Abstract—This paper investigates the problem of $H_\infty$ static output feedback (SOF) control for discrete-time switched linear systems with average dwell time switching. By using the multiple Lyapunov function technique, a switched SOF controller is designed such that the closed-loop system is exponentially stable and achieves a weighted $L_2$-gain. Sufficient conditions for SOF control are derived and formulated in terms of linear matrix inequalities (LMIs). The minimal average dwell time and the corresponding SOF controller are obtained from the LMI conditions for a given system decay degree. Additionally, based on Finsler’s lemma, two sets of slack variables with special structure are introduced to provide extra freedom in the LMI optimization problem, which leads to reduction of the conservatism and improvement of the performance. A numerical example is given to illustrate the effectiveness of the proposed method.

I. INTRODUCTION

As an important class of hybrid systems, switched systems consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems. In recent years, switched systems have received a great deal of attention, see [1]-[5] and references therein. The motivation for studying switched systems comes partly from the fact that switched systems and switched multi-controller systems have numerous applications in control of mechanical systems, process control, automotive industry, power systems, aircraft and traffic control, and many other fields. The problems encountered in switched systems can be classified into three categories [1]. The first one is to find conditions which guarantee that switched systems are asymptotically stable under any switching signal. The second one is to construct a switching signal that makes switched systems asymptotically stable. And the third one, which is of interest in this paper, is to identify certain useful classes of switching signals for which switched systems are asymptotically stable. In the study of switched systems, several approaches have been used such as multiple Lyapunov function approach [6]-[9], dwell time (average dwell time) approach [11]-[15], switched quadratic Lyapunov function approach [23] and so on. Among them, the multiple Lyapunov function technique which was proposed in [6] and later generalized in [7]-[9], has proved to be a powerful and effective tool for stability analysis and synthesis of switched systems. Meanwhile, switched systems with dwell time (or average dwell time) are also called slowly switched systems. And the average dwell time approach is recognized to be more flexible and efficient in stability analysis of switched systems [11][13].

It is also well-known that SOF control is very useful and more realistic, since it can be easily implemented with low cost. The problem has been extensively studied in the past decades and for the SOF control problem of linear systems, there are various approaches to deal with it, see for example [16]-[22] and references therein. The problem of SOF control for discrete-time switched linear systems under arbitrary switching has been studied in [23]-[25] and sufficient existence conditions are obtained in terms of LMIs via the switched quadratic Lyapunov function approach. However, the switched quadratic Lyapunov function approach is not suitable to analyze the slowly switched systems (i.e., switched systems with average dwell time switching) due to the stricter requirements on the Lyapunov values at each switching time. Therefore, the existing methods in [23]-[25] cannot be applied to design SOF controllers for slowly switched linear systems. To the best of our knowledge, few results are available in the open literature to solve this problem.

In this paper, we investigate the problem of $H_\infty$ SOF control for discrete-time switched linear systems with average dwell time switching. By using the multiple Lyapunov function technique combined with average dwell time approach, a switched SOF controller is designed such that the closed-loop switched system is exponentially stable and achieves a weighted $L_2$-gain. Sufficient conditions for SOF control are derived and formulated in terms of LMIs. And consequently the minimal average dwell time and the corresponding SOF controller gains are obtained from the LMI conditions for a given decay degree. In addition, by Finsler’s lemma, two sets of slack variables with special structure are introduced to provide extra freedom in the LMI optimization problem, which lead to reducing the conservatism and improving the performance. A numerical example is given to illustrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section 2 gives preliminaries and the problem statement. Section 3 is
the main result of the paper. First, several essential lemmas are given. Then, based on these lemmas, an SOF controller and the minimal average dwell time are obtained in terms of LMIs. Section 4 gives a numerical example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

Notations: We use standard notations throughout this paper. $M^T$ is the transpose of the matrix $M$. $M > 0 (M < 0)$ means that $M$ is positive definite (negative definite). The symbol $*$ will be used in some matrix expressions to induce a symmetric structure. The Hermitian part of a square matrix $M$ is denoted by $\text{He}(M) := M + M^T$. $\ell^2$ is the Lebesgue space consisting of all discrete-time vector-valued functions that are square-summable over $[0, 1, 2, \ldots, \infty)$. The $\ell_2$-norm of a causal vector signal $x(t)$ with bounded-energy is $\|x(k)\|_{2} = (\sum_{k=0}^{\infty} \|x(k)\|^2)^{1/2}$. $\mathbb{N}$ represents the set of nonnegative integers.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following discrete-time switched linear system

