Abstract—In this paper we apply dynamic feedback linearization to the tracking problem for a turbocharged diesel engine (TDE) equipped with exhaust gas recirculation (EGR) valve and variable geometry turbocharger (VGT). The model used here is the third-order mean-value model, a reduction of the eighth-order mean-value one, see [13], for sake of simplicity. Our goal is to track desired values of suitably chosen outputs. In fact, we first plan to control the input manifold pressure and the compressor mass flow rate instead of the air fuel ratio (AFR) and the EGR fraction. Unfortunately, the former lead to a non-minimum phase system while the latter are not accessible for measurements in a vehicle, see [7]. We thus replace the problem of tracking of desired values of the output $y$ (input manifold pressure and compressor mass flow rate) by that of tracking a suitably constructed modified output for which the values to be tracked are specifically chosen: namely, when the modified output $\tilde{y}$ approaches them, the original output converges to the desired values. Simulation results are presented.

I. INTRODUCTION

In order to comply with more constraining European emission regulations, refer to [1] (Euro norms see [16] and [17]), car manufacturers introduce in some diesel engines two actuators: the exhaust gas recirculation (EGR) valve and the variable geometry turbocharger (VGT). The former permits recirculation of exhaust gas into the intake manifold reducing by this way the formation of NO\textsubscript{x} while the latter permits the improvement of the relatively low power diesel engine density. But some drawbacks have to be underlined: an important reduction of the amount of fresh air leads to an increase in particulate emissions and possibly visible smoke whereas a low amount of EGR fraction leads to an increase in NO\textsubscript{x} emissions, see [7].

To render these two actuators more efficient during combustion, several control design methods have been proposed: polynomial control in [2], dynamic feedback linearization in [12] and [13], optimal nonlinear control in [14], constructive Lyapunov control in [7], indirect passivation in [9], passivation in [8], predictive control in [3], [4] and [11], etc.

The output $y$ of to-be-controlled variables, which consists of the input manifold pressure $p_1$ and the compressor mass flow rate $W_c$, leads to a non-minimum phase system. Therefore we propose another choice of output (whose zero dynamics are trivial) such that if that modified output $\tilde{y}$ tracks a suitably chosen value, then the original output $y(t)$ converges asymptotically to its desired value. In [3] we solved the tracking problem for the TDE model using nonlinear predictive control. In [13], Planos et al. apply dynamic feedback linearization based on the property of flatness to a reduced-order TDE model. Pursuing their work, this paper presents a more general result on the application of the dynamic feedback linearization control design method to the simplified TDE model. Indeed, we see, with notions of geometric control that for each suitably chosen vector output, the third-order nonlinear model remains dynamic feedback linearizable with a trivial zero dynamics. To argue this, we chose a different vector output from that of Planos et al. in [13] for this study.

The paper is organized as follows: in Section II a description of TDE is given and its simplified model is presented with a brief remind on the eighth-order one. Vector output is chosen. In Section III we examine the behavior of zero dynamics and propose a modification of the output. A construction of a dynamic extension of the simplified model and a general result about the TDE outputs choice are given in Section IV. Section V presents the control law derived from feedback linearization theory while simulation results are presented in Section VI. Conclusion and some future research directions are briefly given in Section VII.

II. TURBOCHARGED DIESEL ENGINE: DESCRIPTION, MODEL AND OUTPUTS CHOICE

A. Description of TDE functioning

A schematic diagram (see Fig. 1) of TDE is presented below. At the top of this diagram is the turbocharger composed of the turbine and the compressor. During the functioning of TDE, the turbine takes its energy from the exhaust gas. As it is linked to the compressor via a common shaft, the compressor starts rotating, bringing consequently fresh air into the combustion chambers via the intercooler and the intake manifold. A part of the exhaust gas is recirculated into the combustion chambers to reduce NO\textsubscript{x} formation refer to [7].

