Observer Based Control of Piezoelectric Actuators with Classical Duhem Modeled Hysteresis

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Abstract—An observer-based adaptive control scheme for piezoelectric actuators is proposed. In the developed method the classical Duhem model is adopted to describe the hysteresis exhibited in the piezoelectric actuators. Due to the unavailability of the hysteresis output, an observer-based adaptive controller, with incorporation of a pre-inversion neural network compensator for the purpose of mitigating the hysteretic effects, is designed to guarantee the stability of the adaptive system and tracking error between the position of the piezoelectric actuator and the desired trajectory to a desired precision. Simulation studies illustrate the effectiveness of the proposed method.

I. INTRODUCTION

Hysteresis phenomenon occurs in all smart material-based sensors and actuators, such as shape memory alloys, piezoceramics and magnetostrictive actuators [1, 2]. In order to study this phenomenon, different models were proposed [3, 4]. Normally, hysteresis models are classified into two categories, physics-based model such as Jiles-Atherton model [5] and phenomenology-based model such as Preisach operator [3, 4] and Duhem model [5]. From control systems point of view, hysteresis is generally non-differentiable, nonlinear, and unknown. As a result, systems with hysteresis usually exhibit undesirable inaccuracies or oscillation and even instability. Mitigating the effect of hysteresis becomes necessary and important, thus it has received increasing attention in recent years [6-9]. Many of these studies are related to modeling of hysteresis and their control issues [1-9].

With the development of artificial intelligent (AI), AI is being applied to dealing with nonlinearities in systems [10]. Only a few studies have been carried out by using NN to tackle hysteresis modeling and compensation [11-15]. In [11, 13], a NN model is used to describe the hysteresis behavior in different frequencies with the knowledge of some properties of magnetic materials, such as loss separation property to allow the separate treatment of quasi-static and dynamic hysteretic effects. In [13], a modified LuGreber observer and NN are used to identify a general model of hysteresis. These researches demonstrate that NN can work as an unknown function approximator to describe the characteristics of hysteresis. In [14, 15], the approximation property of NN is applied to coping with the identification of Preisach-type hysteresis in piezoelectric actuator, and the hysteresis estimation problem for piezo-positioning mechanism based on hysteresis friction force function, respectively. It should be noted that the aforementioned results share a common assumption that the output of hysteresis is measurable.

In practical systems, smart actuators are generally used for the purpose of high precision control. However, measurement of output of hysteresis is very difficult, if not impossible. Hence it is a challenge to design an observer for the unavailable output of hysteresis. Due to the unavailability of the output of hysteresis, the major obstacle of pre-inversion compensator for hysteresis is the lack of effective observer design methods for piezoelectric actuators. Especially, the traditional “LuGre-type” nonlinear observer design in [17] or the “high-gain” observer in [18] cannot be applied directly, since the hysteresis is highly nonlinear. The sliding-mode observer was developed to estimate the internal friction states of LuGre model for the servo actuators with friction [19]. This observer needs a low-pass filter to remove the high-frequency components in the estimated state variable, which is not applicable in this paper. In [16], Yang and Lin proposed homogeneous observers design for a class of n-dimensional inherently nonlinear systems whose Jacobian linearization is neither controllable nor observable.

Inspired by NN’s universal approximation property, and the aforementioned facts in observer design, we propose an observer-based adaptive control of piezoelectric actuators with unknown hysteresis in this paper. The main contribution of this paper is the following: First, it applies the NN to on-line approximate complicated piecewise continuous unknown nonlinear functions in the explicit solution to Duhem model. Second, an observer is designed to estimate the output of hysteresis of piezoelectric actuator based on the system input and output. Third, the stability of the controlled piezoelectric actuator with the observer is analyzed by using Lyapunov extension [20].

The organization of the paper is as follows. In Section II, an explicit solution of Duhem model of hysteresis is derived, and the Augmented Multilayer Perceptron (MLP) Neural Network preliminary is introduced. The main results on the observer-based adaptive control scheme for the piezoelectric actuators are presented in Section III. Section IV provides simulation studies and Conclusions are reached in Section V.