$$
x(k + 1) = A_i x(k) + B_i u(k) + B_i^w w(k)
$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input, $w(k) \in \mathbb{R}^m$ is the disturbance input which belongs to $\ell_2[0, \infty)$, $y(k) \in \mathbb{R}^r$ is the measurement, and $z(k) \in \mathbb{R}^q$ is the controlled output. $\sigma(k) : [0, \infty) \rightarrow I = \{1, \ldots, N\}$ is the switching signal which is assumed to be a piecewise continuous function depending on time or state or both. $N > 1$ is the number of subsystems. The $i$th subsystem is denoted by constant matrices $A_i, B_i, B_i^w, C_i^r, D_i, D_i^w, C_i$ with the appropriate dimensions. For the switching time sequence $k_0 < k_1 < k_2 < \ldots$ of switching signal $\sigma$, the holding time $[k_i, k_{i+1})$ is called the dwell time of the currently engaged subsystem, where $i \in \mathbb{N}$.

Without loss of generality, we assume that $C_i, i \in I$ are of full row rank, then there exist nonsingular transformation matrices $T_i$ such that

$$
C_i T_i = \begin{bmatrix} I & 0 \end{bmatrix}
$$

Note that for given $C_i$, the corresponding $T_i$ are generally not unique. Special $T_i$ can be obtained as follows:

$$
T_i = \begin{bmatrix} C_i^T (C_i C_i^T)^{-1} & C_i^\perp \end{bmatrix}
$$

where $C_i^\perp$ denotes an orthogonal basis for the null space of $C_i$.

Definition 1: The equilibrium $x = 0$ of system (1) is said to be exponentially stable under switching signal $\sigma(k)$, if there exist constants $K > 0, 0 < \beta < 1$ such that the solution $x(k)$ of system (1) with $w = 0$ satisfies $\|x(k)\| \leq K/\beta^{k-k_0} \|x(k_0)\|$, $\forall k \geq k_0$.

Definition 2 [13] [15]: For $\gamma > 0$ and $0 < \alpha < 1$, system (1) is said to have a weighted $L_2$-gain, if under zero initial condition $x = 0$, it holds that

$$
\sum_{s = k_0}^{\infty} (1 - \alpha)^s z^T(s) z(s) \leq \sum_{s = k_0}^{\infty} \gamma^2 w^T(s) w(s)
$$

for all nonzero $w(k) \in \ell_2[0, \infty)$.

Definition 3 [11]: For any $k_0 < k_s < k_v$, let $N_{\sigma(k)}(k_s, k_v)$ denotes the switching number of $\sigma(k)$ over $(k_s, k_v)$. If $N_{\sigma(k)}(k_s, k_v) \leq N_0 + (k_v - k_s)/\tau_a$ for $\tau_a > 0, N_0 \geq 0$, then $\tau_a$ is called average dwell time.

In this paper, we are interested in designing a switched SOF controller

$$
u(k) = K_i y(k)
$$

where $K_i, i \in I$ are to be determined. The SOF controller (5) is assumed to be switched synchronously by the switching signal $\sigma$ in system (1).

Under the controller (5), the closed-loop switched system becomes

$$
x(k + 1) = A_{ci} x(k) + B_{ci} w(k)
$$

$$
z(k) = C_{ci} x(k) + D_{ci} w(k), \quad i \in I
$$

where

$$
A_{ci} = A_i + B_i K_i C_i
$$

$$
B_{ci} = B_i^w
$$

$$
C_{ci} = C_i^r + D_i K_i C_i
$$

$$
D_{ci} = D_i^w
$$

Then, the problem of $H_{\infty}$ SOF control to be addressed in this paper is formulated as follows. Given a switched system (1) and a prescribed level of disturbance attenuation $\gamma > 0$, design a switched SOF controller (5) and find out admissible switching signals with the minimal average dwell time such that the closed-loop system (6) is exponentially stable and achieve a prescribed weighted $L_2$-gain.

The following multiple Lyapunov function with the form

$$
V(x_k) \triangleq x_k^T P_{\sigma(k)} x_k, \quad \sigma(k) \in I
$$

will be used in the sequel.

III. MAIN RESULTS

This section gives the main result of the paper. First, several lemmas are given which are essential for later development.