B. Reduced-order model

The following presented nonlinear model (1) is a simplified model from the eighth-order one briefly outlined in [13]:

\begin{align}
\dot{p}_1 &= k_1(W_c + W_{egr} - k_e p_1) + \frac{T_1}{\tau_1} p_1 \\
\dot{p}_2 &= k_2(k_e p_1 - W_{egr} - W_i + W_f) + \frac{T_2}{\tau_2} p_2, \\
\dot{P}_c &= \frac{1}{\eta_m}(p_1 - P_c)
\end{align}

where the compressor (resp. the turbine) mass flow rate is related to the compressor (resp. the turbine) power as follows:

\begin{equation}
W_c = P_c \frac{k_e}{p_1^2 - 1}
\end{equation}
Despite of the fact that the real inputs are the EGR valve and VGT openings, the considered inputs, in this study, for sake of simplicity, are $u_1 = W_{egr}$ and $u_2 = W_i$, see [13].

In the sequel, $T_1$ and $T_2$ are assumed to vanish because their corresponding measured signals $T_1$ and $T_2$ have very slow variations, see [7]. The fuel mass flow rate $W_f$ is regarded as an external disturbance and will not be taken into account for the synthesis of the controller. This yields the following system:

\begin{align*}
\dot{p}_1 &= k_1(W_c + u_1 - k_c p_1) \\
\dot{p}_2 &= k_2(k_c p_1 - u_1 - u_2) \\
\dot{P}_c &= \frac{1}{\tau}(\eta_m P_1 - P_c)
\end{align*}

Replacing $W_c$ and $P_1$ by their expressions (2) and (3) and denoting $K_0 = \frac{\eta_m}{\tau} k_1$ yield the system:

\begin{equation}
\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2,
\end{equation}

where

\begin{equation}
f(x) = \begin{bmatrix}
k_1 k_c \frac{P_2}{p_1} - k_1 k_c p_1 \\
k_2 k_c p_1 \\
0
\end{bmatrix},
\end{equation}

\begin{equation}
g_1(x) = \begin{bmatrix}
k_1 \\
-k_2 \\
0
\end{bmatrix}
\quad \text{and} \quad
g_2(x) = \begin{bmatrix}
0 \\
-k_2 \\
K_0 (1 - \frac{p_2}{\tau}^\gamma)
\end{bmatrix}
\end{equation}

and $x = (p_1, p_2, P_c)$ belong to the set $\Omega$, see [7], defined by

\begin{equation}
\Omega = \{(p_1, p_2, P_c) : 1 < p_1 < p_1^{\max}, 1 < p_2 < p_2^{\max}, 0 < P_c < P_c^{\max}\},
\end{equation}

with the maximal values $p_1^{\max}, p_2^{\max}, P_c^{\max}$ follow from physical limits of TDE.

All the parameters of the model $k_1, k_2, k_c, k_c, \tau$ and $\eta_m$ are identified from the eighth-order mean-value nonlinear model at a constant speed of 1600 RPM and a fueling rate of 7.2 kg/h as said in a previous study, see [13]. The full-order model consists of the equations of the change of pressures, masses and fractions of burned gas in the intake and exhaust manifolds. These six equations are completed by two more: the turbocharger speed and the air mass flow rate in the pipe connecting the compressor outlet and the intake manifold, see [13]. For a detailed description of the full-order model see [7] and [15]. The nomenclature of some TDE variables is summarized in TABLE I, see [7] and [13].