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II. DUHEM MODEL AND PRELIMINARIES

A. Duhem model of hysteresis

Many different mathematical models are built to describe the hysteresis behavior, such as Preisach model, Prandtl–Ishlinskii model and Duhem model [21, 22]. Considering its capability of providing a finite-dimensional differential model of hysteresis, we adopt classical Duhem model to develop the adaptive controller for the piezoelectric actuator.

The Duhem model is a rate independent operator, with input signal \( \nu \), \( \dot{\nu} \) and output signal \( \tau \). The Duhem model describes hysteresis \( H(t) \) by the following mathematical model [21, 22],

\[
\frac{d \tau}{dt} = \alpha \left[ \frac{d}{dt} (f(\nu) - \tau) + \frac{d}{dt} g(\nu) \right]
\]

where \( \alpha \) is a positive number, \( f(\nu) \) and \( g(\nu) \) are prescribed real-valued functions on \( (-\infty, +\infty) \).

It can also be represented as [21, 22]:

\[
\frac{d \tau}{d\nu} = \begin{cases} \alpha \cdot \left[ f(\nu) - \tau \right] + g(\nu), & \nu > 0 \\ -\alpha \cdot \left[ f(\nu) - \tau \right] + g(\nu), & \nu < 0 \end{cases}
\]

where \( \alpha \) is the same positive number in (1), \( g(\nu) \) is the slope of the model, and \( f(\nu) \) is the average value of the difference between upward side and downward side.

It has been shown that Duhem model can describe a large class of hysteresis in various smart materials, such as ferromagnetically soft material, or piezoelectric actuator by appropriately choosing \( f(\nu) \) and \( g(\nu) \) [21-23].

In order to describe the piezoelectric actuator, we choose the same functions \( f(\nu) \) and \( g(\nu) \) as those in [23]

\[
f(\nu) = \begin{cases} a \cdot v_s & \text{for } \nu \geq v_s \\ -a \cdot v_s & \text{for } \nu \leq v_s \end{cases}
\]

\[
g(\nu) = \begin{cases} 0 & \text{for } \nu > v_s \\ \bar{b} & \text{for } \nu \leq v_s \\ 0 & \text{for } \nu < v_s \end{cases}
\]

where \( v_s > 0 \), \( a > 0 \), \( \bar{b} > 0 \) and \( a > \bar{b} > a/2 \). Suppose the parameter \( a \) satisfies \( a \in [a_{\min}, a_{\max}] \), \( a_{\min} \) and \( a_{\max} \) are known constants.

Substituting (3) and (4) into (2), we have

\[
\tau = \begin{cases} \alpha \cdot v_s^2 - \tau & \nu > v_s, \dot{\nu} > 0 \\ \alpha \cdot a \cdot v - \tau + \bar{b} \cdot \dot{\nu} & 0 < \nu \leq v_s, \dot{\nu} > 0 \\ \alpha \cdot \bar{b} \cdot \dot{\nu} - v_s & v_s \leq \nu < 0, \dot{\nu} < 0 \\ \alpha \cdot a \cdot v_s + \tau & v < -v_s, \dot{\nu} < 0 \end{cases}
\]

Equation (5) can be solved for \( \tau \)

\[
\tau = \begin{cases} -f_{21} & \nu > v_s, \dot{\nu} > 0 \\ a \cdot v - f_{22} & 0 < \nu \leq v_s, \dot{\nu} > 0 \\ a \cdot \bar{b} \cdot \dot{\nu} - f_{23} & -v_s \leq \nu < 0, \dot{\nu} < 0 \\ -f_{24} & \nu < -v_s, \dot{\nu} < 0 \end{cases}
\]

where

\[
f_{21} = a \cdot v_s - \tau, \quad f_{22} = a \cdot v - \tau + \bar{b} \cdot \dot{\nu}, \quad f_{23} = \bar{b} \cdot \dot{\nu} - v_s, \quad f_{24} = a \cdot \bar{b} \cdot \dot{\nu} + \tau
\]

Equation (6) can also be expressed as:

\[
\tau = a \cdot \chi(\nu) \cdot v_f - k_1 \cdot \nu - k_2 \cdot \dot{\nu} = 0
\]

where \( \chi_i (i = 1, 2, \ldots, m) \) are indicator functions defined as:

\[
\chi_i = \begin{cases} 1 & \nu \leq v_i, \dot{\nu} < 0 \\ 0 & \nu > v_i \end{cases}
\]

