Modeling and compensation of asymmetric hysteresis in a piezo actuated metrological AFM

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Abstract—The manipulation of samples in atomic force microscopes (AFMs) is often performed using piezoelectric actuators. In this paper, a metrological AFM with a 3 degree-of-freedom (DOF) stage driven by piezo-stack actuators is considered. The piezo actuators exhibit hysteresis, which can change the system dynamics and/or acts as a non-linear disturbance on the system. This deteriorates the performance of the AFM. The 3 DOF stage exhibits asymmetric hysteresis, which is modeled by extending the Coleman-Hodgdon model. The asymmetry includes a scan range dependent offset and an asymmetry between the trace and retrace directions. Non-linear multi-variable optimization is employed to derive the optimal generic model for all scan ranges. The proposed extended Coleman-Hodgdon model describes the asymmetric hysteresis over all scan ranges with an accuracy of 97%. Based on the model, a feedforward compensation method is developed. Experiments on the metrological AFM show that the application of the hysteresis feedforward largely improves the scanning accuracy.

I. INTRODUCTION

Atomic force microscopes (AFMs) are a special type of scanning microscopes. AFMs make an image of a sample by scanning the surface with a very sharp probe, which has a tip with a radius of one atom at the end. The AFM was invented in 1986 by Binning, Quate and Gerber [1]. The sample to be investigated can either be moved under the probe (scanning sample mode) or the probe can be moved over the sample (scanning tip mode). The sample causes the cantilever, to which the tip is attached, to deflect. The deflection can be used to obtain the height information of the sample.

The positioning of the sample under the tip in various degrees-of-freedom (DOFs) is in most cases performed using piezoelectric actuators [2]. Different types of scanners have been developed, such as tube piezo actuators [3], resonant scanners [4], and rigid scanners [5].

In this paper, a metrological AFM is considered. The metrological AFM is used to calibrate transfer standards for commercial AFMs. In contrast to commercial AFMs, the accuracy of the (position and height) measurements is much more important than the scanning speed. Furthermore, the measurements have to be traceable to the standard of length. The manipulation of the sample under the tip is performed using a 3 DOF rigid scanner, driven by three piezoelectric stack actuators. The piezoelectric actuators exhibit non-linear behavior, in particular hysteresis and creep, which limit the positioning accuracy. The increasing interest in AFMs for nano-applications requires a higher precision. Active compensation of the hysteresis can improve the performance of the AFM and the accuracy of the generated images.

The 3 DOF rigid scanner used here has an asymmetric design, which results in asymmetric hysteresis in the system [6]. In literature, several models and methods are proposed for the identification and compensation of (asymmetric) hysteresis. The Preisach model is used in [7] to describe asymmetric hysteresis with 80 parameters, which makes it difficult and time consuming to identify a single overall model for the complete operating range of the AFM. In [8], separate Preisach models are identified for various scan ranges. This requires a lot of models to be identified, one for each possible scan range. Consequently, for the compensation the inversion of and switching between many different models is required. A Bouc-Wen model with nine parameters is identified in [9] using genetic algorithms. However, the asymmetry in the hysteresis is only limited to the regions near zero velocity of the stage. A Bouc-Wen model for symmetric hysteresis combined with a PI feedback compensation for the asymmetric part in [10] does not fully utilize the a-priori knowledge of the asymmetry. In [6], the hysteresis curve is split into six different parts, each described by a generalized Bouc-Wen model. This requires switching between different models at a non-zero velocity of the stage, which can cause discontinuities in the feedforward compensation in the imaging region. A Coleman-Hodgdon model with only five parameters is developed in [11] for the modeling and compensation of symmetric hysteresis in a scanning microscope. In [12], the Coleman-Hodgdon model is extended to incorporate variations in the offset for various scan ranges. However, a new model has to be identified for each scan range.