### TABLE I

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGR</td>
<td>Exhaust gas recirculation</td>
</tr>
<tr>
<td>AFR</td>
<td>Air fuel ratio</td>
</tr>
<tr>
<td>$N$</td>
<td>Engine speed</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Intake manifold burned gas fraction</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Exhaust manifold burned gas fraction</td>
</tr>
<tr>
<td>$m_{11}$</td>
<td>Mass of gas in the intake manifold</td>
</tr>
<tr>
<td>$m_{22}$</td>
<td>Mass of gas in the exhaust manifold</td>
</tr>
<tr>
<td>$p_1$</td>
<td>Gas pressure in the intake manifold</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Gas pressure in the exhaust manifold</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Compressor power</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Turbine power</td>
</tr>
<tr>
<td>$W_c$</td>
<td>Total mass flow rate into the engine</td>
</tr>
<tr>
<td>$W_f$</td>
<td>Turbine mass flow rate</td>
</tr>
<tr>
<td>$W_t$</td>
<td>Fuel mass flow rate</td>
</tr>
<tr>
<td>$W_{egr}$</td>
<td>EGR mass flow rate</td>
</tr>
<tr>
<td>$V_1$</td>
<td>Intake manifold volume</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Exhaust manifold volume</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Intake manifold temperature</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Exhaust manifold temperature</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Compressor temperature</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Temperature of the exhaust from the engine</td>
</tr>
<tr>
<td>$W_{egr}$</td>
<td>EGR temperature</td>
</tr>
<tr>
<td>$\omega_{c}$</td>
<td>Turbocharger speed</td>
</tr>
<tr>
<td>$J_{ec}$</td>
<td>Turbocharger moment of inertia</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Compressor isentropic efficiency</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>Turbine isentropic efficiency</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>Turbocharger mechanical efficiency</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Specific heat ratio</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Specific gas constant</td>
</tr>
</tbody>
</table>

#### C. Vector output choice

The output of to-be-controlled variables consists of the input manifold pressure $p_1$ and the compressor mass flow rate $W_c$ instead of the AFR and EGR fraction because the latter are not accessible for measurements in a vehicle, see [7]. We thus consider the nonlinear system (4) (equivalently, (5)-(7)) with the vector output:

\begin{equation}
y = \begin{bmatrix}
p_1 \\
W_c
\end{bmatrix}
\end{equation}

and the goal is to track desired constant values $p_{1d}$ of $p_1$ and $W_{cd}$ of $W_c$. In the sequel, we suppose that all the components $(p_1, p_2, P_c)$ of the state $x$ measured or estimated.

### III. ZERO DYNAMICS AND CHANGE OF THE OUTPUT

Consider the square-MIMO ($m \times m$) nonlinear system

\begin{equation}
\dot{x} = f(x) + \sum_{j=1}^{m} g_j(x)u_j \\
y = \left(h_1(x), \ldots, h_m(x)\right)^T,
\end{equation}

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ are the vectors state, control, and output, respectively.

To simplify the exposition, the standard geometric notation for Lie derivatives is used in this paper. For a real-valued
function $h$ on $R^n$ and a vector field $f$ on $R^n$, the Lie derivative of $h$ along $f$ at $x \in R^n$ is given by:

$$L_fh(x) = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i}(x)f_i(x).$$

Inductively, we define

$$L^k_fh(x) = L_fL^{k-1}_fh(x) = \frac{\partial L^{k-1}_fh(x)f(x),}$$

with $L^0_fh(x) = h(x)$.

A. Vector relative degree

A system of the form (10) has a vector relative degree $(\rho_1, \cdots, \rho_m)$ if:

(i) for any $x \in R^n$

$$L_{g_i}L^\rho_j h_i(x) = 0,$$

for all $1 \leq i \leq m$, all $1 \leq j \leq m$, and all $0 \leq k < \rho_i - 1$;

(ii) the $m \times m$ matrix (decoupling matrix)

$$A(x) = \begin{bmatrix}
L_{g_1}L^{\rho_1-1}_1 h_1(x) & \cdots & L_{g_m}L^{\rho_1-1}_1 h_1(x) \\
\vdots & \ddots & \vdots \\
L_{g_1}L^{\rho_m-1}_m h_m(x) & \cdots & L_{g_m}L^{\rho_m-1}_m h_m(x)
\end{bmatrix}$$

(11)

is nonsingular for all $x \in R^n$ (see, e.g., [6]).