Following the definition of the indicator functions, we get:

\[
\nu \leq v_1 = 0, \quad \nu \leq v_2 = 1
\]

Let \( F_2 = \int f(\nu) d\nu \), \( F_3 = \int f(\nu) d\nu \), \( \tau_2 = F_2 \), and \( K_a = a \cdot \chi_1 \). \( \tau \) can be rewritten as:

\[
\tau = K_a \cdot \dot{\nu} - F_2
\]

B. Augmented Multilayer Perceptron (MLP) Neural Network

The MLP NN has been explored to approximate any function with arbitrary degree of accuracy [24]. However, it needs a large number of NN nodes and training iterations to approximate nonsmooth functions (i.e. piecewise continuous), such as friction, hysteresis, backlash and other hard nonlinearities. For these piecewise continuous functions, the MLP needs to be augmented as a function approximator. Results for approximation of piecewise continuous functions or functions with jumps are given in [25]. We will use the augmented NN to approximate the piecewise continuous function in the hysteresis model.

Let \( S \) be a compact set of \( \mathbb{R}^n \) and define \( C^n(S) \) be the space such that the map \( f(x) : S \rightarrow \mathbb{R}^n \) is piecewise continuous. The NN can approximate a function \( f(x) \in C^n(S) \), \( x \in \mathbb{R}^n \) which has a jump at \( x = c_j \) and is continuous from the right as

\[
f(x) = W^T \cdot \sigma(V^T \cdot x + F^T \cdot \varphi(V^T \cdot (x - c_j))) + \varepsilon(x)
\]

where \( \varepsilon(x) \) is a functional restructure error vector, \( W^T \), \( W_f \), \( V^T \), and \( \varphi(\cdot) \) are nominal constant weight matrices. \( \sigma(\cdot) \) and \( \varphi(\cdot) \) are activation functions for hidden neurons.

For the hysteresis model (9), the piecewise continuous function \( f_2 \) will be approximated by the augmented NN. In this paper, it is assumed that there exists weight matrix \( W \) such that \( \| f(x) \| \leq \varepsilon_N \) with constant \( \varepsilon_N > 0 \), for all \( x \in \mathbb{R}^n \), and the Frobenius norm of each matrix is bounded by a known constant \( \| W \| \leq W_N \) with \( W_N > 0 \).
III. NN-BASED COMPENSATOR AND CONTROLLER DESIGN

Given the augmented MLP NN and hysteresis model, a NN-based pre-inversion compensator for the hysteresis is designed to cancel out the effect of hysteresis. In this section, a novel approach is developed to compensate the hysteretic nonlinearity and to guarantee the stability of integrated piezoelectric actuator control system.

A. Problem statement

Consider a piezoelectric actuator subject to a hysteresis nonlinearity described by Duham model. It can be identified as a second-order linear model preceded by hysteretic nonlinearities described by Duhem model. It can be identified as follows [28]:

\[ m \cdot \ddot{y}(t) + b \cdot \dot{y}(t) + k \cdot y(t) = k \cdot c \cdot \tau_{pd}(t) \]

\[ \tau_{pr}(t) = H[y(t)] \]  \hspace{1cm} (11)

where \( v(t) \) is the input to piezoelectric actuator, \( y(t) \) denotes the position of piezoelectric actuator, \( m \), \( b \), \( k \) and \( c \) denote the mass, damping, stiffness and effective piezoelectric coefficient, respectively. \( H(\cdot) \) represents the Duham model (1). \( \tau_{pr} \) is output of the hysteresis. In order to eliminate the effect of hysteresis on the piezoelectric system, a NN-based hysteresis compensator is designed to make the output from hysteresis model \( \tau_{pr} \) approach the designed control signal \( \tau_{pd} \). After the hysteresis is compensated by the NN, an observer based adaptive control for piezoelectric actuator is to be designed to ensure the stability of the overall system and the reasonable tracking performance of output tracking error of the piezoelectric actuator with unknown hysteresis.

We consider the tracking problem where \( y(t) \) is asymptotically track a reference signal \( y_d(t) \) having the properties that \( y_d(t) \) and its derivatives up to second derivative are bounded, and \( \dot{y}_d(t) \) is piecewise continuous, for all \( t \geq 0 \). The tracking error of the piezoelectric system is defined as

\[ e_p(t) = y_d(t) - y(t) \]  \hspace{1cm} (12)

A filtered error is defined as

\[ r_p(t) = \dot{e}_p(t) + \lambda_p \cdot e_p(t) \]  \hspace{1cm} (13)

where \( \lambda_p > 0 \) is a design parameter.

