Adaptive Control of Sensor Networks for Detection of Percolating Faults

Abhishek Srivastav Asok Ray Shashi Phoha
axs964@psu.edu axr2@psu.edu sxp26@psu.edu

The Pennsylvania State University
University Park, PA 16802

Keywords: Sensor Network; Adaptive control; Graph Theory; Ising Model; Percolating Faults

Abstract—A complex network of interdependent components is susceptible to percolating faults. Sensor networks deployed for real-time detection and monitoring of such systems require adaptive re-distribution of resources for an energy-aware operation. This paper presents a statistical mechanical approach to adaptive self-organization of a sensor network for detection and monitoring of percolating faults. A complex dynamical system of interdependent components (e.g., computer and social network) is represented as an Ising-like model where component states are modeled as spins, and interactions as ferromagnetic couplings. Using a recursive prediction and correction methodology the sensor network is shown to adaptively self-organize to the dynamic environment and real-time detection and monitoring is enabled. The algorithm is validated on a test-bed simulating the operation of a sensor network for detection of percolating faults (e.g., computer viruses, infectious disease, chemical weapons, and pollution) in an interacting multi-component complex system.

1. INTRODUCTION

Real-time situational awareness is of paramount importance for both military and civilian applications for detection (of known and unknown events) and for control to maintain desired performance. The need for real-time situational awareness has generated interest in the area of distributed sensing and control. Recent advances in the technologies of microcomputers and wireless communications have enabled usage of inexpensive and miniaturized sensor nodes [18], [10], [17] that can be densely deployed in both benign and harsh environments as a sensor network for various applications. A sensor network is essentially a collection of miniature platforms, each of which is equipped with sensing, communication, and computing devices. Often used to monitor large and distributed systems composed of interdependent components, sensor networks have found diverse applications such as Structural Health Monitoring, Military Operations in Urban Terrain (MOUT), weather, habitat, and pollution monitoring [13], [5], [23], [1].

A complex system composed of interacting and interdependent components is susceptible to percolating faults such as power blackouts, spread of computer viruses and infectious diseases [15], [6], [14]. Usage of distributed sensor networks for pattern recognition and detection of percolating faults becomes a challenging task because of the need to process a large volume of generated data in real time [19]. Moreover, sensor nodes are often severely constrained for the use of available resources - such as processing power, energy, communication bandwidth etc. Since events such as the growth of anomalies are usually rare and localized events, data from all sensor nodes need not be processed simultaneously at all times. Thus, the network can often be operated at a reduced capacity by using a only fraction of available resources while extracting the necessary information in real time.

Complex systems of interacting dynamic components found in computer networks, social networks, chemistry, and biology have recently been studied using the concepts of statistical mechanics and graph theory [2]. The tools of statistical mechanics have been applied to investigate the ensemble behavior of a large number of interacting units. For instance, representing nodes in a graph as energy levels and its edges as particles occupying it, complex networks have been shown to follow Bose statistics, where certain known characteristics of the network appear as thermodynamically distinct phases of a Bose gas [4]. A detailed review of statistical mechanics of complex networks is reported by Albért and Barabási [2] and, in the context of statistical physics, Strogatz [22] has explored the behavior of interacting dynamical systems in various disciplines.

Ising’s ferromagnetic spin model [11][8] has been traditionally used to study critical phenomena (e.g., spontaneous
magnetization) in various systems. Essentially, it allows to model the collective behavior of an ensemble of interacting agents. Its ability to model local and global influences on constituting units that make a binary choice (e.g. ±1) has been shown to characterize the behavior of systems in diverse disciplines other than statistical mechanics, e.g., finance [7], biology [3], and sociophysics [12].

This paper extends the application of Ising model to the field of sensor networks to enable resource-aware real-time monitoring of a dynamic environment for detection of percolating faults. Tools of statistical mechanics and graph theory have been applied to formulate an algorithm for self-organization and adaptive control for redistribution of available resources in the sensor network. From these perspectives, contributions of the work reported in this paper are: (1) Adaptive control of sensor network for node activity scheduling, self-learning and adaptation (2) Construction of a state-dependent Hamiltonian function to characterize neighborhood interactions and time-dependent external influences (3) Interdisciplinary approach to sensor networks using statistical mechanical concepts (e.g., Boltzman distribution and thermodynamic equilibrium) and (4) Construction of an importance sampling function for probabilistic activation of sensor nodes.

