Control-Oriented Aerothermoelastic Modeling Approaches for Hypersonic Vehicles

Adam J. Culler, Jack J. McNamara, and Andrew R. Crowell

Abstract—The field of aerothermoelasticity is essential for control-oriented modeling of hypersonic vehicles due to a high degree of coupling between vehicle systems, as well as the presence of aerodynamic heating. In the present study, an efficient aerothermoelastic model is investigated in two ways. First, an approximate aerodynamic heating model is verified using Computational Fluid Dynamic flow analysis. Next, the model is used to gain insight into the degree of coupling between the aerothermal and aeroelastic systems. Results demonstrate that both material property degradation and two-way coupling are important for control-oriented aerothermoelastic modeling. Furthermore, quasi-static and dynamic average approaches for fluid-thermal-structural coupling offer an accurate and efficient approximation for implementing two-way coupling.

NOMENCLATURE

c plate specific heat
c_f local skin friction coefficient
D = E\nu h^3/12(1 - \nu^2), bending stiffness
E modulus of elasticity
H enthalpy
h plate thickness
k plate thermal conductivity
L plate length, streamwise
M Mach number
M_T thermal moment
N_x in-plane load, total
P_r Prandtl number
p pressure
Q_aero aerodynamic heat flux
Q_rad radiation heat flux
q_a = p - p_\infty, aerodynamic pressure
q_\infty = \rho_{\infty} U^2_x/2, dynamic pressure
R_e Reynolds number
S_t Stanton number
T temperature
T_{env} environment temperature
t time
U air velocity
w normal plate displacement
x streamwise coordinate
z normal coordinate

\alpha thermal expansion coefficient
\gamma ratio of specific heats, air
\epsilon emissivity
\nu Poisson’s ratio
\rho air density
\rho_m plate density
\sigma Stefan-Boltzmann constant

Subscripts
\infty freestream
aw adiabatic wall
e edge of the boundary layer
w wall

Superscripts
* reference enthalpy condition

I. INTRODUCTION

Currently, there is a focus by NASA, the United States Department of Defense, and the United States Air Force on the development of hypersonic technologies for next generation reusable launch vehicles and unmanned hypersonic cruise vehicles [1]-[5]. As shown in Fig. 1, modern hypersonic vehicle configurations are typically based on a lifting body, integrated airframe-propulsion concept, where the entire lower vehicle surface is part of a scramjet engine. A challenge with this class of vehicle is a tight-coupling between the aerodynamic, control, structural, and propulsion systems that cannot be neglected during analysis and design [5]-[10]. Furthermore, air-breathing hypersonic vehicles must fly within the atmosphere for sustained periods of time to meet the needs of the propulsion system [7], [11]; resulting in severe aerodynamic heating.

These issues imply that the field of aerothermoelasticity, which involves mutual fluid-thermal-structural interactions in a system, has an important role in control-oriented modeling of hypersonic vehicles[5]. Specifically, structural deflections alter the inflow to the engine, as well as the flow over the outer body; while the stiffness and aeroelastic behavior of the vehicle and control effectors are time and mission dependent. Thus, the incorporation of aerothermoelastic effects is essential to successful guidance, navigation, and control of hypersonic vehicles systems [12]. During the last 15 years, a number of researchers have performed investigations into these effects using multi-disciplinary models of hypersonic vehicles [5], [13]-[28]. This study aims to expand upon this work by improving control-oriented aerothermoelastic...
modeling approaches. The objective of this study is to investi-
gate an efficient procedure for computing the aerodynamic
heating on hypersonic vehicles appropriate for control design
evaluation, and characterize the degree of coupling
between the fluid-thermal-structural interactions. Since this is
a preliminary investigation, a relatively simple configuration
is chosen; namely a flexible panel on the surface of a
hypersonic vehicle.