For the third-order model (5)-(7) with the output (9), the vector relative degree exists and $(\rho_1, \rho_2) = (1, 1)$ for all $(p_1, p_2, P_c) \in \Omega$. The decoupling matrix is:

$$A(x) = \begin{bmatrix}
k_1 & 0 & -k_{0}k_{c} \rho_1^{-\mu} - k_1 \\
\mu k_c k_1 p_1 p_1^{-1} \rho_1^{-\mu} - k_1 & -K_0 k_c \rho_1^{-\mu} - k_1 \\
\rho_1^{-\mu} - k_1 & -K_0 k_c \rho_1^{-\mu} - k_1
\end{bmatrix}.$$

The sum of the vector relative degree components is equal to 2 which is less than 3, the dimension of the state space of system (5)-(7). Therefore one-dimensional zero dynamics exist. An examination of their stability is necessary before deriving the vector control law (assuring tracking the desired output value).

B. Zero dynamics

Since the goal is to track a reference signal, which consists of desired fixed values $p_{1d}$ and $W_{cd}$ of the respective components of the output $y = (p_1, W_c)\',$ define the error

$$e = \begin{bmatrix}
p_1 - p_1d \\
W_c - W_{cd}
\end{bmatrix},$$

(12)

between the to-be-controlled variables $(p_1, W_c)$ and their desired values. The zero dynamics of the error are obtained by applying the control annihilating identically the error $e(t)$ and thus are given by

$$P_2 = k_2 W_{cd} \left[1 - \frac{(p^\rho_{1d})^{-1}}{\eta_m k_c (1-p_2^{-\mu})}\right],$$

(13)

which are unstable. Indeed, (13) has a single equilibrium $P_{2c} = \left[1 - \frac{(p^\rho_{1d})^{-1}}{\eta_m k_c (1-p_2^{-\mu})}\right]^{-\frac{1}{2}} \approx 1.7$ bar and the eigenvalue $\lambda \approx 25$ of the linearization of (13) at $P_{2c}$ is positive, see Fig. 2 and Fig. 3. These numerical values are given for the values $p_{1d} \cong 1.6$ bar and $W_{cd} \cong 0.07$ kg/s that are, indeed, natural for applications. It can be noticed, however, that the instability of the zero dynamics (13) depends neither on the choice of $p_{1d}$ nor of $W_{cd}$. The system is I-O decoupleable via static feedback). So using I-O linearization we can easily find a control law that steers the error $e(t)$, given by (12), asymptotically to zero (see, e.g., [6]). Unstable zero dynamics will result, however, in an undesired property: the internal variable $p_2$ will not remain bounded, it will either reach the limit value $p_2 = 1$ in finite time (if $p_{20} < p_{2c}$, see Fig. 2) or will go to plus infinity with an asymptotically constant velocity (if $p_{20} > p_{2c}$, see Fig. 3). To avoid dealing with unstable zero dynamics, we propose another choice of vector output and a dynamic extension.

C. Change of the vector output

We will overcome the problem of unstable zero dynamics by changing the output (9) such that the modified system has trivial zero dynamics, that is, consisting of a single equilibrium point. This can be achieved by keeping the first component as $p_1$ and replacing the second output component by a function $\tilde{h}_2(x)$, where $x = (p_1, p_2, P_c)$, such that, indeed, $L_{g_2} \tilde{h}_2 = 0$. Resolving this equation gives

$$\tilde{h}_2(x) = P_c + \frac{K_0}{k_2} \left(p_2 - \frac{1}{1 - \mu} p_2^{-\mu}\right),$$

and thus we consider the new output $\tilde{y}(t)$ of to-be-controlled variables defined by

$$\tilde{y} = \tilde{h}(x) = \begin{bmatrix}
p_1 \\
P_c + \frac{K_0}{k_2} \left(p_2 - \frac{1}{1 - \mu} p_2^{-\mu}\right)
\end{bmatrix}. $$