In this paper, we propose an extended Coleman-Hodgdon model to describe the asymmetric hysteresis in the system. The hysteresis identification and compensation is performed for one DOF only, but the approach is also applicable to the multi DOF case. Since the effect of creep is mainly present at low frequencies, the creep is assumed to be...
compensated for by feedback control and not incorporated in the model. The model includes a scan range dependent offset and asymmetry between the trace and retrace scan directions. Only 16 parameters are required to describe the hysteresis for all scan ranges.

The contributions of this paper are twofold. Firstly, an extended Coleman-Hodgdon model is derived to incorporate the asymmetry present in the hysteresis of the 3 DOF stage of the metrological AFM. Identification of the model parameters is performed by a least-squares data-fitting optimization using experimental data. Secondly, the proposed model is used to develop a feedforward compensation method of the asymmetric hysteresis. The combination of the feedforward compensation with a stabilizing feedback controller allows the sample to be positioned with a tracking error within the sensor bound of 5 nm for scans with a velocity up to 7.2 µm/s.

This paper is organized as follows. The metrological AFM is discussed in Section II together with the nature of the asymmetric hysteresis. An extension of the Coleman-Hodgdon model to describe the asymmetry in the hysteresis and the model identification are treated in Section III. The design of the feedforward compensation method is discussed in Section IV. The results of open-loop and closed-loop experiments on the metrological AFM with the hysteresis feedforward are presented in Section V. Finally, conclusions are drawn in Section VI.

II. METROLOGICAL AFM

The metrological AFM, shown in Fig. 1, consists of a Topometrix AFM head, a piezo-stack driven 3 DOF stage and a ZYGO interferometer to measure the stage position in all DOFs. The deflection of the cantilever in the AFM head is measured by an optical sensor consisting of a laser and a photodiode. The 3 DOF PI P517.3CL stage [13] is a rigid stage containing three piezo-stack actuators, which can move the stage through a flexure mechanism in a range of 100 µm in x and y directions and in a range of 20 µm in z direction. The resolution and noise bounds of the different sensors are given in Table I.

A schematic representation of the metrological AFM is shown in Fig. 2. For clarity, the flexure mechanisms between the piezo-stack actuators and the stage are not shown. The sample can be moved under the tip by applying a voltage \( V_i \) (V), \( i \in \{x, y\} \). The position of the stage in x (µm) and y (µm) directions is measured using the lasers in corresponding directions. The bending of the cantilever in z (µm) direction is measured using the laser and photodetector. In z direction, the deflection of the cantilever is controlled to a constant deflection by applying a voltage \( V_z \) (V) to the piezo-stack actuator. The constant deflection of the cantilever has the advantage that the orientation of the tip compared to the sample topography remains constant. The mirrors and lasers of the interferometer are aligned such that the laserspots in all DOFs are aligned with the tip of the cantilever, thus eliminating Abbe errors. The height of the sample is measured directly using the interferometer in z direction.

The piezoelectric actuators in the 3 DOF stage suffer from hysteresis, which can result in a change of the system dynamics and/or act as non-linear disturbances on the system. Hysteresis can contribute to loss of robustness, performance degradation or even instability in feedback controlled piezoelectric devices [14]. For scans in one DOF, the voltage to the piezo-actuator varies as \( V \in [V_m, V_M] \) (V), where \( V_m \) (V) and \( V_M \) (V) are the minimum and maximum voltage respectively. For symmetric scans in terms of input voltage, which are generally performed, \( V_M = -V_m \). The voltage range is defined as \( V_R = V_M - V_m \) (V). Furthermore, let the trace direction be denoted by \( V = \frac{d}{dt}(V) \geq 0 \) and the retrace direction by \( V < 0 \).

