and bearing, \( \alpha_{jad}, \beta_{jad} \), respectively. Next, using (6) and (9), define the position tracking error as

\[
e_{jd} = \rho_{i} - \rho_{j} - s_{jad} R_{adj}^T \Xi_{jad} \in \mathbb{E}^a \tag{10}
\]

which can be measured using local sensor information. To form the position tracking error dynamics, it is convenient to rewrite (10) as

\[
\dot{e}_{jd} = R_{ij} \nu_j - R_{ij} \nu_j - \dot{R}_{adj}^T s_{jad} \Xi_{jad} \quad \text{revealing}
\]

\[
\dot{e}_{jd} = R_{ij} v_j - R_{ij} v_j - \dot{R}_{adj}^T s_{jad} \Xi_{jad} \tag{11}
\]

Next, select the desired translational velocity of follower \( j \)

\[
v_{jd} = [v_{jd}, v_{jd}, v_{jd}]^T \in \mathbb{E}^e, \quad \text{to stabilize (11) is written as}
\]

\[
v_{jd} = R_{ij}^T (R_{ij} v_i - s_{jad} R_{adj}^T \Xi_{jad} + K_{jd} \nu_{jd}) \tag{12}
\]

where \( K_{jd} = \text{diag} \{k_{jx}, k_{jy}, k_{jz} \} \in \mathbb{R}^{3 \times 3} \) is a diagonal positive definite design matrix all with positive design constants. Next, the translational velocity tracking error dynamics for follower \( j \) and leader \( i \) is defined as

\[
e_{jd} = [e_{jd}, e_{jd}, e_{jd}]^T = v_{jd} - v_{jd}
\]

and \( e_{jd} = v_{jd} - v_{jd} \), respectively. Applying (12) to (11) while observing \( v_{jd} = v_{jd} - e_{jd} \), and similarly \( v_{jd} = v_{jd} - e_{jd} \), reveals the closed loop position error dynamics to be rewritten as

\[
\dot{e}_{jd} = -K_{jd} e_{jd} + R_{ij} e_{jd} - R_{ij} e_{jd} \quad \text{and finally}
\]

\[
\dot{e}_{jd} = -K_{jd} e_{jd} + R_{ij} e_{jd} - R_{ij} e_{jd}
\]

Next, the translational velocity tracking error dynamics are developed. Differentiating (13), observing

\[
\dot{v}_{jd} = -S(\omega_j) \nu_j + R_{jd}^T (R_{jd} S(\omega_j) \nu_j + R_{jd} \dot{v}_i - \dot{R}_{adj}^T s_{jad} \Xi_{jad}),
\]

and substituting the translational velocity dynamics in (1) allows the velocity tracking error dynamics to be written as

\[
\dot{e}_{jd} = \dot{v}_{jd} - \dot{v}_{jd} = -N_{jd}(\nu_j) / m_j - S(\omega_j) e_{jd} - G(R_j) / m_j \tag{15}
\]

Adding and subtracting \( R_{jd}^T (K_{jd} (R_{jd} v_i + R_{jd} v_j)) \), and substituting the translational velocity dynamics in (1) allows the velocity tracking error dynamics to be written as

\[
\dot{e}_{jd} = \dot{v}_{jd} - \dot{v}_{jd} = -N_{jd}(\nu_j) / m_j - S(\omega_j) e_{jd} - G(R_j) / m_j
\]

Remark 1: Examining the velocity tracking error dynamics (15), it is observed that the derivative of the leader’s control velocity, \( \dot{v}_{lad} \), is required as a result of using \( \dot{v}_{lad} \) in (12). If the measured velocity of the leader, \( v_i \), had been used instead of \( \dot{v}_{lad} \) in (12), the tracking error dynamics (15) would be dependent on \( \dot{v}_{lad} \) which are considered to be unknown by the follower \( j \) in this work. In the following development, a NN is introduced to learn the unknown quantities of (15); however, to effectively approximate the leader’s dynamics, \( \dot{v}_{lad} \), terms like the leader’s control thrust and rotational torques would be required to be communicated to each follower in addition to the leader’s measured linear and angular velocities so that the terms could be included in the NN input of the follower.

Moving on, the velocity \( v_{jbd} \) is directly controllable with the thrust input. However, in order to control the translational velocities \( v_{jbd} \) and \( v_{jbd} \), the pitch and roll must be controlled, respectively, thus redirecting the thrust. Thus, we now seek to find expressions for the desired pitch, \( \theta_{jd} \), and roll, \( \phi_{jd} \). Moreover, it is desirable to specify the maximum desired pitch and roll angles to be tracked by the follower UAV.