(14)

In the next subsection we will show that the system (5)-(7) with the output (14) has, indeed, trivial zero dynamics.
As we specified, our problem is to track desired constant values \( p_{1d} \) of \( p_1 \) and \( W_{cd} \) of \( W_c \). A natural question is how to reformulate the problem in terms of the components of the new output \( \tilde{y} \), given by (14), in order to achieve a solution of the original tracking goal. Notice that \( W_c, P_c, \) and \( p_1 \) are linked via the relation \( W_c = P_c \frac{k_c}{p_1^{\rho_{c1}} - 1} \) and hence the desired tracking values \( p_{1d} \) and \( W_{cd} \) determine uniquely the desired value \( P_{cd} \) of \( P_c \) as

\[
P_{cd} = W_{cd} \frac{p_{1d}^{\rho_{c1} - 1}}{k_c}.
\]

Notice that given any fixed \( p_{1d} \) and \( P_{cd} \), there exists a unique point \( x_e = (p_{1e}, p_{2e}, P_{ce}) \), satisfying \( p_{1e} = p_{1d} \) and \( P_{ce} = P_{cd} \), and unique control values \( u_e = (u_{1e}, u_{2e}) \) such that the right hand side of (5)-(7) has an equilibrium at \( x_e \) when the controls are evaluated at \( u_e \). To see this, recall that the equilibrium set of (5) consists of the points at which we can create an equilibrium by a suitable feedback and thus \( E = \{ x : f(x) \in \text{span}\{g_1(x), g_2(x)\} \} \). Observe that the zero dynamics manifold of the error \( e \), given by (12), is \( Z^* = \{ p_1 = p_{1d}, W_c = W_{cd} \} \) and is thus transversal to the equilibrium set \( E \). More precisely, the intersection \( \{ x_e \} = E \cap Z^* \) consists of a single equilibrium point \( x_e = (p_{1d}, p_{2e}, P_{cd}) \), with uniquely defined \( p_{2e} \), and there exist unique control values \( u_e = (u_{1e}, u_{2e}) \in \mathbb{R}^2 \) such that

\[
f(x_e) + u_{1e}g_1(x_e) + u_{2e}g_2(x_e) = 0.
\]

We will define the desired tracking value \( \tilde{h}_{2d} \) of \( \tilde{h}_2 \), the second component of the new output \( \tilde{y} \) (given by (14)), as

\[
\tilde{h}_{2d} = \tilde{h}_2(x_e) = P_{cd} + \frac{K_0}{k_2} \left( p_{2e} - \frac{1}{1 - \mu_2} p_{1d}^{\rho_{c1} - \mu_2} \right).
\]

Notice that the zero dynamics manifold \( Z^*_d \) of the error

\[
\dot{e} = \begin{bmatrix}
    p_1 - p_{1d} \\
    \tilde{h}_2(x) - \tilde{h}_{2d}
\end{bmatrix}
\]

passes through the equilibrium point \( x_e \). Now a crucial observation is that when the new output \( \tilde{y}(t) \) tracks asymptotically the constant value \( (p_{1d}, \tilde{h}_{2d}) \) and the overall system approaches the equilibrium point \( x_e \), then the original output \( y(t) \) tracks asymptotically the desired values \( (p_{1d}, W_{cd}) \). It follows that in order to solve the tracking problem, it is enough to show that the zero dynamics corresponding to the new error (15) are asymptotically stable (for instance, trivial consisting of \( x_e \) only). The main idea of changing the output is illustrated in Fig. 4. The zero dynamics, evolving on \( Z^*_d \), of the error (between the original output \( y(t) \) and its desired value) are unstable. We look for a new output \( \tilde{y} \) and for its desired value such that the zero dynamics of the new error are asymptotically stable and their manifold \( Z^*_d \) intersect \( Z^*_d \) at an equilibrium point \( x_e \). A solution can be either asymptotically stable dynamics on \( Z^*_d \) (as illustrated by Fig. 4) or trivial zero dynamics reduced to the equilibrium \( x_e \) (which will be the case of our TDE model).