The measured hysteresis of the metrological AFM is shown in Fig. 3 for separate symmetric scans in x direction and voltage ranges \( V_R \in \{2, 10, 20, 40, 60, 80, 100\} \) V while controlling the stage in y and z direction to a constant value.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Resolution</th>
<th>Noise bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZYGO x</td>
<td>0.15 nm</td>
<td>±5 nm</td>
</tr>
<tr>
<td>ZYGO y</td>
<td>0.15 nm</td>
<td>±4 nm</td>
</tr>
<tr>
<td>ZYGO z</td>
<td>0.15 nm</td>
<td>±2 nm</td>
</tr>
<tr>
<td>Head z</td>
<td>0.05 nm</td>
<td>±0.15 nm</td>
</tr>
</tbody>
</table>

Table I: RESOLUTION AND NOISE BOUND OF THE DIFFERENT SENSORS.
The general solution of (3) can be formulated as
\[ x(V) = bV - s(V)\alpha^{-1}(b - u) \left( 1 - C_1 e^{-s(V)\alpha V} \right). \] (4)

In order to have a closed hysteresis loop, consider the following boundary conditions
\[ x_t(V_m) = x_t(V_m), \] (5)
\[ x_t(V_M) = x_t(V_M). \] (6)

With (5) and (6), the constant \( C_1 \) of (4) becomes
\[ C_1 = 2(e^{-s(V)\alpha V_m} + e^{-s(V)\alpha V_M})^{-1}. \]

For the AFM of Fig. 1, the sensitivity of the hysteresis curve, \( bV \) in (4), is not linear as function of \( V_R \) [16]. Expansion of the model with an exponential asymptotic sensitivity gives
\[ x(V) = \left( b - ce^{-s(V)dV_R} \right) V - s(V)\alpha^{-1}(b - u) \left( 1 - C_1 e^{-s(V)\alpha V} \right). \] (7)

The hysteresis model (7) is centered around the origin with respect to the position, i.e. \( x(V_m) = -x(V_M) \). To incorporate the voltage range dependent offset and the asymmetry between the trace and retrace directions, we propose an extended Coleman-Hodgdon model as
\[ x(V) = \tilde{c}(V_R) + \left( \tilde{b} - \tilde{c}e^{-s(V)dV_R} \right) V - s(V)\tilde{\alpha}^{-1}(b - u) \left( 1 - \tilde{C}_1 \tilde{\alpha} e^{-s(V)\tilde{\alpha} V} \right), \] (8)
where the tilde (\( \tilde{\cdot} \)) denotes scan direction dependent constant parameters, which are for the trace direction denoted with the subscript \( t \) and for the retrace direction with the subscript \( r \). The scan direction dependent parameter \( \tilde{b} \) is defined as
\[ \tilde{b} = \frac{1}{2} [s(V) + 1] b_t - \frac{1}{2} [s(V) - 1] b_r. \] (9)

The other scan direction dependent parameters are defined in a similar manner. Based on Fig. 3, the offset \( \tilde{c}(V_R) \) is chosen as
\[ \tilde{c}(V_R) = \tilde{f}V_R^2 + \tilde{g}V_R + \tilde{h}. \] (10)

The influence of the different model parameters on the hysteresis curve is shown in Fig. 4. The positions at \( V = 0 \) equal \( x(0) = \tilde{c}(V_R) - s(V)\tilde{\alpha}^{-1}(b - u)(1 - \tilde{C}_1) \), dependent on the scan direction as defined in (2). At the end of a scan in each direction the hysteresis curve converges via an exponential decay of \( e^{-s(V)\tilde{\alpha} V} \) to a straight line \( \tilde{e} + \tilde{q}_1 V - \tilde{q}_0 \), where \( q_1 = (\tilde{b} - \tilde{c}e^{-s(V)dV_R}) \) and \( q_0 = s(V)\tilde{\alpha}^{-1}(b - u) \).

B. Identification

The parameters of the asymmetric hysteresis model described by (8) and (10) are identified using non-linear least-squares data-fitting. The optimal parameters \( p = [\tilde{b}, \tilde{c}, \tilde{d}, \tilde{u}, \tilde{\alpha}, \tilde{f}, \tilde{g}, \tilde{h}] \) are determined using the measured first-order reversal hysteresis data of Fig. 3 and by optimizing
\[ \min_{p} F(p), \]
The hysteresis feedforward is tested by means of experiments on the metrological AFM in both open and closed loop manners.

