

\( H_\infty \) unwinding web tension control of a strip processing plant using a pendulum dancer

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Abstract—All flexible materials such as textiles, papers, polymers or metals are handled on rollers during their processing. Maintaining web tension in the entire processing line under changing web speed is a key factor for achieving good final product quality. Two approaches are mostly used for tension control: load cells which provide direct web tension measurement or dancers which indirectly regulate web tension via regulation of dancer position. Dancers are mobile mechanisms located in specific area of the processing line to regulate web tension. The focus of this paper is to propose an improved alternative to web processing industry practice (PI controllers) based on \( H_\infty \) synthesis for pendulum dancers. Even though pendulum dancers are widely used on industrial plants, they have been studied very rarely. The nonlinear and linearized phenomenological models of the subsystem containing the dancer are first proposed. The position controller of the dancer is then synthesized using the standard \( H_\infty \) framework and the mixed sensitivity approach. The performance of the proposed approach is illustrated by experimental results carried out on a large experimental web handling platform containing four driven rollers and the pendulum dancer in the unwind section. These are the first published results of successful application of an \( H_\infty \) controller to a real plant containing a pendulum dancer.

I. INTRODUCTION

Any continuous and flexible material whose width is significantly less than its length and whose thickness is significantly less than its width can be described as a web. The unwinder-winder systems handling web material such as textiles, papers, polymers or metals are very common in the industry, because they represent a more convenient way of transporting and processing a product from one form to another. Printing, coating and drying are examples of operations that can be performed in sections of a line. Web tension and speed are two key variables in order to achieve the expected final product quality. One of the main objectives in web handling machinery is to reach an expected web speed while maintaining the web tension within a close tolerance band over the entire processing line. This tolerance depends obviously on the type of material that has to be processed. Due to a number of sources of disturbance, i.e. web vibrations, temperature and moisture, strong coupling between speed and tension is introduced by the web mechanical behavior, especially in the case of large scale systems with many actuators. This coupling makes regulation of web tension under speed changes difficult. It has been common practice in industrial web transport systems to use decentralized PI-type controllers. But recently, to answer increasing requirements, more efficient control strategies have been proposed to improve tension control.

Modeling of the longitudinal dynamics of the web has now been studied for several decades, [3], [11], [12], [14]. Overviews of problems and solutions in tension control can be found in [9], [15], [16], [18]. Typically a web tension measurement is obtained in a span of the processing line and is used as a feedback signal for a driven roller. The most commonly used methods to perform tension control are load cells and dancers. Both have been compared in [6]. Tension control is performed using direct measurement of web tension in the case of load cells. Several centralized and distributed robust control strategies based on \( H_\infty \) controllers, LQG controllers or state feedback are proposed in [2], [7], [8], [9], [14], [15] using load cells mounted on idler rollers. But the focus of this paper is to discuss a class of \( H_\infty \)-based control strategies using a pendulum dancer.

Contrary to load cells, the use of dancer mechanisms is an indirect method to ensure tension control. The variable that is regulated is not web tension but the position of the dancer which provides regulation of tension indirectly. Usually web handling plant designers have recourse to dancer mechanisms close to the unwinder and the rewinder because they act naturally as mechanical filters to attenuate various disturbances. These disturbances are mainly eccentricity and non-circularity of web material rolls. Mechanically, two kinds of dancer structures can be distinguished: linear and pendulum dancers (a picture of the pendulum dancer used in this work is shown in Fig. 2). The first is the most commonly studied in the literature, [5], [13]. Nevertheless, pendulum dancers seem to be more used than linear dancers in industrial processing plants because they are more convenient to manufacture. First models of the pendulum dancer have been introduced in [10], [17]. The use of dancers can be divided into two categories: active and passive dancers. A comparative study is proposed in [5] for web tension disturbance attenuation using a linear dancer. Active dancers are mainly used for tension disturbance attenuation requiring a tension measurement provided by a load cell, like in [13]. In such a case, the position of dancer is imposed by an actuator that acts directly on the dancer. On the other side, passive dancers can be seen in some cases as “free-to-move” roller whose
role is to avoid fast large tension variations thanks to their damping characteristics. But this paper investigates another common application of passive dancers, i.e. tension control. The principle consists in the position control of the dancer: a force is applied to the dancer by a cylinder that is compensated by the expected web tension in a specific position. During the unwinding-rewinding process, the pressure in the cylinder remains constant while the position feedback is used to adjust the speed of a driven roller which is most of the time the unwinder or the rewinder next to the dancer.