The paper is organized in five sections including the present one. Section 2 presents a brief review of graph theory and Ising Model formulation that form the backbone of the sensor network algorithm. Section 3 formulates the algorithm for self organization of sensor networks based on the principles of statistical mechanics and graph theory. Section 4 presents the simulation results for validation of the algorithm. Section 5 summarizes and concludes the paper with recommendations for future research.

2. Ising Model formulation

Let \( G = (V, E, W) \) be a weighted graph, where \( V = \{v_1, v_2, \ldots, v_N\} \) is the set of individual components of the system under consideration; an edge \((v_i, v_j) \in E\) is a two-element subset of \( V \) representing interdependence between a pair of components \( v_i \) and \( v_j \); and the function \( W : E \to \mathbb{R} \) yields \( W((v_i, v_j)) = w_{ij} \) as the strength of interaction between the components \( v_i \) and \( v_j \). A weighted graph has been used to represent interacting components in a large system as the choice of this framework naturally leads to an Ising-like formulation.

The Ising’s spin model was originally conceived to explain the onset of spontaneous magnetization in ferromagnetic materials. In this model a ferromagnet is considered as a collection of large number of spins placed at the crystal lattice sites. Each spin can be in two states +1 (up) or −1 (down). A nearest neighbor interaction model is assumed where the interaction energy of the system is a function of only nearest neighbor spin configurations.

Every interaction in \( G \), identified as an edge and its weight, is now represented in a nearest neighbor model in the sense of Ising. However, unlike a conventional Ising model, \( G \) can potentially be a directed graph where elements of \( E \) are 2-tuples and interactions between components may not be symmetric. Self loops are assumed to be absent in \( G \) as they are not meaningful in this context.

For the weighted graph \( G \), its weights are used to construct a time-dependent Hamiltonian \( \mathcal{H}^\tau \) as

\[
\mathcal{H}^\tau = - \sum_{<i,j>} w_{ij} \sigma_i \sigma_j - \mathcal{B}^\tau \sum_i \sigma_i \tag{1}
\]

where \(<i,j>\) denotes a pair of nearest neighbor spins; \(w_{ij}\) and \(\mathcal{B}^\tau\) are neighborhood interactions and time-dependent external field, respectively. Each node is assigned a spin \(\sigma_i\) to represent its current state. The spin \(\sigma_i\) of node \(v_i\) is ±1 to represent its state as functional (+1) or failed (−1). In general, the interactions \(w_{ij}\) in Eq. (1) can be time-dependent but they are assumed to be constants here. Every \(w_{ij}\) is taken to be strictly positive; thus, being in the same spin state as its neighbors is energetically favorable for a node. This assumption is representative of a typical multi-component system where malfunctioning neighbors make a node more likely to change its state from functional (+1) to failed (−1) under similar external influences, which is analogous to ferromagnetic influences in an Ising model.

Let \( S = \{S_1, S_2, \ldots, S_N\} \) be a sensor network, where every component \(v_i \in V\) is being monitored by a sensor node \(S_i\) which is represented as a function \(S_i : \mathcal{P} \to \mathbb{R}^n\) that maps the physical space \(\mathcal{P}\) of observable parameters into the measurement space \(\mathbb{R}^n\). The pattern of a sensor data sequence, generated from a component \(v_i \in V\), could be statistically characterized as the state probability vector \(p_i^t\) of a finite state machine, and a (non-negative scalar) distance measure \(\mu_t \triangleq d(p_i^t, q_i^t)\) of \(p_i^t\) from a given reference pattern \(q_i^t\) can be computed [19]. Instead of directly incorporating sensor data sequences, the scalar measures \(\mu_t\) are used for construction of a Hamiltonian in the Ising model to make the computation independent of specific sensor modalities as described below.

The field \(\mathcal{B}^\tau\) in Eq. (1) is representative of external influences and points in the negative direction. Thus enforcing the components of the system to flip to their respective failed (−1) state. The value of \(\mathcal{B}^\tau\) at node \(v_k\) and time \(\tau\) is estimated as:

\[
\mathcal{B}^\tau (k) = - \sum_{i=1}^{N} B_i (\mu_i^\tau, \{\mu_j^\tau\}) \delta_i (k) \quad \tag{2}
\]
where $B_i(\mu_i^v, \{\mu_j^v\})$ is magnitude of the local field at node $v_i$, which is a function of the measure $\mu_i^v$ at node $v_i$ and the set of measures $\{\mu_j^v\}$ of its nearest neighbors $v_i$. The functional form of $B_i$ is taken to be identical for all nodes $v_i$, and $\delta_i(k)$ is the unit impulse function, i.e., $\delta_i(k) = 1$ if $k = i$ and $\delta_i(k) = 0$ if $k \neq i$.