Fig. 5. Errors between the modeled and measured hysteresis of the trace (light grey) and retrace (dark grey) directions for various voltage ranges $V_R \in \{2, 10, 20, 40, 60, 80, 100\}$ V.

Fig. 4. Schematic overview of the influence of the model parameters on the hysteresis curve.

The extended Coleman-Hodgdon model of (8) describes the asymmetric hysteresis of the metrological AFM for all voltage ranges $V_R \in \{2, 10, 20, 40, 60, 80, 100\}$ V.

The selection of the scan variables according to the scan direction of (2) changes for the feedforward compensation models is performed at standstill of the stage, i.e. outside the imaging region.

Since the hysteresis is asymmetric with respect to the trace and retrace direction, the hysteresis feedforward consists of two parts, one for each direction. The switch between the two parts $V_t(r_x)$ and $V_f(r_x)$ depends on the direction of the reference position, i.e. on the sign of the reference velocity. For increasing reference position ($r_x \geq 0$), the feedforward of the trace part $V_t(r_x)$ is used and for decreasing reference position ($r_x < 0$) the retrace hysteresis feedforward part $V_f(r_x)$ is used. The switch between the two hysteresis feedforward models is performed at standstill of the stage, i.e. outside the imaging region.

The selection of the scan variables according to the scan direction of (2) changes for the feedforward compensation depending on the reference position $r_x$ as

$$\tilde{b}_{FF} = \frac{1}{2} [s(\tilde{r}_x) + 1] b_t - \frac{1}{2} [s(\tilde{r}_x) - 1] b_r.$$

**V. RESULTS**

The hysteresis feedforward is tested by means of experiments on the metrological AFM in both open and closed loop manners.
loop. In this section, the results of the experiments for a scan in \( x \) direction are presented. The reference position \( r_x \) is a sawtooth shaped signal over a range of 36 \( \mu \text{m} \) with a velocity of 7.2 \( \mu \text{m/s} \). At the start of the experiment, the voltage equals zero.

### A. Open loop

For the open loop experiment, the resulting voltage as function of the reference position \( V(r_x) \) is applied directly to the piezo-stack actuator in \( x \) direction. Fig. 6 shows the reference position \( r_x \), the measured stage position in \( x \) direction, the error \( e_x = r_x - x \), and the input voltage \( V(r_x) \). The resulting voltage of the feedforward (11) also has an offset and asymmetry in order to obtain the desired symmetric stage movement. The stage position \( x \) closely matches the reference position \( r_x \), with a maximum absolute error \( \max(|e_x|) = \max(|r_x - x|) = 0.2941 \, \mu\text{m} \).

### B. Scanning motion

The combination of the hysteresis feedforward with a feedback controller can further improve the performance of the metrological AFM. A schematic overview of the implementation of the hysteresis feedforward in combination with a feedback controller is shown in Fig. 7 for the \( x \) direction.

Although the metrological AFM is inherently a MIMO system, from the relative gain array (RGA) it can be concluded that for feedback control at relatively low bandwidths the system can be considered as three separate SISO systems [17]. Therefore, also the hysteresis is assumed to be decoupled between the 3 DOFs. In this paper, we use the definition of bandwidth \( f_{\text{BW}} \) (Hz) as the crossover frequency of the loop gain \( L_{xx}(f) = P_{xx}(f)C_{xx}(f) \). The feedback controller \( C_{xx} \) is a stabilizing PI controller designed employing loopshaping techniques as

\[
C_{xx}(s) = k \frac{1}{s} \frac{2\pi f_{\text{LP}}}{s + 2\pi f_{\text{LP}}},
\]

where \( k = 0.0779 \) denotes the controller gain and \( f_{\text{LP}} = 50 \) Hz denotes the frequency where the integrating action ends. The feedback controlled system has a bandwidth \( f_{\text{BW}} = 7.87 \) Hz, a phase margin of 78.4 deg and an amplitude margin of 19.6 dB. The position and tracking error of an experiment with only feedback control, i.e. without the hysteresis feedforward, are shown in Fig. 8 by the light-grey line. The maximum absolute tracking error equals \( \max(|e_x|) = 160.9 \, \mu\text{m} \).