IV. Dynamic Extension of the Simplified Model and Generalization about the TDE Outputs Choice

In this section we will follow notions of geometric nonlinear control (see, e.g., [6] and [10]). The system (5)-(7) with the output (14) has the decoupling matrix

\[
\hat{A}(x) = \begin{bmatrix}
    k_1 - \frac{\mu_1 k_1 P_1^{-\rho_{c1}}}{p_1^{\rho_{c1}} - 1} & 0 \\
    -\frac{\mu_2 k_1 P_1^{-\rho_{c1}}}{p_1^{\rho_{c1}} - 1} & 0
\end{bmatrix},
\]

which is not invertible, and thus the system has no a vector relative degree. We can, however, construct a suitable dynamic extension with a well defined relative degree. To this end, put \( z = u_1 = W_{egr}, \dot{z} = v_1 \) and apply the new vector control \([v_1, v_2]^T\), where \( v_2 = u_2 = W_t \). This yields the following extended nonlinear system

\[
\begin{align*}
\dot{v}_1 &= k_1 (W_c + z - k_c p_1) \\
\dot{v}_2 &= k_2 (k_c p_1 - z - v_2) \\
\dot{P}_c &= \frac{1}{\rho_{c1}} (\eta_{pc} P_t - P_c) \\
\dot{z} &= v_1
\end{align*}
\]

which we can rewrite, denoting its extended state by \( x^e = (p_1, p_2, P_c, z)^T \in \Omega \times [W_{egr}, W_{egr}] \) (where \( W_{egr}^\text{min} \) and \( W_{egr}^\text{max} \) are the minimum and maximum values of the EGR mass flow rate), as

\[
\dot{x}^e = f^e(x^e) + g_1^e(x^e)v_1 + g_2^e(x^e)v_2,
\]

where

\[
f^e(x^e) = \begin{bmatrix}
    k_1 (k_c p_2 - k_c p_1 + z) \\
    k_2 (k_c p_1 - z) \\
    -\frac{k_2}{p_2^{\rho_{c2} - 1}} \\
    0
\end{bmatrix},
\]

\[
g_1^e(x^e) = \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    1
\end{bmatrix}
\]

and

\[
g_2^e(x^e) = \begin{bmatrix}
    0 \\
    k_2 (1 - p_2^{\rho_{c2}}) \\
    0 \\
    0
\end{bmatrix}.
\]

The extended system (18)-(20) with the output (14) has the vector relative degree \((\rho_{c1}^e, \rho_{c2}^e) = (2, 2)\) and the invertible
decoupling matrix

\[ A^e(x^e) = \begin{bmatrix}
    k_1 & k_1k_0\frac{1-p_2^{-\mu}}{p_1^{1-\mu}} \\
    \mu k_2K_0(z - k_1p_1)p_2^{-\mu-1} & + \frac{K_0}{p_1}\end{bmatrix}. \]

The extended system is thus I-O decouplable and has trivial zero dynamics (since the sum of the components of its vector relative degree is \( p_1^2 + p_2^2 = 2 + 2 = 4 \), the dimension of the state space of the extended system). Notice that the original system (5)-(7), with the output (14), has trivial zero dynamics too because the latter does not depend on invertible endogenous feedback.