Given the spin states and anomaly measures at a given time instant, it follows from Eq. (2) that self-organization of a sensor network for redistribution of resources reduces to estimation of the probabilities of the possible subsequent state of the system. A statistical mechanical approach that is used to compute the probability of subsequent states of the observed system is given in the next section.

3. Algorithm for Self Organization of Sensor Networks

A statistical mechanical representation of the sensor network is formulated as follows. The thermodynamic state $I$ of the system represented by the graph $G$ can be given by the spin sequence $(\sigma_1, \sigma_2, \ldots, \sigma_N)$; the probability $P_I$ that the system is in this state is given by the Gibbs Distribution [11]:

$$P_I(\sigma_1, \sigma_2, \ldots, \sigma_N) = \frac{1}{Z_N} \exp(-\beta E_I)$$

where the energy $E_I$ of the thermodynamic state $I$ is derived from the Hamiltonian in Eq. (1); the parameter $\beta$ is proportional to the inverse temperature and $Z_N$ is the associated partition function defined as:

$$Z_N = \sum_I \exp(-\beta E_I)$$

where the sum runs over all possible spin sequences or thermodynamic states $I$. The partition function may not be computationally tractable, especially, for systems with an irregular lattice and a large number of interacting nodes. The situation is simplified by the following assumptions [16]:

- *Markov dynamics*, i.e., the future state depends only on the present state.
- *Quasi-static equilibria at all time instants*, i.e., the probability of state transitions corresponding to large changes in energy is assumed to be zero and the system follows the single-spin dynamics.
- *Detailed balance*: Let $P_I$ be the probability of being in thermodynamic state $I$ and $p_{ij}$ be the probability of transition from the thermodynamic state $I$ to state $J$. Then, detailed balance [16] implies that

$$\left(P_I p_{ij} = p_{ij} p_{ji} \right) \Rightarrow \left(P_I = \frac{p_{ij}}{p_{ji}} \right)$$

and $p_{ij} = p_{ji} \exp(-\beta(E_j - E_i))$ follows from Eq. (4).

Equation (5) eliminates the partition function $Z_N$ and tells how the ratios of transition probabilities $p_{ij}$ and $p_{ji}$ should behave but it does not provide the solution for $p_{ij}$. For this purpose, $p_{ij}$ is derived from the so-called heat-bath dynamics [16] as:

$$p_{ij} = \frac{\exp\left(-\beta(E_j - E_i)\right)}{1 + \exp\left(-\beta(E_j - E_i)\right)} > 0$$

Irreducibility of the state transition matrix $[p_{ij}]$ is ensured because of strict positivity of each $p_{ij}$.

The change in energy $\Delta E_{flip}$ due to a single-spin-flip (from +1 to −1 or vice versa) at a node $v_i$ is given by:

$$\Delta E_{flip} = 2 \sum_{i,j} w_{ij} \sigma_i \sigma_j + 2 \beta \sigma_i$$

where $i, j$ denotes the nearest neighbor of node $i$. The flip probability $p_{flip}^i$ of node $v_i$ is obtained by using Eq. (6) as:

$$p_{flip}^i = \frac{\exp(-\beta \Delta E_{flip}^i)/[1 + \exp(-\beta \Delta E_{flip}^i)]}$$