Since web handling systems are large scale systems, decentralized control strategies with each subsystem containing only one actuator (i.e. one driven roller) are commonly used. The emphasis of this paper is on the unwind section of a large experimental platform containing four driven rollers as shown in Fig. 1. The roller labeled as “PD” in the unwind section is the pendulum dancer roller – picture shown in 2. The angular position of the pendulum dancer is used as a feedback signal to provide a speed reference correction of the unwind roll. It remains common in industrial practice to tune manually a PI-controller for dancer position regulation. But in this contribution, the pendulum angular position control will be performed with a SISO $H_\infty$ controller calculated from a simplified phenomenological model of the unwind section. The controller is synthesized using the standard $H_\infty$ framework based on the mixed sensitivity approach. This paper presents the first experimental results of an $H_\infty$ controller applied to the unwinding section of a web handling system using a pendulum dancer (some preliminary simulation results are given in [10]).

The rest of the paper is organized as follows. Section II recalls the main physical laws to model web handling systems. A model of the pendulum dancer is also presented. A LTI model is derived in order to design the $H_\infty$ controller. Section III deals with the discussion of the synthesis of a full-order $H_\infty$ controller for the unwind section together with the pendulum dancer. Experimental results are also shown and discussed in Section III.

II. SYSTEM MODELING

Figure 1 is an example of a web handling system which exhibits the classical structure of an industrial processing line. Such a line is usually divided into several subsections that are controlled independently either in tension or in speed. Each subsection contains a driven roller and a web span, i.e. a span between two consecutive driven rollers whose purpose is to ensure web tension control, except in one case where the subsection is only composed of a driven roller. It is usually called the master speed roller and its purpose is to regulate web speed over the entire processing line.

The system in Fig. 1 which has been used to experiment the controllers is composed of an unwinder, two intermediate driven rollers and a rewinder. Tension control is performed using load cell (LC) feedback in the pull-roll and rewind sections. The load cell in the unwind section is only used for monitoring. The intermediate driven rollers (denoted as S-Wrap Section) are the master speed rollers.

The nonlinear model of a web transport system, [11], [12], is built from the equations describing web tension behavior between two consecutive rollers and the velocity of each roller. Moreover, the pendulum dancer dynamics has to be taken into account to complete the behavior of the whole system. It is a common assumption to neglect free roller dynamics in the development of a model because they have only effects during transient states. This assumption is reasonable and will be used in this paper. The only free roller whose dynamics is included in the model is of the pendulum dancer in order to consider the effects of dancer rotation on
the tensions of the spans downstream and upstream dancer roller.

A. Web Velocity Dynamics

The velocity dynamics of the \( k \)th roll is given by torque balance on it. Assuming the absence of slippage between the web and the roll, the web velocity \( V_k \) is equal to the peripheral velocity of the roller. The velocity dynamics is:

\[
d\left( J_k \omega_k \right) = R_k (T_{k+1} - T_k) + K_k U_k - C_{rk} \tag{1}
\]

where \( \omega_k = V_k/R_k \) is the angular velocity of the \( k \)th roll, \( T_k \) is the web tension between the \( k \)th and the \((k+1)\)th rolls, \( K_k U_k \) is the motor torque (if the roll is driven), \( C_{rk} \) corresponds to all the friction torques, \( J_k \) is the roll inertia and \( R_k \) is the roll radius.

B. Web Tension Dynamics

The calculation of web tension between two consecutive rollers is based on three laws [11], [12]: Hooke’s law, Coulomb’s law and mass conservation law. The equation of continuity applied to the web transport systems gives:

\[
d \left( \frac{L_k}{1 + \epsilon_k} \right) = -\frac{V_{k+1}}{1 + \epsilon_k} + \frac{V_k}{1 + \epsilon_k-1} \tag{2}
\]

where \( L_k \) is the web length between the \( k \)th and the \((k+1)\)th rollers and \( \epsilon_k \) is the strain in the corresponding web span. Finally this relationship yields the following tension dynamics[11], [12]:

\[
L_k \frac{dT_k}{dt} = ES (V_{k+1} - V_k) + T_{k+1} V_k - T_k (2V_k - V_{k+1}) \tag{3}
\]