To accomplish these design objectives, we first define the scaled desired orientation vector, \( \bar{\Omega}_{jd} = [\bar{\Theta}_{jd} \bar{\Phi}_{jd}]^T \), where \( \bar{\Theta}_{jd} = \pi \theta_{jd} / (2 \theta_{max}) \), \( \bar{\Phi}_{jd} = \pi \phi_{jd} / (2 \phi_{max}) \), where \( \theta_{max} \in (0, \pi/2) \) and \( \phi_{max} \in (0, \pi/2) \) are design constants used to specify the maximum desired roll and pitch, respectively. Next, we rewrite translational rotation matrix (2) in terms of \( \bar{\Omega}_{jd} \), and define \( R_{jd} = R_{ij} \bar{\Omega}_{jd} \).

Then, add and subtract \( G(R_{jd}) / m_j \) and \( R_{jd} \Lambda_j \) with

\[
\Lambda_j = R_{jd} v_i - R_{jd}^T s_{jad} \Xi_{jad} + K_{jd} R_{jd} e_{jd} - K_{jd} e_{jd}
\]

and utilizing (15) to yield

\[
\dot{e}_{jd} = -S(\omega_j) e_{jd} - G(R_j) / m_j + R_{jd}^T (\Lambda_j + f_{\theta_{jd}}(x_{\theta_{jd}}))
\]

\[
- u_j E_{jd} / m_j - K_{jd} R_{jd} e_{jd} - R_{jd} e_{jd}
\]

\[
+ R_{jd}^T (K_{jd} (R_{jd} \nu_j - R_{jd} \nu_j - \dot{R}_{adj}^T s_{jad} \Xi_{jad})) \tag{16}
\]

where \( f_{\theta_{jd}}(x_{\theta_{jd}}) = R_{jd} G(R_{jd}) / m_j - (G(R_j) / m_j + R_{jd}^T (\Lambda_j + f_{\theta_{jd}}(x_{\theta_{jd}}))) \)

\[
+ R_{jd} E_{jd} (1 - K_{jd} e_{jd})
\]

\[
+ R_{jd}^T K_{jd} (1 - K_{jd} e_{jd}) E_{jd}
\]

\[
+ R_{jd}^T K_{jd} R_{jd} e_{jd} - N_{jd}(\nu_j) / m_j + R_{jd}^T R_{jd} S(\omega_j) \nu_j \tag{17}
\]

is an unknown function which can be rewritten as

\[
f_{\theta_{jd}}(x_{\theta_{jd}}) = f_{\theta_{jd}}(x_{\theta_{jd}}) = f_{\theta_{jd}}(x_{\theta_{jd}}) = f_{\theta_{jd}}(x_{\theta_{jd}})
\]

\[
\in \mathbb{R}^3
\]

In the forthcoming development, the approximation properties of NN will be utilized to estimate the unknown function \( f_{\theta_{jd}}(x_{\theta_{jd}}) \) by bounded ideal weights \( W_{T_{\theta_{jd}}}^T, V_{T_{\theta_{jd}}}^T \) such that \( \|W_{T_{\theta_{jd}}}\|_p \leq W_{M_{\theta_{jd}}} \) for an unknown constant \( W_{M_{\theta_{jd}}} \), and written as

\[
f_{\theta_{jd}}(x_{\theta_{jd}}) = W_{T_{\theta_{jd}}}^T \sigma(V_{T_{\theta_{jd}}} x_{\theta_{jd}}) + \nu_{\theta_{jd}} \]

where \( \nu_{\theta_{jd}} \leq \epsilon_{\theta_{jd}} \) is the bounded NN approximation error where \( \epsilon_{\theta_{jd}} \) is a known constant. The NN estimate of \( f_{\theta_{jd}}(x_{\theta_{jd}}) \) is written as

\[
\hat{f}_{\theta_{jd}} = \hat{W}_{\theta_{jd}}^T \sigma(\hat{V}_{\theta_{jd}} x_{\theta_{jd}}) = \hat{W}_{\theta_{jd}}^T \sigma(\hat{V}_{\theta_{jd}} x_{\theta_{jd}}) = \hat{W}_{\theta_{jd}}^T \sigma(\hat{V}_{\theta_{jd}} x_{\theta_{jd}})
\]

2992
where $\hat{W}_{j1}^T$ is the NN estimate of $W_{j1}^T$, $\hat{W}_{j1}^T$, $i = 1, 2, 3$ is the $i^{th}$ row of $\hat{W}_{j1}^T$, and $x_{j1}$ is the NN input

$$x_{j1} = [1 \Theta^T \Theta^T \alpha^T \alpha^T v_{j1}^T v_{j1}^T \psi_{j1}^T \psi_{j1}^T \phi_{1j} \phi_{1j} \phi_{2j} \phi_{2j}^T \phi_{3j} \phi_{3j}^T]^T.$$  