Differentiating \( r_p(t) \) and combining it with the system dynamics Eq. (11), one obtains:

\[ m \cdot \ddot{r}_p = -\frac{b}{k} \cdot \dot{r}_p - \frac{m}{k} \cdot (\ddot{y}_d + \lambda_p \cdot \dot{r}_p) + \frac{b}{k} \cdot (\dot{y}_d + (\lambda_p - \frac{k}{b}) \cdot r_p) + \frac{1}{c} \cdot y_d. \]  \hspace{1cm} (14)

The tracking error dynamics can be re-written as

\[ \frac{m}{k} \cdot \ddot{r}_p = -\frac{b}{k} \cdot \dot{r}_p - \tau_{pr} + Y_d^T \cdot \Theta_p \]  \hspace{1cm} (15)

where \( Y_d = \begin{bmatrix} \ddot{y}_d + (\lambda_p - \frac{k}{b}) \cdot \dot{r}_p & \dot{y}_d + \lambda_p \cdot r_p & y_d \end{bmatrix}^T \) is a regression vector and \( \Theta_p = \begin{bmatrix} m \frac{b}{k} \cdot \dot{r}_p \end{bmatrix}^T \in \mathbb{R}^3 \) is a unknown parameter vector with \( \theta_{p_{min}} \leq \theta_p \leq \theta_{p_{max}} \) \( i = 1, 2, 3 \) where \( \theta_{p_{min}} \) and \( \theta_{p_{max}} \) are some known real numbers.

B. NN-based Compensator for Hysteresis

In presence of the unknown hysteresis nonlinearity, the desired control signal \( \tau_{pd} \) for the piezoelectric actuator is different from the real control signal \( \tau_{pr} \). Define the error as

\[ \tilde{\tau}_p = \tau_{pd} - \tau_{pr} \]  \hspace{1cm} (16)

where \( \tau_{pd} \) will be defined in the late development. Differentiating (16), yields

\[ \dot{\tilde{\tau}}_p = \dot{\tau}_{pd} - \dot{\tau}_{pr} \]  \hspace{1cm} (17)

thus, we have

\[ \dot{\tilde{\tau}}_p = \tilde{\tau}_{pd} - K_v \dot{\tilde{\tau}}_p + F_2 \]  \hspace{1cm} (18)

Here we utilize the MLP in (12) to approximate the nonlinear function \( F_2 \).

\[ F_2 = W_{T_2} \cdot \sigma(V_{T_2} \cdot h) + W_{f_{21}} \cdot \phi_{21}(V_{f_{21}} \cdot h) \]

\[ + W_{f_{22}} \cdot \phi_{22}(V_{f_{22}} \cdot h - v_s) \]

\[ + W_{f_{23}} \cdot \phi_{23}(V_{f_{23}} \cdot h + v_s) + e_1(h) \]  \hspace{1cm} (19)

where \( h = [\tilde{\tau}_{pd} \quad \tau_{p_0} \quad v \quad v \quad v] \), \( \tau_{p_0} \) is the initial value of the control signal, \( v_{T_2}^T \), \( v_{f_{21}}^T \), \( v_{f_{22}}^T \), and \( v_{f_{23}}^T \) are input-layer weight matrices, \( w_{T_2}^T \), \( w_{f_{21}}^T \), \( w_{f_{22}}^T \), and \( w_{f_{23}}^T \) are output-layer weight matrices, \( 0, v_s \), and \( -v_s \) are jump points on the output layer, and \( \sigma() \), \( \phi_{21}(\cdot) \), \( \phi_{22}(\cdot) \), and \( \phi_{23}(\cdot) \) are the activation functions, and \( e_1(h) \) is the functional restructure error in which inversion error is included. Output-layer weight matrices \( w_{T_2}^T \), \( w_{f_{21}}^T \), \( w_{f_{22}}^T \), and \( w_{f_{23}}^T \) are trained so that the output of NN approximates to the nonlinear function \( F_2 \). Let