Lemma 1: (Finsler’s Lemma) Let that $\xi \in \mathbb{R}^n, \mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n \times n}$ and $\mathcal{H} \in \mathbb{R}^{n \times n}$ such that $\text{rank}(\mathcal{H}) = r < n$, then the following statements are equivalent:

i) $\xi^T \mathcal{P} \xi \leq 0$, for all $\xi \neq 0, \mathcal{H} \xi = 0$;

ii) $\exists \chi \in \mathbb{R}^{n \times n}$ such that $\mathcal{P} + \chi^T \mathcal{H} + \mathcal{H}^T \chi < 0$.

Remark 1: Note that the condition ii) remains sufficient for i) to hold even arbitrary constraints are imposed to the scaling matrices $\chi$.

Lemma 2 [2][15]: Consider the discrete-time switched system $x_{k+1} = f_\sigma(x_k), \sigma \in I$ and let $0 < \alpha < 1, \mu > 1$
be given constants. Suppose that there exists a Lyapunov function candidate \( V(x) = \{V_\sigma(x)\}, \sigma \in I \) satisfying the following properties:

\[
\begin{align*}
\Delta V_\sigma(k)(x_k) & \triangleq V_\sigma(k)(x_{k+1}) - V_\sigma(k)(x_k) \leq -\alpha V_\sigma(k)(x_k), \\
\forall k & \in [k_1, k_{l+1}) \\
V_\sigma(k)(x_k) & \leq \mu V_\sigma(k_{l-1})(x_k)
\end{align*}
\]

then the system is exponentially stable for any switching signal with the average dwell time

\[
\tau_a \geq \tau_a^* = \text{ceil}\left[\frac{-\ln \mu}{\ln(1 - \alpha)}\right]
\]

(11)

where function ceil(\( v \)) represents rounding real number \( v \) to the nearest integer greater than or equal to \( v \).

**Lemma 3:** Let \( 0 < \alpha < 1, \gamma > 0 \) and \( \mu > 1 \) be given constants. If the following inequalities are satisfied

\[
\Delta V_\sigma(k)(x_k) + \alpha V_\sigma(k)(x_k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0,
\]

\forall k \in [k_1, k_{l+1})

(12)

\[
V_\sigma(k)(x_k) - \mu V_\sigma(k_{l-1})(x_k) \leq 0
\]

then the system (6) has a weighted \( L_2 \)-gain for any switching signal with the average dwell time satisfying (11).

**Proof:** Due to the limit of the space, it is omitted.

Based on Lemmas 1-3, the following theorem is given to solve the \( H_{\infty} \) SOF control problem.

**Theorem 1:** Let \( \alpha > 0, \gamma > 0 \) and \( \mu > 1 \) be given constants. If there exist symmetric matrices \( P_i \in \mathbb{R}^{n \times n} \), scalar \( \lambda \) and matrices \( G_i \in \mathbb{R}^{n \times n}, F_i \in \mathbb{R}^{n \times n}, L_i \in \mathbb{R}^{m \times n}, \) \( i \in I \) with the following structure

\[
G_i = \begin{bmatrix} G_{i11} & 0 \\ G_{i21} & G_{i22} \end{bmatrix}, \quad F_i = \begin{bmatrix} \lambda G_{i11} & 0 \\ F_{i21} & F_{i22} \end{bmatrix},
\]

\[
L_i = \begin{bmatrix} L_{i1} & 0 \end{bmatrix}
\]

(14)

satisfying the following inequalities

\[
\begin{bmatrix}
\Xi_{11} & * & * & * \\
0 & -I & * & * \\
\Xi_{31} & T_i^{-1}B_i^w & \Xi_{33} & * \\
\Xi_{41} & D_i^w & \Xi_{43} & -\gamma^2 I
\end{bmatrix} < 0
\]

(15)

\[
P_i - \mu P_j \leq 0
\]

(16)

where

\[
\Xi_{11} = T_i^{-1}P_i T_i^{-T} - G_i - G_i^T
\]

\[
\Xi_{31} = T_i^{-1}A_i T_i G_i + T_i^{-1}B_i L_i - F_i^T
\]

\[
\Xi_{33} = \text{He} \{(T_i^{-1}A_i T_i F_i + \lambda T_i^{-1}B_i L_i)\}
\]

\[-(1 - \alpha)T_i^{-1}P_i T_i^{-T}
\]

\[
\Xi_{41} = C_i^T G_i + D_i L_i
\]

\[
\Xi_{43} = C_i^T F_i + \lambda D_i L_i
\]

and \( T_i \) are given by (3), then the closed-loop system (6) is exponentially stable and has a weighted \( L_2 \)-gain for any switching signal with the average dwell time satisfying (11). Moreover, if (15)-(24) are feasible, then the switched SOF controller can be given by

\[
K_i = L_i G_i^{-1}
\]