Next, the virtual control inputs $\theta_{jd}$ and $\phi_{jd}$ are identified to control to control the translational velocities $v_{j1b}$ and $v_{j1b}$, respectively. The key step in the development is identifying the desired closed loop velocity tracking error dynamics. For convenience, the desired translational velocity closed loop system is selected as

$$\dot{v}_r = -S(\omega_j) \varepsilon_{jv} - K_{jv} \varepsilon_{jv} - \tau_{jd} - K_{jv} R_{e\omega} \varepsilon_{jv} \tag{18}$$

where $K_{jv} = \text{diag} \{k_{j11} \cos(\bar{\theta}_{jd}), k_{j22} \cos(\phi_{2j}), k_{j33}\}$ is a diagonal positive definite design matrix with each $k_{ii} > 0$, $i=1, 2, 3$, and $\tau_{jd} = \varepsilon_{jv} / m_j$. In the following development, it will be shown that $\bar{\theta}_{jd} \in (-\pi / 2, \pi / 2)$, $\bar{\phi}_{jd} \in (-\pi / 2, \pi / 2)$; therefore, it is clear that $K_{jv} > 0$. Then, equating (16) and (18) while considering only the first two velocity error states reveals

$$
\begin{align*}
\begin{bmatrix}
-c_{\phi_{jd}} s_{\phi_{jd}} & c_{\phi_{jd}} s_{\phi_{jd}} \\
-c_{\phi_{jd}} s_{\phi_{jd}} & c_{\phi_{jd}} s_{\phi_{jd}} \\
-s_{\phi_{jd}} & -s_{\phi_{jd}} \\
\end{bmatrix}
\begin{bmatrix}
\hat{\omega}_{jd} \\
\hat{\phi}_{jd} \\
\end{bmatrix}
=
\begin{bmatrix}
-A_{j1} \hat{f}_{j1} \hat{f}_{j1} + A_{j2} \hat{f}_{j1} + A_{j3} \hat{f}_{j1} \\
A_{j1} \hat{f}_{j1} + A_{j2} \hat{f}_{j1} + A_{j3} \hat{f}_{j1} \\
\end{bmatrix}
\end{align*}
$$

where $A_{j} = [A_{j1} \ A_{j2} \ A_{j3}]^T$. Then, applying basic math operations, the first line of (19) can be rewritten as

$$c_{\phi_{jd}}(s_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) + s_{\phi_{jd}}(A_{j2} + \hat{f}_{j1}) + k_{jv} e_{jv}) = -s_{\phi_{jd}}(A_{j3} + \hat{f}_{j1} - g). \tag{20}$$

Similarly, the second line of (19) can be rewritten as

$$c_{\phi_{jd}}(s_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) + s_{\phi_{jd}}(A_{j2} + \hat{f}_{j1}) + k_{jv} e_{jv}) = -s_{\phi_{jd}}(A_{j3} + \hat{f}_{j1} - g). \tag{21}$$

Next, (20) is solved for the desired pitch $\theta_{jd}$ while (21) can be solved for the desired roll $\phi_{jd}$. Using the NN estimates, $\hat{f}_{j1}$, the desired pitch $\theta_{jd}$ can be written as

$$\theta_{jd} = (2\theta_{\max} / \pi) \arctan(N_{\theta_{jd}} / D_{\theta_{jd}}) \tag{22}$$

where

$$N_{\theta_{jd}} = c_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) + s_{\phi_{jd}}(A_{j2} + \hat{f}_{j1}) + k_{jv} e_{jv}$$

and

$$D_{\theta_{jd}} = A_{j3} + \hat{f}_{j1} - g. \tag{23}$$

Next, (20) is solved for the desired roll $\phi_{jd}$ while (21) can be solved for the desired roll $\phi_{jd}$. Using the NN estimates, $\hat{f}_{j1}$, the desired roll $\phi_{jd}$ can be written as

$$\phi_{jd} = (2\phi_{\max} / \pi) \arctan(N_{\phi_{jd}} / D_{\phi_{jd}}) \tag{24}$$

where

$$N_{\phi_{jd}} = c_{\phi_{jd}}(A_{j2} + \hat{f}_{j1}) - s_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) + k_{jv} e_{jv}$$

and

$$D_{\phi_{jd}} = c_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) - s_{\phi_{jd}}(A_{j1} + \hat{f}_{j1}) + k_{jv} e_{jv}.$$  

Remark 2: The expressions for the desired pitch and roll in (22) and (23) will always produce desired values in the stable operation regions of the UAV since $\tan(\theta)$ approaches $\pm \pi / 2$ as its argument increases. Thus, introducing the scaling factors in $\bar{\theta}_{jd}$ and $\bar{\phi}_{jd}$ results in $\theta_{jd} \in (-\theta_{\max}, \theta_{\max})$ and $\phi_{jd} \in (-\phi_{\max}, \phi_{\max})$, and the aggressiveness of the UAVs maneuvers can be managed. Further, if the unscaled desired orientation vector were used in the development of (16), the maximum desired pitch and roll would still remain within the stable operating regions.