C. Pendulum Dancer Modeling

A sketch of the pendulum dancer mechanism is shown in Fig. 3. It is composed of an articulated pivoted arm whose end point contains a free roller \( R_2 \). A cylinder imposes a force \( F_j \) to the dancer. This force depends on the expected web tension and varies with tension references. In operation, the objective of the regulation is to maintain the dancer in an angular equilibrium position so that the web tensions \( T_2 \) and \( T_3 \) of the downstream and upstream strips compensate the force applied by the cylinder. This angular position is denoted by \( \alpha \) and its dynamics is derived based on torque balance which is expressed as:

\[
J_d \frac{d^2 \alpha}{dt^2} = F_j L_j \cos (\alpha + \alpha_j) - (L_d - R_2) T_2 \cos (\alpha - \theta_3) - (L_d + R_2) T_3 \cos (\alpha - \theta_2) - M_d g L_{gc} \sin \alpha - C_f \tag{4}
\]

where \( \alpha \) corresponds to the angular position of the dancer, \( \alpha_j \) is the angular position of the cylinder, \( \theta_2 \) and \( \theta_3 \) are the angular displacements of the downstream and upstream strips from their initial horizontal positions. \( J_d, L_d, L_j, L_{gc} \) and \( M_d \), respectively, are the dancer inertia, the length of the dancer arm, the distance of the pneumatic cylinder to the pivot point, the distance to the center of gravity, and the dancer mass. \( T_2 \) and \( T_3 \), respectively, are web tensions in the upstream and downstream spans. In steady state, the purpose is usually to have the dancer in a vertical position. Consequently, if \( T_0 \) is the nominal tension in the web (i.e. tension reference), the nominal force \( F_j \) that has to be applied by the cylinder to maintain a reference tension in both upstream and downstream spans will be:

\[
F_j = \frac{2L_d T_0}{L_j} \tag{5}
\]

The movement of the pendulum dancer mechanism implies some changes in the values of the system parameters. These are \( L_2, L_3, \theta_2, \theta_3 \) and \( \alpha_j \). These changes are based on geometrical considerations (more details on this issue can be found in [17]). Moreover the angular velocity of the pendulum creates a difference in the tangential velocities of the roller \( R_2 \) for the downstream and upstream webs which can be formulated as [17]:

\[
V_{2d} = V_2 + \dot{\alpha} (L_d - R_2) \tag{6}
\]

\[
V_{2u} = V_2 - \dot{\alpha} (L_d + R_2) \tag{7}
\]

where \( V_{2d} \) is the downstream web speed, \( V_{2u} \) is the upstream web speed, \( V_2 \) is the roller circumferential velocity calculated with the relationship (1) and \( \dot{\alpha} \) is the angular velocity of the dancer.

D. Linear Plant

The relationship (6) that gives the angular position dynamics of the dancer can be linearized assuming that the position \( \alpha \) varies slowly and is very small (i.e. \( \alpha << 1 \)):

\[
J_d \frac{d^2 \alpha}{dt^2} = F_j L_j - (L_d - R_2) T_2 - (L_d + R_2) T_3 - PL_{gc} \alpha - C_f \tag{8}
\]

The linear model of web tension is deduced from (3) by considering a plant working point [11]. If \( V_0 \) and \( T_0 \) are respectively the nominal web velocity and tension, the relationship (3) becomes in the general case:

\[
L_k \frac{dT_k}{dt} = (ES + T_0) (V_{k+1} - V_k) + V_0 (T_{k-1} - T_k) \tag{9}
\]

But it is slightly different for the web tensions downstream and upstream of the dancer because, as mentioned above,
it has to take into account the dancer rotation. The speed \( V_2 \) is replaced in (9) by the expression of \( V_{2d} \) for \( k = 2 \) (\( T_2 \)-dynamics) and \( V_{2u} \) for \( k = 3 \). These relations can be formulated as follows:

\[
L_3 \frac{dT_2}{dt} = (ES + T_0)(V_2 - V_1 + (L_d - R_2)\dot{\alpha}) + V_0(T_1 - T_2) \tag{10}
\]

\[
L_3 \frac{dT_3}{dt} = (ES + T_0)(V_3 - V_2 + (L_d + R_2)\dot{\alpha}) + V_0(T_2 - T_3) \tag{11}
\]

Note that the lengths used in these equations are considered as constant according to the previous assumption that the variations in the dancer angular position are very small.