A natural question is whether a dynamic extension is necessary. In other words, is it possible to choose another output, say \( y = \tilde{h}(x) \), such that the original system (5)-(7), with the output \( \tilde{y} \), would have the vector relative degree \((\tilde{p}_1, \tilde{p}_2) = (2, 1)\). In this case, the original system would be static feedback I-O decouplable with trivial zero dynamics (with respect to the output \( \tilde{y} \)) and no extension would be needed. The answer is: this is impossible, independently of the choice of the output \( y = h(x) \). Indeed, if such an \( \mathbb{R}^2 \)-valued function \( \tilde{h} \) exists, then the original system (5)-(7) would be static feedback linearizable. This is not the case, however, because the distribution \( \mathcal{D} = \text{span}\{g_1, g_2\} \) is not involutive since the Lie bracket

\[ [g_1, g_2] = \begin{bmatrix} 0 & 0 \\
    0 & -\mu k_2K_0p_2^{-\mu-1} \end{bmatrix} \]

is independent of \( g_1 \) and \( g_2 \).

Another way of looking at the extension procedure that we propose is to observe that although the system (5)-(7) is not static feedback linearizable it is, however, flat, refer to [5] and [13] (that is, dynamic feedback linearizable) since any 3-dimensional system with non involutive distribution \( \mathcal{D} = \text{span}\{g_1, g_2\} \) is so and the components \( p_1 \) and \( \tilde{h}_2 \), given by (14), or any other suitably chosen outputs, see [13], of the new output \( \tilde{y} \) are actually flat outputs (linearizing outputs) of the system (5)-(7). Indeed, we can express the state components \( p_1, p_2, P_e \) as well as the controls \( u_1 \) and \( u_2 \) of the system using \( p_1 \) and \( \tilde{h}_2 \) and their time derivatives; when calculating \( u_2 \) we will have to differentiate \( u_1 \) which confirms that the system is dynamically (but not statically) linearizable.

V. Control Law

In [3] we solved the tracking problem for the simplified TDE model using nonlinear predictive control. In this section we will linearize the extended system which will allow us to calculate the control law that assures tracking desired values of the modified output (14) which, in turn, guarantees tracking the desired values of the original output (9) (as we explained in Section III). The extension (18)-(20) of the TDE model, together with the output (14), is I-O decouplable and has trivial zero dynamics so the linearizing coordinates and the linearizing feedback can be calculated as follows, see, e.g., [6] and [10]. Introduce new coordinates

\[ \varphi_1^e = \tilde{h}_1(x^e) = p_1 \]

\[ \varphi_2^e = \tilde{h}_2(x^e) = -\frac{P_e}{k_2} + K_0(k_1p_1 - z_1) \]

followed by the feedback

\[ v(x) = A^e(x^e)^{-1}[-b(x^e) + w] \]

where \( A^e(x^e) \) is the decoupling matrix (21), and \( b(x^e) \) is given by:

\[ b(x^e) = \begin{bmatrix}
    k_1^2 \left( k_e + \frac{\mu k_e}{p_1(1-\mu)} \right) \left( k_e + \frac{P_e}{p_1(1-\mu)} \right) \left( k_e + \frac{P_e}{p_1(1-\mu)} \right) \\
    -\frac{P_e}{k_2} - k_1k_0(1-\mu) \left( k_e + \frac{P_e}{p_1(1-\mu)} \right) \left( k_e + \frac{P_e}{p_1(1-\mu)} \right) \\
    -\mu k_2k_0p_2^{-\mu-1}(k_e - k_2p_2^{-\mu}) \end{bmatrix}. \]

This yields the following decoupled system:

\[ \begin{align*}
    \dot{\varphi}_1^e &= \dot{\varphi}_2^e \\
    \dot{\varphi}_2^e &= w_2 \\
    \dot{\varphi}_1^e &= w_1 \\
    \dot{\varphi}_2^e &= w_2
\end{align*} \]