Now that the desired orientation has been found, next define the attitude tracking error as

$$e_{\theta_{jd}} = \Theta_{\hat{\theta}_{jd}} - T_{\omega_{jd}}, \tag{24}$$

where dynamics are found using (4) to be

$$\begin{align*}
\dot{\omega}_{jd} &= T_{j}^{-1}(\hat{\Theta}_{jd} + K_{j\omega} e_{\theta_{jd}}) \tag{25} \\
\dot{\Theta}_{jd} &= T_{j}^{-1}(\hat{\Theta}_{jd} + K_{j\omega} e_{\theta_{jd}})
\end{align*}$$

where $K_{j\omega} = \text{diag}(k_{j\omega1}, k_{j\omega2}, k_{j\omega3}) \in \mathbb{R}^{3 \times 3}$ is a diagonal matrix of positive design constants. Define the angular velocity tracking error as

$$\varepsilon_{\omega_{jd}} = \omega_{jd} - \omega_{j}, \tag{26}$$

and observing $\omega_{j} = \omega_{jd} - \varepsilon_{\omega_{jd}}$, the closed loop orientation tracking error system can be written as

$$\dot{\varepsilon}_{\omega_{jd}} = -K_{j\omega} e_{\theta_{jd}} + T_{j} e_{\omega_{jd}}. \tag{27}$$

Examining (25), calculation of the desired angular velocity requires knowledge of $\Theta_{\hat{\theta}_{jd}}$; however, $\Theta_{\hat{\theta}_{jd}}$ is not known in view of the fact $\hat{f}_{j1}$ and $\hat{f}_{j1}$ are not available. Further, development of $u_{j2}$ in the following section will reveal $\phi_{jd}$ is required which in turn implies $\hat{f}_{j1}$ and $\hat{f}_{j1}$ must be known. Since these requirements are not practical, the universal approximation property of NN is invoked to estimate $\omega_{jd}$ and $\hat{\phi}_{jd}$. [1]

To begin the NN virtual control development, we rearrange (25) to observe the dynamics of the ideal virtual controller to be

$$\begin{align*}
\dot{\hat{\Theta}}_{jd} &= T_{j}(\omega_{jd} - T_{j}^{-1} K_{j\omega} e_{\theta_{jd}}) \tag{28} \\
\dot{\Theta}_{jd} &= T_{j}^{-1}(\hat{\Theta}_{jd} + K_{j\omega} e_{\theta_{jd}}) + T_{j}^{-1}(\hat{\Theta}_{jd} + K_{j\omega} e_{\theta_{jd}})
\end{align*}$$

For convenience, we define a change of variable as $\Omega_{\hat{\phi}_{jd}} = \omega_{jd} - T_{j}^{-1} K_{j\omega} e_{\theta_{jd}}$, and the dynamics (28) become

$$\begin{align*}
\dot{\hat{\Theta}}_{jd} &= T_{j} \Omega_{\hat{\phi}_{jd}} \tag{29} \\
\dot{\Omega}_{\hat{\phi}_{jd}} &= T_{j}^{-1}(\hat{\Theta}_{jd} + T_{j}^{-1}(\hat{\Theta}_{jd} = f_{j\Omega}(\phi_{jd}) = f_{j\Omega}
\end{align*}$$

Defining the estimates of $\Theta_{\hat{\phi}_{jd}}$ and $\Omega_{\hat{\phi}_{jd}}$ to be $\hat{\Theta}_{jd}$ and $\hat{\Omega}_{jd}$.
respectively, and the estimation error \( \hat{\Theta}_{jd} = \Theta_{jd} - \hat{\Theta}_{jd} \), the dynamics of the proposed NN virtual control inputs become

\[
\dot{\hat{\Theta}}_{jd} = \dot{\hat{\Theta}}_{jd} + T^j \hat{\Theta}_{jd} + K_{\beta31} \hat{\Theta}_{jd} \tag{30}
\]

\[
\hat{\Theta}_{jd} = T^j \hat{\Theta}_{jd} + K_{\beta31} \hat{\Theta}_{jd} + \hat{\Theta}_{jd} + K_{\beta31} T^j \hat{\Theta}_{jd}
\]

where \( K_{\beta31} \) and \( K_{\beta31} \) are positive constants. The estimate \( \hat{\Theta}_{jd} \) is then written as

\[
\hat{\Theta}_{jd} = \hat{\Theta}_{jd} + T^j K_{\beta31} e_{\Theta_{jd}} + K_{\beta31} T^j \hat{\Theta}_{jd} \tag{31}
\]

where \( K_{\beta31} \) is another positive constant. Observing

\[
\tilde{\Theta}_{jd} = \omega_{jd} - \hat{\omega}_{jd} = \Omega_{jd} - \omega_{jd} \tag{32}
\]

subtracting (30) from (29), as well as adding and subtracting

\[
T^j \tilde{\Theta}_{jd} \text{ and } K_{\beta31} T^j \tilde{\Theta}_{jd}, \text{ the virtual controller estimation error dynamics are found to be}
\]

\[
\dot{\hat{\Theta}}_{jd} = T^j \tilde{\Theta}_{jd} - (K_{\beta31} - K_{\beta31} T^j) \tilde{\Theta}_{jd} \tag{33}
\]

\[
\dot{\tilde{\Theta}}_{jd} = f_{\beta1}(x_{jd}) - f_{\beta1}(x_{jd}) - K_{\beta31} T^j \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} + K_{\beta31} T^j \tilde{\Theta}_{jd}
\]

where \( f_{\beta1}(x_{jd}) = f_{\beta1}(x_{jd}) + T^j \tilde{\Theta}_{jd} - K_{\beta31} T^j \tilde{\Theta}_{jd} \) is an unknown function.