### III. \( H_\infty \) TENSION CONTROL

The typical control scheme for tension regulation in each subsystem of an industrial plant is proposed in Fig. 4. \( K_1 \) and \( K_2 \) are scaling factors which depend on unwinder radius and gear ratio between the roller and motor shafts. But the product of these two factors cancels the effects of time-varying radius. The control scheme is composed as follows: an inner loop ensures speed control whereas an outer loop performs tension control, i.e. position control in the case of a dancer. Note that the output of the controller of the outer loop is used to adjust speed reference which is the speed reference for the entire line. Industrial web handling systems use commonly two PI or PID controllers to regulate speed reference and angular velocity, \( \omega_B \) and \( \omega_B \) are respectively drive angular velocity and gear ratio between the roller and motor shafts. But the weighting function \( W_p(s) = \frac{s}{M + \omega_B} \) is as follows [1]:

\[
W_p(s) = \frac{s}{M + \omega_B} + \omega_B \epsilon_0
\]

where \( M \) is the maximum peak magnitude of the sensitivity. The calculation of this feedforward term at the output of the speed controller provides improved speed loop performance. This feedforward term is adaptive and is required to compensate for roll inertia (and its variations) and frictions (i.e. static and dynamic frictions). The calculation of this feedforward term has been explored in [7] and [14]. Assuming that inertia and friction are compensated, the drive can be seen as an integrator and the closed-loop as a second-order transfer function expressed as follows:

\[
G(s) = \frac{\Omega}{\Omega_{ref}} = \frac{k(s + \omega)}{s^2 + ks + k\omega} \tag{12}
\]

where \( \Omega_{ref} \) and \( \Omega \) are respectively drive angular velocity reference and angular velocity, \( k \) and \( \omega \) are respectively the proportional gain and the lead frequency of the PI controller.

#### A. \( H_\infty \) Problem Formulation

The computation of the SISO \( H_\infty \) position controller is performed using the scheme of the Fig. 4. The inner speed loop is taken as in (12). The simplified model neglecting idler rollers which is used to describe the unwinder and dancer section is illustrated in Fig. 5. \( L_1 \) and \( L_2 \) are the lengths between the unwinder and the dancer roller and between dancer roller and master speed roller respectively. The system can be formulated using the classical state space model:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*} \tag{13}
\]

where \( x^T = [x_1 \ x_2 \ T_1 \ V_2 \ T_2 \ \alpha \ \dot{\alpha}] \) is the state vector. Note that \( x_1 \) and \( x_2 \) refer to the speed loop state variables. \( u = [\Delta V \ F_y] \) is the system inputs which are control value for speed reference adjustment and cylinder force in our case and \( y = \alpha \) is the dancer position measurement.

The decentralized position SISO \( H_\infty \) controller has been synthesized using the framework of Fig. 6 with output weighting. By decomposing the system \( G \) as follows:

\[
G = \begin{bmatrix} G_{\Delta V \rightarrow y} & G_{F_y \rightarrow y} \end{bmatrix}
\]

the frequency weighting functions \( W_p, W_u, W_t \) and \( W_d \) appear in the closed-loop transfer function matrix in the following manner:

\[
T_{rd \rightarrow z} = \begin{bmatrix} W_pS & -W_pRW_d \\
W_uKS & -W_uKRWD \\
W_tT & W_tRW_d \end{bmatrix}
\]

where \( S \) is the sensitivity function, \( S = (1 + KG_{\Delta V \rightarrow y})^{-1} \), \( T \) is the complementary sensitivity function, \( T = KG_{\Delta V \rightarrow y}S \), and \( R = GF_yS \). \( r \) is the position reference and \( d \) can be seen as a disturbance on the force \( F_y \) that is applied by the cylinder on the dancer. The \( H_\infty \) problem consists of finding a stabilizing controller \( K \) which minimizes the \( H_\infty \)-norm of the transfer function \( T_{rd \rightarrow z} \) between external inputs \( r \) and \( d \) and performance outputs \( z \):

\[
\|T_{rd \rightarrow z}\|_\infty = \sup_\omega (\sigma(T_{rd \rightarrow z})) \leq \gamma \tag{16}
\]