The desired tracking value \( p_1d \) of \( \varphi_1^e \) is given while desired tracking value \( \tilde{h}_d \) of \( \varphi_2^e \) is deduced from \( W_{cd} \) and from the requirement that \( x^e = (p_1d, p_2e, P_{cd}, \tilde{h}_d) \) is an equilibrium point (see Section III). This yields

\[ \begin{align*}
    z_d &= u_1d = k_2p_1d - W_{cd} \\
    p_2e &= \left( 1 - \frac{P_{cd}}{\tau K_0 W_{cd}} \right)^{-\frac{1}{\mu}} \\
    \tilde{h}_d &= P_{cd} + \frac{K_0}{k_2} \left( p_2e - \frac{1}{1-\mu} P_{cd}^{-\mu} \right).
\end{align*} \]

The tracking control law is now calculated as

\[ w = \begin{bmatrix}
    K_1^1(\varphi_1^e - p_1d) + K_1^2 \varphi_2^e \\
    K_2^1(\varphi_1^e - \tilde{h}_d) + K_2^2 \varphi_2^e \\
    K_1^1 \varphi_1^e + K_2^1 \varphi_2^e + K_1^2 \tilde{h}_d \\
    K_2^1 \varphi_1^e + K_2^2 \tilde{h}_d
\end{bmatrix} \]

where \( K_1^j, K_2^j \in \mathbb{R} \) are chosen so that, for \( i = 1, 2 \), the polynomials \( \lambda^2 - K_1^j \lambda - K_2^j \) are Hurwitz. This control law guarantees that output \( \tilde{y}(t) \) tracks asymptotically the constant values \( (p_1d, \tilde{h}_d) \) and therefore the original output \( y(t) \) tracks asymptotically the desired values \( (p_1d, W_{cd}) \).

VI. Simulation Results

We chose as the output \( y(t) \) to be tracked a concatenation of constant desired values \( p_1d \) of \( p_1 \) and \( W_{cd} \) of \( W_e \), corresponding to specific rate of emission of the engine, in the neighborhood of that chosen by Plianos et al., see [13]. The eigenvalues of the closed loop system are picked up as -3 and -4 for the first subsystem and -2 and -1 for
The second. The simulations have been done under Matlab Simulink V 7.0, with the following characteristics: all "step sizes" and "tolerances" are under the position "auto" and the used solver is "Ode 23s (stiff/Mod. Rosenbrock). The time of simulation is 40 s. We show in Figures 5, 6, 7 (zoom of Fig. 6) and 8 the behavior of the to-be-controlled output \( y(t) = (p_1(t), W_r(t)) \) as well as that of the second component \( h_2(z(t)) \) of the modified output \( \hat{y}(t) \).

**Fig. 5.** Extended model (4th order): Output \( p_1 \) and its reference signal \( P_{1,d} \)

**Fig. 6.** Extended model (4th order): Output \( h_2 \) (denoted by \( H \)) and its reference signal \( h_{2,d} \) (denoted by \( H_d \))

**Fig. 7.** Extended model (4th order): Zoom in on output \( h_2 \) (denoted by \( H \)) and its reference signal \( h_{2,d} \) (denoted by \( H_d \))

**Fig. 8.** Extended model (4th order): Output \( W_r \) and its desired value \( W_{r,d} \)

**VII. CONCLUSIONS AND FUTURE WORKS**

**A. Conclusions**

In this paper, a 3-dimensional nonlinear TDE model is considered. In order to avoid dealing with unstable zero dynamics we propose a solution of the tracking problem based on two basic ingredients: change of the output (which yields a system with a trivial zero dynamics) and dynamic extension (which allows a simple calculation of a tracking controller).

**B. Future Works**

In our study we suppose that all states are accessible for measurements which may not always be the case in practice (for instance, the gas pressure in the exhaust manifold is not accessible for measurements). Therefore in our future works we are planning to use a (nonlinear) observer for that state of the turbocharged diesel engine TDE and to do a comparative study between our controller based dynamic feedback linearization and that of Plianos et al., see [13]. We are planning also to study robustness of the control law with respect to the fuel mass flow rate \( W_f \) considered as an external perturbation in this study.

**REFERENCES**