II. Method of Solution

The aerothermoelastic modeling approach used in this
study is illustrated in Fig. 2. The aerothermal problem
consists of interaction between the aerodynamic heating
and structural heat transfer, while the aeroelastic problem
consists of fully-coupled inertial-elastic-aerodynamic inter-
actions. The coupled aerothermoelastic model includes the
influence of aerodynamic heating on structural deformation
(Mechanism 1) and feedback from the aeroelastic solution
to the aerothermal problem (Mechanism 2).

A. Fluid Model

The panel considered in this study is located on an inclined
surface of a wedge-shaped body, as shown in Fig. 3. The
inclined surface before and after the panel is assumed to be
flat and rigid, thus the inviscid flow properties at the leading
edge of the panel (Location 3) are the same as those behind
the leading edge shock [29].

The unsteady inviscid pressure over the panel is computed
using third order piston theory [30], which has been used
extensively in hypersonic aeroelastic research [12]. Since the
panel is located on an inclined surface, the “freestream” flow
conditions used in (1) to compute the pressure distribution
over the panel are those at the leading edge of the panel. The
inviscid flow temperature and Mach number distributions
near the panel are computed using isentropic flow relations
[29] based on the total condition at the leading edge of the
panel and the pressure distribution from (1).

\[
q_a = 2 q_\infty \left[ \left( \frac{1}{U_\infty} \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \right) + \frac{\gamma + \frac{1}{4}}{M_\infty} \left( \frac{1}{U_\infty} \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \right)^2 + \frac{\gamma + 1}{12} M_\infty^2 \left( \frac{1}{U_\infty} \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \right)^3 \right]
\]

(1)

Aerodynamic heating is modeled using Eckert’s reference
enthalpy method [31]. The reference enthalpy method uses
boundary layer equations from incompressible flow theory,
but with flow properties evaluated at a reference enthalpy to
account for the effects of compressibility. Using reference
parameters, heat flux at the wall, \( Q_{aero} \), is given by (2).

For turbulent flow the Stanton number, \( St^* \), is determined
using the Colburn-Reynolds analogy shown in (3), and the
local skin friction coefficient, \( \epsilon_f^* \), is calculated using the
Schultz-Grunow formula given in (4) [31]. Coupling from
the structure to the aerodynamic heating model is achieved
by updating the edge flow properties as the structure deforms,
and also by updating the surface temperature of the panel as
it is heated.

\[
Q_{aero} = St^* \rho U_c (H_{aw} - H_w)
\]

(2)

\[
St^* = \epsilon_f^* \frac{1}{2 (Pr^*)^{2/3}}
\]

(3)

Fig. 1. Schematic of the NASA X-43 Experimental Aircraft.

Fig. 2. Fully-coupled modeling approach for aerothermoelastic systems.
\[ c_f^* = \frac{0.370}{(\log_{10} Re_x^*)^{2.584}} \]  

(4)

B. Thermal Model

Transient temperature distributions in the panel, \( T(x,z,t) \), are computed using the two-dimensional heat equation (5) to include both chordwise and through-thickness conduction paths. The thermal model includes both a thermal protection system (TPS) and the plate structure. The TPS is modeled as thermal insulation with a high-emissivity upper surface. The boundary condition along the upper surface includes aerodynamic heating, \( Q_{aero}(x,t) \), from (2), and thermal radiation, \( Q_{rad}(x,t) \), given by (6). Thermal radiation is modeled by considering the upper surface to be non-black, diffuse, and enclosed by the environment [32]. An adiabatic boundary condition is applied to the lower surface and edges of the panel.

\[ \rho_m c \frac{\partial T}{\partial t} = k_x \frac{\partial^2 T}{\partial x^2} + k_z \frac{\partial^2 T}{\partial z^2} \]  

(5)

\[ Q_{rad} = \sigma \epsilon (T^4 - T_{env}^4) \]  

(6)

C. Structural Model

The panel structure is shown graphically in Fig. 4 and is modeled using von Kármán plate theory [33], [34] with (7), which includes in-plane thermal force, thermal moment, and unsteady aerodynamic pressure. Chordwise variation of the elastic modulus, \( E(x) \), and the thermal expansion coefficient, \( \alpha(x) \), are included. The panel is supported by immovable, simple supports and only transverse vibrations are considered.