(17)

**Proof:** Firstly, we establish the exponential stability of system (6). Pre- and post-multiplying

\[
\begin{bmatrix}
T_i & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & T_i & 0 \\
0 & 0 & 0 & I
\end{bmatrix}
\]

(18)

and its transpose to (15) obtains

\[
\begin{bmatrix}
\Lambda_{11} & * & * & * \\
0 & -I & * & * \\
\Lambda_{31} & B_i^w & \Lambda_{33} & * \\
\Lambda_{41} & D_i^w & \Lambda_{43} & -\gamma^2 I
\end{bmatrix} < 0
\]

(19)

where

\[
\begin{align*}
\Lambda_{11} & = P_i - T_i G_i T_i^T - T_i G_i T_i^T T_i^T \\
\Lambda_{31} & = A_i T_i G_i T_i^T + B_i L_i T_i^T - T_i F_i^T T_i^T \\
\Lambda_{33} & = \text{He} \{A_i T_i F_i T_i^T + \lambda B_i L_i T_i^T\} - (1 - \alpha)P_i \\
\Lambda_{41} & = C_i^T T_i G_i T_i^T + D_i L_i T_i^T \\
\Lambda_{43} & = C_i^T T_i F_i T_i^T + \lambda D_i L_i T_i^T 
\end{align*}
\]

It follows from (7), (14) and (17) that

\[
A_i T_i G_i T_i^T + B_i L_i T_i^T
\]

(20)

In the same way, we can obtain

\[
A_i T_i F_i T_i^T + \lambda B_i L_i T_i^T = A_i c_i T_i F_i T_i^T
\]

(21)

\[
C_i^T T_i G_i T_i^T + D_i L_i T_i^T = C_i c_i T_i G_i T_i^T
\]

(22)

\[
C_i^T T_i F_i T_i^T + \lambda D_i L_i T_i^T = C_i c_i T_i F_i T_i^T
\]

(23)

Substituting (20)-(23) into (19) obtains

\[
\begin{bmatrix}
\Upsilon_{11} & * & * & * \\
0 & -I & * & * \\
\Upsilon_{31} & B_{i,i} & \Upsilon_{33} & * \\
C_i c_i T_i G_i T_i^T & D_{i,i} & C_i c_i T_i F_i T_i^T & -\gamma^2 I
\end{bmatrix} < 0
\]

(24)

where

\[
\begin{align*}
\Upsilon_{11} & = P_i - T_i G_i T_i^T - T_i G_i T_i^T T_i^T \\
\Upsilon_{31} & = A_i c_i T_i G_i T_i^T - T_i F_i^T T_i^T \\
\Upsilon_{33} & = \text{He} \{A_i c_i T_i F_i T_i^T\} - (1 - \alpha)P_i 
\end{align*}
\]
Pre- and post-multiplying
\[
\begin{bmatrix}
I & 0 & 0 & 0 \\
0 & 0 & I & 0 \\
0 & I & 0 & 0 \\
0 & 0 & 0 & I
\end{bmatrix}
\text{and its transpose to (24) obtains}
\[
\begin{bmatrix}
\Upsilon_{11} & \ast & \ast & \ast \\
\Upsilon_{31} & \ast & \ast & \ast \\
0 & B_{cli}^T & -I & \ast \\
C_{cli} T_i G_i T_i^T & C_{cli} T_i F_i T_i^T & D_{cli} & -\gamma^2 I
\end{bmatrix} < 0
\] (25)
From (25) we have
\[
\begin{bmatrix}
\Upsilon_{11} & \ast \\
\Upsilon_{31} & \ast
\end{bmatrix} < 0
\] (26)
which can be rewritten as follows
\[
\begin{bmatrix}
P_i & 0 \\
0 & -(1-\alpha) P_i
\end{bmatrix} + \mathbf{He}\left\{\begin{bmatrix}
T_i G_i^T T_i^T & -I \\
T_i F_i & A_{cli}^T
\end{bmatrix}\right\} < 0
\]
(27)
Consider the dual system of (6) with \(w = 0\)
\[
x(k+1) = A_{cli}^T x(k)
\] (28)
and rewrite it in the form
\[
\begin{bmatrix}
-I & A_{cli}^T \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x(k+1) \\
x(k)
\end{bmatrix} = 0
\] (29)
Based on Finsler's lemma, if (27) holds then the following inequality holds
\[
\begin{bmatrix}
x(k+1) \\
x(k)
\end{bmatrix}^T
\begin{bmatrix}
P_i & 0 \\
0 & -(1-\alpha) P_i
\end{bmatrix}
\begin{bmatrix}
x(k+1) \\
x(k)
\end{bmatrix} < 0
\]
(30)
which is equivalent to
\[
x(k+1)^T P_i x(k+1) - x(k)^T P_i x(k) < -\alpha x(k)^T P_i x(k)
\] (31)
then (9) is satisfied. In addition, it follows from (24) that (10) is satisfied. From Lemma 2, the closed-loop system without disturbances is exponentially stable for any switching signal with the average dwell time satisfying (11).