Since the piezo-stack actuators act as position actuators, alternatively a simple position feedforward can be used to already improve the performance of the stage [17]. The control input of the position feedforward can be added to the output of the feedback controller \( u_x \), resulting in a new input to the system \( u_x^* \) as

\[
u_x^*(t) = u_x(t) + F_x r_x(t),
\]
where $F_x$ is the feedforward gain. The results for a closed-loop experiment with a position feedforward $F_x = 11 \text{ V}/\mu\text{m}$ are shown in Fig. 8 by the dark-grey line. The position feedforward reduces the tracking error to $\max(|e_x|) = 86.11 \text{ nm}$.

Finally, the results of the experiment with feedback control and the hysteresis feedforward of (11) are shown in Fig. 8 by the black line. Compared to the position feedforward the hysteresis feedforward reduces the tracking error even further. At the turnaround point, the tracking error shows a momentary peak due to the discontinuity in the hysteresis feedforward compensation at this point. This is caused by the switch between the two models. However, the tracking error reduces very quickly to the noise bound of $\pm 5 \text{ nm}$ with the hysteresis feedforward.

Fig. 8 also shows the square root of the cumulative power spectral densities (PSDs) $\sqrt{C_{XX}(f)}$ of the tracking errors for the various experiments. For frequencies $f \to \infty$, the cumulative PSD of the error converges to the squared root-mean-square (rms) value of the error. The rms values of the errors equal for the experiment without feedforward $\text{rms}(e_{\text{no FF}}) = 99.57 \text{ nm}$, with the position feedforward $\text{rms}(e_{\text{pos FF}}) = 19.34 \text{ nm}$ and with the hysteresis feedforward $\text{rms}(e_{\text{hyst FF}}) = 11.08 \text{ nm}$. The additional hysteresis feedforward improves the tracking performance compared to the feedback only case by 89% and compared to the case with feedback control and position feedforward by 43%.

C. Discussion

Increasing the feedforward gain $F_x$ does not further reduce the tracking error. The improvement of the hysteresis compensation is mainly due to the difference in the direction of the curve at the turnaround points compared to a position feedforward, i.e. to a straight line through the origin.

The discontinuity in the hysteresis model results in a small step in the feedforward compensation. The discontinuity can be reduced by adding constraints to the optimization procedure or by rewriting the optimization to close the retrace directions and of an offset that is dependent on the scan range.

The voltage range $V_R$ is proportional to the position range through the static piezo extension factor. The hysteresis feedforward (11) can be used for arbitrary setpoint profiles $x_s$ by replacement of the voltage range dependency in (11) by a setpoint dependency using

$$V_R = \gamma x_s,$$

where $\gamma$ (m/V) is the piezoelectric extension coefficient.

VI. CONCLUSIONS

The asymmetric design of the 3 DOF stage of the metrological AFM results in asymmetric hysteresis. The asymmetry consists of a different behavior between the trace and retrace directions and of an offset that is dependent on the scan range.

An extended Coleman-Hodgdon model is proposed to model the asymmetric hysteresis in one DOF. Separate models are identified for the trace and retrace directions, resulting in a generic model with a total of 16 parameters. Non-linear optimization yields the model parameters based on measured data. The generic extended model describes the hysteresis of all scan ranges with an accuracy of 97%.

A feedforward hysteresis compensation method is developed using inverse models. The switch between the models is performed at standstill of the stage outside the imaging region. The application of the hysteresis feedforward improves the tracking performance by 89% compared to using only feedback control and by 43% compared to using feedback control and a position feedforward. The method is also applicable to other DOFs.

With the presented control method, the AFM can perform scans with a velocity up to 7.2 $\mu$m/s and a tracking error within the sensor bound of $\pm 5 \text{ nm}$ in the imaging region, except for the turnaround points.

REFERENCES