\[ D \frac{\partial^4 w}{\partial x^4} - N_x \frac{\partial^2 w}{\partial x^2} + h \rho_m \frac{\partial^2 w}{\partial t^2} + q_a + \frac{\partial^2 M_T}{\partial x^2} = 0 \]  

(7)

D. Coupled Aerothermoelastic Panel Solution

The panel deformation is computed using Galerkin’s method [34], [35] in conjunction with a fourth order Runge-Kutta time-integration procedure [36]. An explicit finite difference approach [32], [37] is used to solve the heat equation; transient temperatures in the panel are computed at discrete points through the thickness and along the length.

\[ \frac{\partial}{\partial x} (U(x,t)) = 0 \]  

\[ Q_a(x,t) \]  

Fig. 4. Simply supported plate structure.

\[ \frac{\partial}{\partial x} (X(t)) = 0 \]  

\[ Z(\text{ft}) \]  

Fig. 5. Computational grid for the forebody of an air-breathing hypersonic vehicle geometry.
prediction. Over the lower surface, the agreement is slightly better. Specifically, the REM is within 10% of the Baldwin-Lomax prediction, and 25% of the Wilcox \( k - \omega \) prediction. Note that the average percent variation between the Baldwin-Lomax and Wilcox \( k - \omega \) predictions is 21% over the upper surface, and 19% over the lower surface. Therefore, the percent difference between the REM relative to the CFD models is on the same order as the percent variation between the different CFD models. Based on these results, the REM is considered appropriate for the current application. Control design using this model, however, must account for at least 30% uncertainty in the predicted aerodynamic heat flux.

B. Fluid-Thermal-Structural Coupling

In order to assess the degree of fluid-thermal-structural coupling in hypersonic flow, the dynamic stability boundary (aka flutter boundary) of the panel was computed using several different cases listed in Table II. Cases ‘B’ correspond to the neglect of material property degradation in the analysis. Cases ‘B’ and ‘C’ utilize quasi-static fluid-thermal-structural coupling, i.e. the panel deformation is computed without inertial terms in the equation of motion. The temperature distribution is then frozen, and panel stability is determined from a response test of the system using the fully dynamic equations of motion. Cases ‘D’ solve the dynamic equations of motion, therefore panel stability is inherently included in the solution. A ‘1’ corresponds to one-way aerothermal-aeroelastic coupling, while ‘2’ corresponds to two-way coupling. Case ‘D-2’ represents a tightly coupled solution to the complete dynamic equations, thus this case is considered the “truth model.” The ‘D-3’ case represents a solution procedure where the aeroelastic and aerothermal solutions are marched forward in time on separate time scales (several time steps of aeroelastic simulation per one time step of aerothermal simulation). The dynamic response of the panel deformation is averaged and passed to the aerothermal solution to update the aerodynamic heating.

While normal operation of hypersonic vehicles avoid operation near or beyond the onset of flutter, computation of the boundary provides a convenient single metric for assessing the importance of fluid-thermal-structural coupling. The parameters used in the panel analysis is listed in Table III. Temperature dependent material properties of the thermal insulation [45] and the titanium plate [46], [47] are included.