Now we consider the weighted \(L_2\)-gain of system (6).

The inequality (24) can be rewritten as follows
\[
\mathcal{P} + \mathcal{X} \mathcal{H} + \mathcal{H}^T\mathcal{X}^T < 0
\]
(32)
where
\[
\mathcal{P} = \begin{bmatrix}
P_i & 0 & 0 & 0 \\
0 & I & 0 & 0 \\
0 & 0 & -(1-\alpha) P_i & 0 \\
0 & 0 & 0 & -\gamma^2 I
\end{bmatrix},
\]
\[
\mathcal{X} = \begin{bmatrix}
T_i G_i^T T_i^T & 0 \\
0 & T_i F_i & 0 & 0
\end{bmatrix},
\]
\[
\mathcal{H} = \begin{bmatrix}
-I & 0 & A_{cli}^T & C_{cli}^T \\
0 & -I & B_{cli}^T & D_{cli}^T
\end{bmatrix}
\]
(33)
Consider the dual system of (6)
\[
x(k+1) = A_{cli}^T x(k) + C_{cli}^T w(k)
\]
\[
z(k) = B_{cli}^T x(k) + D_{cli}^T w(k)
\]
(34)
and define the augmented signal \(\xi\) as
\[
\xi = \begin{bmatrix}
x(k+1) \\
z(k) \\
x(k) \\
w(k)
\end{bmatrix}
\]
(35)
then, (34) can be rewritten in the form
\[
\mathcal{H} \xi = 0
\]
(36)
By Finsler’s lemma, if (32) holds then the following inequality holds
\[
\xi^T \mathcal{P} \xi < 0
\]
(37)
Substituting (33) into (37), we have
\[
x(k+1)^T P_i x(k+1) - (1-\alpha)x(k)^T P_i x(k) + z(k)^T z(k) - \gamma^2 w(k)^T w(k) < 0
\]
(38)
which is nothing but (12). Additionally, (13) is satisfied due to (24). Based on Lemma 3, the system has a weighted \(L_2\)-gain \(\gamma\). And thus the proof is completed.

Letting \(F_i = 0\), Theorem 1 reduces to the following corollary:

**Corollary 1:** If there exist symmetric matrices \(P_i \in \mathbb{R}^{n \times n}\) and matrices \(G_i \in \mathbb{R}^{n \times n}\), \(L_i \in \mathbb{R}^{m \times n}\), \(i \in I\) with the following structure
\[
G_i = \begin{bmatrix}
G_{i11} & 0 \\
G_{i12} & G_{i22}
\end{bmatrix},
\]
(39)
satisfying the following inequalities
\[
\begin{bmatrix}
\Gamma_{11} & \ast & \ast & \ast \\
0 & -I & \ast & \ast \\
\Gamma_{31} & T_i^{-1} B_i^T & -T_i^{-1} P_i T_i^{-T} & \ast \\
\Gamma_{41} & D_i & 0 & -\gamma^2 I
\end{bmatrix} < 0
\]
(40)
where
\[
\begin{align*}
\Gamma_{11} &= T_i^{-1} P_i T_i^{-T} - G_i - G_i^T \\
\Gamma_{31} &= T_i^{-1} A_i T_i G_i + T_i^{-1} B_i L_i \\
\Gamma_{41} &= C_i^T T_i G_i + D_i L_i
\end{align*}
\]
and \(T_i\) are given by (3), then the closed-loop system (6) is exponentially stable and has a weighted \(L_2\)-gain for any switching signal with the average dwell time satisfying (11). Moreover, if (40)-(49) are feasible, then the switched SOF controller can be given by \(K_i = L_i G_i^{-1}\).