In (30), universal approximation property of NN has been utilized to estimate the unknown function \( f_{\beta1}(x_{jd}) \) by bounded ideal weights \( W_{M2}^T, V_{M}^T \) such that \( \|W_{M2}\| \leq W_{M2} \) for a known constant \( W_{M2} \), and written as

\[
f_{\beta1}(x_{jd}) = W_{M2}^T \sigma(V_{M}^T x_{jd}) + e_{\beta1}
\]

where \( e_{\beta1} \) is the bounded NN approximation error such that \( \|e_{\beta1}\| \leq e_{\beta1} \) for a known constant \( e_{\beta1} \). The NN estimate of \( f_{\beta1}(x_{jd}) \) is written as

\[
f_{\beta1}(x_{jd}) = W_{M2}^T \sigma(V_{M}^T x_{jd}) + \hat{e}_{\beta1}
\]

where \( \hat{\sigma}_{\beta1} = \sigma_{\beta1} + W_{M2}^T \tilde{\Theta}_{jd} \) is the NN input written in terms of the virtual control estimates, desired trajectory, and the UAV velocity. The NN input is chosen to take the form of

\[
\tilde{\Theta}_{jd} = [1 N_j \Theta_{jd} \tilde{\Theta}_{jd} v_{jd}^T].
\]

Next, differentiating (32), using (33) as well as adding and subtracting \( W_{M2}^T \hat{\sigma}_{\beta1} \) reveals

\[
\dot{\hat{\Theta}}_{jd} = -K_{\beta31} \hat{\Theta}_{jd} + \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} + T^j \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} + T^j \tilde{\Theta}_{jd}
\]

where \( \tilde{\Theta}_{jd} = \omega_{jd} - \hat{\omega}_{jd} = \Omega_{jd} - \omega_{jd} \).

\[
\dot{\tilde{\Theta}}_{jd} = \dot{f}_{\beta1}(x_{jd}) - \dot{f}_{\beta1}(x_{jd}) - K_{\beta31} T^j \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} + K_{\beta31} T^j \tilde{\Theta}_{jd}
\]

\[
\dot{\tilde{\Theta}}_{jd} = \dot{f}_{\beta1}(x_{jd}) - \dot{f}_{\beta1}(x_{jd}) - K_{\beta31} T^j \tilde{\Theta}_{jd} - T^j \tilde{\Theta}_{jd} + \hat{\sigma}_{\beta1} + \tilde{\Theta}_{jd} - K_{\beta31} \tilde{\Theta}_{jd}
\]

\[
\tilde{\Theta}_{jd} = \hat{\sigma}_{\beta1} + T^j \tilde{\Theta}_{jd} - K_{\beta31} \tilde{\Theta}_{jd}
\]

\[
\|\tilde{\Theta}_{jd}\| \leq \tilde{\Theta}_{\beta1} \text{ with } \tilde{\Theta}_{\beta1} = \epsilon_{\beta1} + 2W_{M2}^T N_{\Omega} \text{ a computable constant with } N_{\Omega} \text{ the constant number of hidden neurons in the virtual control NN and the fact } \|\tilde{\Theta}_{jd}\| \leq \sqrt{N_{\Omega}} \text{ was used.}
\]

Examination of (33) and (34) reveals \( \dot{\hat{\Theta}}_{jd} = 0, \hat{\Theta}_{jd} = 0 \), and \( \tilde{\Theta}_{jd} = 0 \) to be equilibrium points of the estimation error dynamics when \( \|\tilde{\Theta}_{\beta1}\| = 0 \).

To this point, the desired translational velocity for follower \( j \) has been identified to ensure the leader-follower objective (8) is achieved. Then, the desired pitch and roll were derived to drive \( v_{jd} \rightarrow v_{jd} \) and \( v_{jd} \rightarrow v_{jd} \), respectively. Then, the desired angular velocity was found to ensure \( \theta_j \rightarrow \theta_j \).

What remains is to identify the UAV thrust to guarantee \( v_{jd} \rightarrow v_{jd} \) and rotational torque vector to ensure \( \omega_j \rightarrow \omega_j \). First, the thrust is derived.