Several important observations related to control-oriented aerothermoelastic modeling can be made from Fig. 7. It is evident by comparing the ‘B’ cases to the ‘C’ and ‘D’ cases that inclusion of material property degradation (e.g. softening of the structure due to heating) is important for aerothermoelastic modeling over extended trajectories. Furthermore, note that there is little difference between the time to onset of panel flutter for the one-way versus two-way coupling solutions for the high Mach number cases. However there are significant differences in time to onset of flutter for the lower Mach numbers considered. This can be attributed with the fact that for lower Mach numbers the panel is farther from dynamic instability. It takes longer operation in hypersonic flow for aerodynamic heating to degrade the panel to the onset of flutter. Thus two-way coupling is an important effect for extended exposure of structures to hypersonic flows, where errors introduced through one-way thermal coupling increase with time. These lower Mach number results are most applicable for control-oriented aerothermoelastic mod-
eling since air-breathing hypersonic vehicles will operate for extended periods of time under exposure to aerodynamic heating. Finally, note that both the quasi-static and dynamic averaged cases produce excellent agreement with the ‘D-2’ case. These approaches to fluid-thermal-structural coupling are 170 and 26 times faster, respectively, than a one-to-one time stepped, dynamic solution for the aerothermoelastic problem. An issue with the quasi-static case, however, is evident for the Mach 8.0 operating condition, where the nonlinear equation solver failed to converge to a solution. Therefore, for cases with linear or moderately nonlinear deformations, the quasi-static fluid-thermal-coupling strategy may be satisfactory. For moderate to high nonlinear deformations, the dynamic averaged approach may be required.

**TABLE II**

**AEROTHERMOELASTIC MODELING CASES.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Coupling Type</th>
<th>Aeroheating Panel Shape</th>
<th>Material Degradation</th>
<th>Aerothermoelastic Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>1-way</td>
<td>Flat</td>
<td>None</td>
<td>Quasi-Static</td>
</tr>
<tr>
<td>B-2</td>
<td>2-way</td>
<td>Inst. Def.</td>
<td>None</td>
<td>Quasi-Static</td>
</tr>
<tr>
<td>C-1</td>
<td>1-way</td>
<td>Flat</td>
<td>$E(T), \alpha(T)$</td>
<td>Quasi-Static</td>
</tr>
<tr>
<td>C-2</td>
<td>2-way</td>
<td>Inst. Def.</td>
<td>$E(T), \alpha(T)$</td>
<td>Quasi-Static</td>
</tr>
<tr>
<td>D-1</td>
<td>1-way</td>
<td>Flat</td>
<td>$E(T), \alpha(T)$</td>
<td>Dynamic</td>
</tr>
<tr>
<td>D-2</td>
<td>2-way</td>
<td>Inst. Def.</td>
<td>$E(T), \alpha(T)$</td>
<td>Dynamic</td>
</tr>
<tr>
<td>D-3</td>
<td>2-way</td>
<td>Avg. Def.</td>
<td>$E(T), \alpha(T)$</td>
<td>Dynamic</td>
</tr>
</tbody>
</table>

**TABLE III**

**AEROTHERMOELASTIC PANEL FLUTTER STUDY PARAMETERS.**

- Altitude: 30 km
- Freestream Mach Number: 8 – 14
- Nondimensional Dynamic Pressure: 73 – 203
- Forebody Surface Inclination: 5.0°
- Transition to Turbulence Upstream of Panel: 1.0 m
- Panel Length: 1.5 m
- Plate Thickness: 5.0 mm
- Initial Panel Temperature: 300 K

**IV. CONCLUSIONS AND FUTURE WORK**

Control-oriented modeling in the hypersonic regime requires coupling aerothermal and aeroelastic systems. The fully-coupled aerothermoelastic model used in this study incorporates the reference enthalpy method for aerodynamic heating. The reference enthalpy method is shown to be in good agreement with 2-D CFD flow analysis at the representative operating condition, and is computationally efficient. Aerothermoelastic simulation illustrate that the effects of material property degradation and two-way thermal coupling is important for control-oriented modeling of hypersonic vehicles. The latter requirement implies significant penalties in computational expense of aerothermoelastic modeling. However two approaches are introduced that significantly reduce this expense, namely quasi-static or dynamic averaged fluid-thermal-structural coupling. Future work will extend the aerothermoelastic modeling approach used here to a three-dimensional vehicle to further explore these effects on a more representative configuration.

**V. ACKNOWLEDGMENTS**

This research is funded in part by NASA award NNX08AB32A with Mr. Peter Ouzts as technical monitor, an AFRL summer research fellowship with Dr. David Doman as technical monitor, and by a National Defense Science and Engineering Graduate Fellowship to Adam Culler.

**REFERENCES**