**Proof:** The proof of this corollary can be done using the same technique and arguments as in the proof of Theorem 1. Thus it is omitted here.

**Remark 2:** When \(\lambda\) in \(F_i\) is set to be fixed parameter, the condition in Theorem 1 becomes convex and can be solved by LMI Control Toolbox [27]. In Theorem 1, by
using the multiple Lyapunov function technique combined with Finsler’s lemma, two sets of slack variables $G_i$, $F_i$ with special structure are introduced to provide extra free dimensions in the solution space. This directly leads to reduction of the conservativeness of the solutions and improvement of the performance. Compared to Theorem 1, Corollary 1 is more conservative since only one set of variables are introduced.

IV. NUMERICAL EXAMPLE

In this section, an example is given to illustrate the effectiveness of the proposed method.

Consider a discrete-time switched linear system consisting of three subsystems described as follows

$$
A_1 = \begin{bmatrix}
-0.5871 & -0.8441 & -0.0092 \\
-0.6865 & -0.5090 & -0.8561 \\
0.0974 & 0.4523 & -0.2280
\end{bmatrix},
B_1 = \begin{bmatrix}
0.1930 & -0.4204 \\
-0.7359 & 0.0346 \\
0.5073 & -0.9077
\end{bmatrix},
C_1^w = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix},
D_1^w = \begin{bmatrix}
1 \\
1 \\
0
\end{bmatrix},
C_1 = [ 1 \ 0 \ 1 ];
$$

$$
A_2 = \begin{bmatrix}
0.1089 & 0.2458 & -0.9035 \\
0.3998 & -0.9213 & -0.4161 \\
0.6745 & -0.5750 & 0.7138
\end{bmatrix},
B_2 = \begin{bmatrix}
-0.4164 & 0.0244 \\
0.8297 & -0.4366 \\
-0.0900 & -0.8416
\end{bmatrix},
B_2^w = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix},
C_2^w = \begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix},
D_2^w = \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix},
C_2 = [ 0 \ 1 \ 1 ];
$$

$$
A_3 = \begin{bmatrix}
0.3049 & 0.4247 & 0.8979 \\
0.8848 & 0.2485 & -0.4161 \\
0.6981 & 0.1034 & 0.2403
\end{bmatrix},
B_3 = \begin{bmatrix}
0.2458 & 0.7409 \\
0.2501 & 0.1580 \\
0.1709 & 0.7205
\end{bmatrix},
B_3^w = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
C_3^w = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix},
D_3^w = \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix},
C_3 = [ 0 \ 1 \ 1 ].
$$

Note that $A_1$-$A_3$ are all unstable. Let $\mu = 2$, $\alpha = 0.5$, then we obtain $\tau_{a}^* = 1$. By using Theorem 1, the following control gains are obtained

$$
K_1 = \begin{bmatrix}
-1.1519 \\
-0.8379
\end{bmatrix},
K_2 = \begin{bmatrix}
0.4797 \\
0.2214
\end{bmatrix},
K_3 = \begin{bmatrix}
0.5627 \\
-0.7362
\end{bmatrix}
$$

and the optimal weighted $L_2$-gain $\gamma_{min} = 4.0200$. The closed-loop state response with initial states chosen as $x(0) = [-4 \ 3 \ 5]^T$ and the disturbance chosen as $w(k) = 1/(20k + 1)$ is shown in Fig. 2. It is clear that the switched system has been stabilized by the SOF Controller under the switching signal shown in Fig. 1. In addition, the condition in Corollary 1 gives the following control gains

$$
K_1 = \begin{bmatrix}
-1.1467 \\
-0.6949
\end{bmatrix},
K_2 = \begin{bmatrix}
0.4487 \\
0.2693
\end{bmatrix},
K_3 = \begin{bmatrix}
0.6375 \\
-0.7068
\end{bmatrix}
$$

and the optimal weighted $L_2$-gain $\gamma_{min} = 4.2310 > 4.0200$. This shows that Theorem 1 is less conservative than Corollary 1 due to the fact that slack variables $F_{i21}, F_{i22}$ in $F_i$ provides extra freedom in the LMI optimization problem in Theorem 1.
V. CONCLUSIONS

This paper considers the problem of $H_\infty$ SOF control for discrete-time switched linear systems with average dwell time switching. By the aid of the multiple Lyapunov function technique combined with Finsler’s lemma, a switched SOF controller has been designed. An example has also been given to illustrate the effectiveness of the proposed method.

REFERENCES