Consider again the translational velocity tracking error dynamics (16), as well as the desired velocity tracking error dynamics (18). Equating (16) and (18) and manipulating the third error state, the required thrust is found to be

\[
u_{jd} = m_j \left[ \frac{e_{\omega_{jd}} s_{\omega_{jd}} e_{\omega_{jd}} s_{\omega_{jd}}}{e_{\omega_{jd}} s_{\omega_{jd}}} \Lambda_j + \dot{f}_{\beta1} \right] + m_j k_{\beta1} e_{\omega_{jd}} + \dot{f}_{\beta1} \tag{35}
\]

where \( \dot{f}_{\beta1} \) is the NN estimate in (17) previously defined. Substituting the desired pitch (22), roll (23), and the thrust (35) into the translational velocity tracking error dynamics (16) yields

\[
\dot{\epsilon}_{\omega_{jd}} = -K_{\omega} \epsilon_{\omega_{jd}} + R_{\omega} \left( \epsilon_{\omega_{jd}} \right) - T_{\omega} \epsilon_{\omega_{jd}} - \tau_{\omega}
\]

where \( \epsilon_{\omega_{jd}} = R_{\omega} \epsilon_{\omega_{jd}} - \tau_{\omega} \). Additionally, in the formulation of (36), the expressions for the desired pitch and roll (22) and (23), respectively, were first written in the form of (20) and (21), so that sine and cosine of the angles could be substituted as opposed to substituting the arc tangent expressions directly into the sine or cosine function.

Next, the rotational torque vector \( u_{j2} \) will be addressed.

First, multiply the angular velocity tracking error (26) by the constant inertial matrix \( A_j \), take the first derivative with respect to time, add and subtract \( T_{\omega} \epsilon_{\omega_{jd}} \), and substitute the UAV dynamics (1) to reveal

\[
J_{j2} \epsilon_{\omega_{jd}} = J_{j2} \dot{\epsilon}_{\omega_{jd}} - u_{j2} - T_{\omega} \epsilon_{\omega_{jd}} - \tau_{\omega}
\]

where \( J_{j2} \epsilon_{\omega_{jd}} = J_{j2} \dot{\epsilon}_{\omega_{jd}} - S(J_j \omega_j) \omega_j - N_{j2} (\omega_j) + T_{\omega} \epsilon_{\omega_{jd}} \).

Examining \( f_{\beta2}(x_{jd}) \), it is clear that the function is nonlinear and contains unknown terms; therefore, the universal approximation property of NN is utilized to estimate the function \( f_{\beta2}(x_{jd}) \) by bounded ideal weights \( W_{M2}^T, V_{M}^T \) such that \( \|W_{M2}\| \leq W_{M2} \) for a known constant \( W_{M2} \) and written as

\[
f_{\beta2}(x_{jd}) = W_{M2}^T \sigma(V_{M}^T x_{jd}) + e_{\beta2}
\]

where \( e_{\beta2} \) is the bounded NN functional reconstruction error such that \( \|e_{\beta2}\| \leq e_{\beta2} \) for a known constant \( e_{\beta2} \). The NN estimate of \( f_{\beta2}(x_{jd}) \) is given by \( \hat{f}_{\beta2}(x_{jd}) = W_{M2}^T \sigma(V_{M}^T + e_{\beta2}) \) where \( W_{M2}^T \sigma_{\beta2} = \tilde{W}_{M2}^T \sigma_{\beta2} \) where \( \tilde{W}_{M2}^T \sigma_{\beta2} \) is the NN estimate of \( W_{M2}^T \sigma_{\beta2} \).
The input to the NN is the estimated angular velocity tracking error as \( \hat{\dot{\omega}}_{\text{ref}} = \hat{\omega}_{\text{ref}} - \dot{\omega}_j \). Now, using the NN estimate \( \hat{\dot{\omega}}_{\text{ref}} \), the rotational torque control input is written as

\[ u_{\text{ref}} = \hat{\dot{\omega}}_{\text{ref}} + K_{\text{ref}} \hat{\dot{\omega}}_{\text{ref}} \]  

(38)

and substituting the control input (38) into the angular velocity dynamics (37) reveals

\[ J \dot{\hat{\dot{\omega}}}_{\text{ref}} = J \dot{\dot{\omega}}_{\text{ref}} - J \dot{\dot{\omega}}_{\text{ref}} - T_j \dot{e}_\text{ref} - \tau_{\text{ref}} \]  

(39)

Now, adding and subtracting \( \tilde{W}_{\text{ref}}^j \Delta \sigma_{\text{ref}} \) and observing \( \hat{\dot{\omega}}_{\text{ref}} = e_j - \omega_{\text{ref}} \) the closed loop dynamics (39) become

\[ J \dot{\hat{\dot{\omega}}}_{\text{ref}} = -K_{\text{ref}} e_j + \tilde{W}_{\text{ref}}^j \Delta \sigma_{\text{ref}} + K_{\text{ref}} e_j - \omega_{\text{ref}} + \tau_{\text{ref}} \]  

(40)

where \( \tilde{W}_{\text{ref}}^j = W_{\text{ref}}^j - \tilde{W}_{\text{ref}}^j \) \( e_j = e_j + \Delta \sigma_{\text{ref}} - \tau_{\text{ref}} \), and \( \Delta \sigma_{\text{ref}} = \sigma_{\text{ref}} - \sigma_{\text{ref}} \). Further, \( \| \Delta \sigma_{\text{ref}} \| \leq \varepsilon_{\text{ref}} \) for a computable constant \( \varepsilon_{\text{ref}} = e_j + 2W_{\text{ref}} \text{Me}_2 + \tau_{\text{ref}} \), where \( N_{\text{ref}} \) is the number of hidden layer neurons.