Thus for a node $v_i$, $p_{flip}^i$ is the likelihood that it would change its state from $\sigma_i \rightarrow -\sigma_i$ given the states of its neighbors and magnitude of external influence. The expected probability $\hat{p}_{flip}$ that a randomly chosen node would flip and generate information in the next sampling instant is obtained as:

$$\hat{p}_{flip} = \frac{1}{N} \sum_{i=1}^{N} p_{flip}^i$$

A high (low) value of $\hat{p}_{flip}$ indicates, irrespective of the chosen node, a higher likelihood of new information being available (unavailable) when a randomly chosen sensor node is set to be active. The number $N_s$ of sensors that should be activated in the next sampling instant, out of a total of $N$ sensors, is therefore a monotonically increasing function of $\hat{p}_{flip}$. The choice of a piecewise linear function for $N_s$ yields:

$$N_s = \min \left( M, \max \left( (1 + \epsilon) N \hat{p}_{flip}, m \right) \right)$$

where $0 < \epsilon \ll 1$ is a detection threshold; $m$ is the minimum number of sensors that must remain active at all time instants; and $M \leq N$ is the maximum capacity of the sensor network. Given the flip probabilities $p_{flip}^i$ for each node and $N_s \leq N$ computed by Eq. (10), the optimal solution for choosing the nodes that should be activated would yield $N_s$ sensor nodes with the highest probability to be active at any instant. This deterministic method where the most probable $N_s$ sensor nodes are activated would be blind to any activities at the nodes not chosen by the method. Thus, a
suboptimal approach is followed where the sensor nodes are not chosen deterministically but $N_s$ nodes are sampled from an importance sampling function. This ensures that there is a finite probability of activating every sensor node at all time instants, although the sampling probability is higher for nodes with high expected activity and vice versa.

The importance sampling function is defined as:

$$p_i^m = \frac{p_i^{\text{flip}}}{\sum_{i=1}^{N_s} p_i^{\text{flip}}}$$

(11)

to provide larger sampling weights to nodes with higher expected activity in the next instant. In essence, $N_s$ nodes are drawn from this distribution at each instant and are scheduled to be activated in the next sampling instant. The collected data are used to modify the flip probability $p_i^{\text{flip}}$ of the node $v_i$ and the importance sampling function $p_i^m$ is re-calculated. The state of a node is changed from $+1$ to $-1$ when it is detected to have reached its respective failed state. The failed state of a node is signalled when the normalized measure $\mu_i$ computed for that node saturates and reaches a value of unity [21]. Unless updated by the sensor network, $\mu_i$ and spin $\sigma_i$ are maintained at their last recorded values. Thus at all times the network works with partial information gathered by a fraction of sensor nodes.

Following a recursive procedure, collected information is used to predict the likelihood of new activities. This estimate is then used to reorganize the network to adapt to the changing environment and collect new data, which is used to modify previously made predictions. This approach is similar to recursive filters (e.g., Kalman filter) where predictions are made using a model and the measurement history, which are then corrected based on new data.

It is noteworthy that the model predicts a non-zero probability of a $-1 \rightarrow +1$ flip, which corresponds to a less likely event of healing or on-line repair. Due to the field pointing in the negative direction, the probability of a $-1 \rightarrow +1$ flip is very small. Thus, the sensor network has a non-zero probability of activating a sensor for a failed component, but it may do so with a very small probability.

The parameter $\beta$ serves the role of bias control for $p_i^m$ (see Eqs. (8) and (11)). Low (high) values of $\beta$ cause $p_i^m$ to move towards uniform distribution ($\delta$-distribution) independent of $\Delta E_{\text{flip}}$.

It must be noted that irrespective of the state of sensors (i.e., active or inactive), the underlying system evolves in time, whereas a limited number, $N_s \leq N$, of sensor nodes are activated at each sampling instant. From this perspective, the efficiency $\eta$ is defined as:

$$\eta = \frac{\sum_{i=1}^{N_s} \mu_i}{\sum_{i=1}^{N_s} \mu_i} \bigg/ \frac{\sum_{i=1}^{N} \mu_i}{\sum_{i=1}^{N} \mu_i}$$

(12)

where the subscripts ‘$m$’ and ‘$a$’ imply measured and actual values, respectively.

Unlike a typical computational statistical mechanical problem, the goal here is not to compute macroscopic parameters but to estimate the probabilities of future thermodynamic states of the system when a particular state is sensed at time $\tau$. In this respect the following two control issues are addressed for the sensor network at all sampling instants: (1) the number of nodes to activate and (2) their distribution in the sensor network.