The weighting function \( W_p \) is usually taken with a high gain at low frequency in order to reject low frequency disturbances. The form of \( W_p \) is as follows [1]:

\[
W_p(s) = \frac{\frac{s}{M} + \omega_B}{s + \omega_B\epsilon_0}
\]

where \( M \) is the maximum peak magnitude of the sensitivity.
$S$, $\omega_B$ is the required frequency bandwidth $\epsilon_0$ is the steady state error allowed. Considering a small value for $\omega_B\epsilon_0$ is usually a sufficient condition to obtain a pole close to zero. The weighting function $W_u$ is used to avoid large control signals and the weighting function $W_t$ increases the roll-off at high frequencies. $W_d$ is selected to minimize the effect of $d$ on $z_1$ mainly; it has been taken as the identity matrix in this work.

### B. Frequency Analysis

This SISO subsystem describing the unwind section is analyzed in the frequency domain. Figure 7 illustrates the plots of the subsystem alone (a), the controller (b), the closed-loop system (c) and the closed-loop sensitivity function.

The open loop system consists of an integrator and second-order resonance model. The integral effect comes from the relationship (8) which provides dancer position dynamics. The bandwidth of the regulated system has been chosen small in order to reject the low frequency disturbances coming from rollers’ out-of-roundness and eccentricity. Note that the resonance remains present in the closed-loop system, but its gain compared to the gain in steady state is now acceptable. As a result this resonance does not play a role on the sensitivity function frequency response.

### C. Experimental Results

The order of the resulting controller is nine. The controller has been implemented on the experimental setup shown in Fig. 1 by its discrete state-space representation using Tustin’s approximation and a sampling period of 10 ms. During the experiments, the only other section where tension is controlled is the rewind zone using a basic industrial PI controller. Tension is not regulated in the pull-roll section, the roller is only driven by a speed loop. Experimental results are shown in Fig. 8. The air pressure applied inside the cylinder of the pendulum dancer corresponds to a web tension of 21 lbf (93 N) if the dancer remains in its expected vertical position. Web tension in the unwind section is monitored using the available load cells in the Unwind and Dancer Section (see fig. 1). The processing line is run for velocity reference that varies between 100 fpm and 300 fpm (approximately 30 m/min and 92 m/min, see fig. 8(c)).

The data plotted in fig. 8 are obtained using the following sequence. First, since the web is slack (zero tension), the dancer is pushed to its extreme position (-11 degrees) as the air pressure is commanded into the cylinder. Then, at $t=20$ s, the outer position loop (see fig. 4) is closed with dancer position feedback for the unwind roll. It results in bringing the dancer back to its expected vertical down position and in regulating web tension at the level of 20 lbf (fig. 8(b)). Finally, at $t=70$ s, the line is run following the speed profile shown in fig. 8(c). Results illustrate the efficiency of the computed full-order controller in reference tracking. During steady state operations, the dancer remains in a band of less than 0.1 degree. Moreover, the occurrence of disturbances caused by speed reference changes has small effects on dancer position despite the well-known strong coupling that exists between web speed and web tension. As a comparison,
many industrial setups can tolerate dancer position variations up to 10 degrees. Nevertheless, small oscillations occur around this average constant tension value during all the unwinding-rewinding process. But their amplitude does not vary with speed. These oscillations are consequences of the rotation of the intermediate rollers. A second phenomenon can be observed on the tension response. The average values of web tension when the line is stopped and when the line is running are different. This offset is due to static and dynamic friction of the rollers on their shaft and illustrates the role played by idle rollers on web tension over the entire processing line.

IV. CONCLUSION
This contribution has focused on web tension regulation in the unwind section using a pendulum dancer. This paper for the first time presents experimental results of an $H_{\infty}$ controller computed using a simplified linear model of the unwind section of a large plant containing a pendulum dancer. A linear state space model of the unwind section containing the pendulum dancer has been derived to compute the SISO $H_{\infty}$ position controllers. The $H_{\infty}$ synthesis is carried out using the standard mixed sensitivity approach. The experimental results carried out on a large plant illustrate the efficiency of the controller. It has been observed that the coupling between line speed and dancer position is nearly nonexistent despite large velocity changes. Future work should investigate the interactions of the other subsystems on the unwinding section containing the pendulum dancer in the controller synthesis.

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REFERENCES