As a final step, we define the augmented variables \( \hat{e}_j = [e_j^T \hat{e}_j^T]^T \), \( \tilde{W}_{\text{ref}}^j = [\tilde{W}_{\text{ref}}^j 0; 0 \tilde{W}_{\text{ref}}^j] \) and \( \Delta \sigma_{\text{ref}} = [\Delta \sigma_{\text{ref}} \Delta \sigma_{\text{ref}}]^T \). In the following theorem, the stability of the follower \( j \) is shown while considering \( \dot{e}_j = 0 \). In other words, the position, orientation, and velocity tracking errors are considered along with the estimation errors of the virtual controller and the NN weight estimation errors of each NN for follower \( j \) while ignoring the interconnection errors between the leader and its followers. This assumption will be relaxed later.

**Theorem 1:** (*Follower UAV System Stability*) Given the dynamic system of follower \( j \) in the form of (1), let the desired translational velocity for follower \( j \) to track be defined by (12) with the desired pitch and roll defined by (22) and (23), respectively. Let the NN virtual controller be defined by (30) and (31), respectively, with the NN update law given by

\[ \tilde{W}_{\text{ref}}^j = F_{\text{ref}} \hat{\Delta} \sigma_{\text{ref}} - \kappa_{\text{ref}} F_{\text{ref}} \tilde{W}_{\text{ref}}^j \]  

(41)

where \( F_{\text{ref}} = F_{\text{ref}} > 0 \) and \( \kappa_{\text{ref}} > 0 \) are design parameters. Let the dynamic NN controller for follower \( j \) be defined by (35) and (38), respectively, with the NN update given by

\[ \tilde{W}_{\text{ref}}^j = F_{\text{ref}} \hat{\Delta} \sigma_{\text{ref}} - \kappa_{\text{ref}} F_{\text{ref}} \tilde{W}_{\text{ref}}^j \]  

(42)

where \( A_{\text{ref}} = [A_{\text{ref}} 0; 0 A_{\text{ref}} 3] \) is the control matrix, and \( F_{\text{ref}} > 0 \) and \( \kappa_{\text{ref}} > 0 \) are constant design parameters. Then there exists positive design constants \( K_{\text{ref}}, K_{\text{ref}}, K_{\text{ref}} \), and positive definite design matrices \( K_{\text{ref}}, K_{\text{ref}}, K_{\text{ref}}, K_{\text{ref}} \), such that the virtual controller estimation errors \( \hat{\Theta}_{\text{ref}}, \hat{\Theta}_{\text{ref}}, \hat{\Theta}_{\text{ref}}, \hat{\Theta}_{\text{ref}} \) and the virtual control NN weight estimation errors, \( \tilde{W}_{\text{ref}}^j, \) the position, orientation, and translational and angular velocity tracking errors, \( e_j, e_j, e_j, e_j \), respectively, and the controller NN weight estimation errors, \( \tilde{W}_{\text{ref}}^j, \) are all semi-globally uniformly ultimately bounded (SGUUB).

**Proof:** Consider the following positive definite Lyapunov candidate

\[ V_j = K_{\text{ref}} t_{\text{ref}} V_{\text{ref}} + V_{\text{ref}} \]  

(43)

where \( K_{\text{ref}} \) is the maximum singular value of \( K_{\text{ref}} \) and

\[ V_{\text{ref}} = 1 \]  

(44)

whose first derivative with respect to time is given by \( \dot{V}_j = V_{\text{ref}} + V_{\text{ref}} \).

Finally, (44) is less than zero provided

\[ K_{\text{ref}} > K_{\text{ref}} + N_{\text{ref}} / K_{\text{ref}} \]  

(45)

and the following inequalities hold:

\[ \| \hat{\Theta}_{\text{ref}} \| > \eta_j \]  

(46)

or

\[ \| e_{\text{ref}} \| > \eta_j \]  

(47)

or

\[ \| e_{\text{ref}} \| > \eta_j \]  

(48)

or

\[ \| e_{\text{ref}} \| > \eta_j \]  

(49)

Therefore, it can be concluded using standard extensions of Lyapunov theory [9] that \( V_j \) is less than zero outside of a compact set, revealing the virtual controller estimation errors, \( \Theta_{\text{ref}}, \Theta_{\text{ref}}, \Theta_{\text{ref}}, \Theta_{\text{ref}}, \) and the NN weight estimation errors, \( \tilde{W}_{\text{ref}}^j, \)
the position, orientation, and translational and angular velocity tracking errors, $e_{j\rho}, e_{jv}, e_{j\theta}$, respectively, and the dynamic controller NN weight estimation errors, $\hat{W}_{ic}$, are all $SGUUB$.