**Algorithm 1:** Algorithm for self-organization of sensor network for detection of pervasive faults

1. **while** (1) **do**
   1. 1: Collect data from active sensor nodes.
   2. 2: Compute local measure $\mu_i^\tau$ for active nodes at current time $\tau$ and update their spin states.
   3. 3: Compute the local field $B_i^\tau$ (Eq. 13)
   4. 4: Compute change in energy $\Delta E$ (Eq. 7) and calculate new $p_i$ (Eq. 6) for each node $i$. 
   5. 5: Update importance function $p^\tau$ (Eq. 11) and calculate $N_s$ (Eq. 10)
   6. 6: Draw $N_s$ samples from $p^\tau$ and to activate sensors nodes in the next time step.

4. **SIMULATION RESULTS AND DISCUSSION**

A simulation test bed has been constructed to validate the proposed methodology. The test bed consists of a 2-dimensional $(25 \times 25)$ array of sensor nodes (i.e., a total of 625 nodes) with four nearest neighbors in the orthogonal directions. All nodes begin with a functional state (i.e., $\sigma_i = +1$) and a small number ($= 5$) of randomly chosen nodes are injected (seeded) with faults that slowly continue to grow until the node reaches the failed state (normalized anomaly measure = 1). Nodes with a critical value of normalized anomaly measure ($\geq 0.8$ here) infect their neighbors with a probability, called transmission probability, equal to the fraction of the total number of failed nodes present in the system at that instant of time $\tau$. Thus, the transmission probability increases as the fault percolates through the system.

The simulation scenario resembles the Susceptible Exposed Infected Removed (SEIR) model of epidemic spread [9] (e.g. infectious diseases and computer viruses).

Each node $v_i$ in the simulation test bed is monitored by a sensor whose $\tau$-dependent data are compressed into a scalar measure $\mu_i^\tau$, and the local field $B_i$ for each node $v_i$ is
identically modeled as:

\[ B(\mu_i^\tau, \{\mu_i^\tau\}_j) = B_0 \left( \mu_i^\tau + \sum_{i_j} \mu_i^\tau_j \exp(-\alpha|i - i_j|) \right) \]

(13)

where \( |i - i_j| = 1 \) for nearest neighbor interactions.

The interactions \( w_{ij} \) are chosen to be 0.8 for each neighbor pair \( (v_i, v_j) \); and the parameters \( N, M, m, \epsilon, B_0, \alpha, \) and \( \beta \) set at 625, 320, 20, 0.001, 5, 1, and 0.333, respectively, for the simulation experiments.

The plots of algorithm efficiency \( \eta \) for three typical simulation runs are shown in Fig. 1. At the beginning, \( \eta \) has a relatively low trend when the sensor network learns the fault pattern; then \( \eta \) approaches unity as the network dynamically adapts itself to the fault pattern. Figure 2 shows the fraction of active sensor nodes at any given time. The number of active sensor nodes dynamically changes and is always a small fraction (e.g., < 25%) of the total number of sensor nodes in these tests. Fig. 1 and 2 show the ability of the sensor network to self-organize to adapt to the dynamic environment. The proposed control algorithm enables real-time detection and monitoring working with a small fraction of sensor nodes to conserve valuable resources.

Figure 3 displays a snapshot of the operational environment. It is seen that a large number of sensor nodes, neighboring a node detected to have failed \( \sigma_i = -1 \) (marked as solid blue dots), were activated. Network resources were directed more towards functional nodes to activate theirs sensors than for nodes detected as failed. As the fault pattern evolves in time, the sensor network dynamically adapts to the changes by making corrections to its predicted estimates while using partial information at all times.

5. Summary and Conclusions

This paper introduces a concept of adaptive self-organization of sensor networks by using weighted graphs and an Ising-like model based on the principles of Statistical Mechanics. Given past measurements, probabilities of future states are computed to construct an importance sampling function to probabilistically activate a small fraction of sensor nodes. Numerical simulation has been conducted on
a test bed of interacting multi-component systems to demonstrate the adaptive self-organizing capability of the proposed control methodology for sensor networks. Simulation results show that the proposed algorithm is capable of detecting percolating faults in a resource-aware manner.

Further research is recommended to utilize information-theoretic concepts, such as mutual information to determine the strength of interactions $w_{ij}$ between interconnected neighbors and transfer entropy [20], to ascertain direction of information flow. The use of multi-state spin models such as the Potts model for systems where a binary state assumption of Ising is not valid needs to be investigated. Also, further research needs to be done for construction of the Hamiltonian in Eq. (1) directly from the patterns of state probability vectors [19] rather than compressing the sensed information into a scalar measure $\mu$.

REFERENCES