B. Formation Leader Control Law

The dynamics and kinematics for the formation leader are defined similarly to (1) and (4), respectively. In our previous work [1], an output feedback control law for a single quadrotor UAV was designed to ensure the robot tracks a desired path, $\rho_{id} = [x_{id}, y_{id}, z_{id}]^T$, and desired yaw angle, $\psi_{id}$. Using the design methods described in this work as well as in [1], a state feedback control law for the formation leader can be derived which ensures the leader’s error systems are $SGUUB$.

Next, the stability of the formation consisting of 1 leader and $N$ followers is considered in the following theorem while considering the interconnection errors between the leader and its followers.

C. Quadrotor UAV Formation Stability

Before proceeding, it is convenient to define the following augmented error systems consisting of the position and translational velocity tracking errors of leader $i$ and $N$ follower UAVs as

$$
e_{j\rho} = \begin{bmatrix} e_{j\rho}^T, e_{j\rho}^T, \ldots, e_{j\rho}^T \end{bmatrix}_{j=1}^{N} \in \Re^{N(N+1)}$$

$$
e_v = \begin{bmatrix} e_v^T, e_v^T, \ldots, e_v^T \end{bmatrix}_{j=1}^{N} \in \Re^{N(N+1)}.$$

Next, the transformation matrix (2) is augmented as

$$
R_{c} = diag \{ R_{c}, R_{c}, \ldots, R_{c} \}_{j=1}^{N} \in \Re^{N(3N+3)}
$$

while the NN weights for the translational velocity error system are augmented as

$$
\hat{W}_{i\rho} = diag(\hat{W}_{i\rho1}, \hat{W}_{i\rho2}, \ldots, \hat{W}_{i\rhoN}) \in \Re^{N(N+1)}$$

$$
\hat{\sigma}_{i\rho} = [\hat{\sigma}_{i\rho1}^T, \hat{\sigma}_{i\rho2}^T, \ldots, \hat{\sigma}_{i\rhoN}^T]_{j=1}^{N} \in \Re^{N(N+1)}.$$

Now, using the augmented variables above, the augmented closed loop position and translational velocity error dynamics for the entire formation are written as

$$
\dot{e}_{\rho} = -K_{\rho}e_{\rho} + (I - G_{r})R_{c}e_{v},
$$

$$
\dot{e}_{v} = -K_{v}e_{v} + A_{\rho} \hat{W}_{i\rho} \hat{\sigma}_{i\rho} - K_{\rho}G_{r}R_{c}e_{v} + \xi_{c},
$$

where $A_{\rho} = diag \{ A_{\rho1}, A_{\rho2}, \ldots, A_{\rhoN} \}$ with $A_{\rho}$ defined similarly to $A_{\rho}$ in terms of $\Theta_{\rho}$, $\xi_{c}$ is an appropriately defined vector consisting of $\xi_{c1}, \xi_{c2}, \ldots, \xi_{cN}$, etc.,

$$
K_{\rho} = diag \{ K_{\rho1}, K_{\rho2}, \ldots, K_{\rhoN} \}_{j=1}^{N},
$$

$$
K_{v} = diag \{ K_{v1}, K_{v2}, \ldots, K_{vN} \}_{j=1}^{N},
$$

$G_{r}$ is a constant matrix relating to the formation interconnection errors defined as $G_{r} = [0; F_{r}] \in \Re^{N+1}$, and $F_{r} \in \Re^{N}$ is dependent on the specific formation topology. For instance, in a string formation where each follower follows the UAV directly in front of it, follower $i$ tracks leader $i$, follower $2$ tracks follower $1$, etc., and $F_{r}$ becomes the identity matrix.

Theorem 2: (UAV Formation Stability) Given the leader-follower criterion of (8) with 1 leader and $N$ followers, let the hypotheses of Theorem 1 hold. Let the virtual control system for the leader $i$ be defined similarly to (14) and (15) with the virtual control NN update law defined similarly to (34). Let the control velocity, desired pitch and roll long with the thrust and rotation torque vector for the leader be given by [1] using state feedback, and let the control NN update law be defined identically to (35). Then, the position, orientation, and velocity tracking errors as well as the virtual control estimation errors for the entire formation are all $SGUUB$.

Proof: Proof of Theorem 2 is omitted whereas it follows as in Theorem 1.

IV. CONCLUSIONS

A new framework for quadrotor UAV leader-follower formation control was presented along with a novel NN formation control law which allows each follower to track its leader without the knowledge of dynamics. All six DOF are successfully tracked using only four control inputs while in the presence of unmodeled dynamics and bounded disturbances. Lyapunov analysis guarantees $SGUUB$ of the entire formation, and numerical results, although not shown, confirm the theoretical conjectures.

REFERENCES