\[ \Theta(h,v_s)=[\sigma(V_{T_2}^T \cdot h) \quad \phi_{21}(V_{f_{21}}^T \cdot h) \quad \phi_{22}(V_{f_{22}}^T \cdot h - v_s) \quad \phi_{23}(V_{f_{23}}^T \cdot h + v_s)]^T \]

and \( w_{T_2} = [w_{T_2}^T \quad w_{f_{21}}^T \quad w_{f_{22}}^T \quad w_{f_{23}}^T] \). The nonlinear function \( F_2 \) is expressed as:

\[ F_2 = W_{T_1}^T \Theta(h,v_s) + e_1(h) \]  \hspace{1cm} (20)

It is assumed that the Frobenius norm of weight matrix \( W_1 \) is bounded, i.e., \( \| W_1 \| \leq W_{1N} \) with a known constant \( W_{1N} > 0 \) and \( \| e_1(h) \| \leq \epsilon_{1N} \) with constant \( \epsilon_{1N} > 0 \) for all \( x \in \mathbb{R}^n \). The estimated nonlinear function \( \hat{F}_2 \) is constructed by using the neural network with the estimated weight matrix \( \hat{W}_1 \):

\[ \hat{F}_2 = \hat{W}_1^T \Theta(h,v_s) \]  \hspace{1cm} (21)

Hence the restructure error between the nonlinear functions \( F_2 \) and \( \hat{F}_2 \) is derived as:

\[ \tilde{F}_2 = F_2 - \hat{F}_2 = \hat{W}_1^T \Theta(h,v_s) + e_1(h) \]  \hspace{1cm} (22)
where \( \ddot{w}^T_1 = W^T_1 - \ddot{w}^T_1 \).

C. Controller Design Using Estimated Hysteresis Output

It is noticed that the output of hysteresis is not normally measurable for the plant subject to unknown hysteresis. However, considering the whole system as a dynamic model preceded by Duhem model, we could design an observer to estimate the output of hysteresis based on the input and output of the plant.

We denote the observed output of hysteresis as \( \hat{\tau}_{pr} \), and define the error between the actual output of actuator \( y \) and the estimated output of actuator \( \hat{y} \):

\[
e^{-\hat{y}} = y - \hat{y}
\]

Denote the error between the output of hysteresis \( \tau_{pr} \) and the observed \( \hat{\tau}_{pr} \) is defined as \( e_{2} = \tau_{pr} - \hat{\tau}_{pr} \). Then the observer is designed as:

\[
\dot{\hat{y}} = \ddot{y} + L_{e} \dot{e} \leq \eta
\]

\[
\hat{\tau}_{pr} = K_{a} \ddot{\hat{y}} - F_{2} + L_{e} \dot{e} + K_{pr} \tau_{pr}
\]

where \( \ddot{y}(t) \) is the velocity of the actuator assumed measurable.

The error dynamics of the observer is obtained based on the actuator model and hysteresis model.

\[
\dot{e}_{1} = -L_{e} \dot{e}
\]

\[
\dot{e}_{2} = K_{a} \ddot{\hat{y}} - F_{2} - L_{e} \dot{e} + K_{pr} \tau_{pr}
\]

where the parameter error is defined as \( \hat{K}_{a} = K_{a} - K_{a} \).

By using the observed hysteresis output \( \hat{\tau}_{pr} \), we may define the signal error between the adaptive control signal \( \tau_{pd} \) and the estimated hysteresis output as:

\[
\bar{\tau}_{pe} = \tau_{pd} - \hat{\tau}_{pr}
\]

The derivative of the error signal is:

\[
\dot{\bar{\tau}}_{pe} = \dot{\tau}_{pd} - \hat{\tau}_{pr}
\]

Hence, Noticing equations (16)-(20) and (25)-(28), we design the observer based adaptive controller as follows:

\[
\dot{\hat{\mu}} = \hat{\mu} \cdot (k_{h} \cdot \hat{\tau}_{pe} + \tau_{pd} + \hat{F}_{2} + r_{p})
\]

\[
\dot{\hat{\nu}} = P \text{ proj} (\hat{\mu}, \eta \cdot \hat{\tau}_{pe}; \{\tau_{pd} + \hat{W}_{1}^{T} \Theta (h, \nu_{s}) + r_{p}\})
\]

\[
\hat{\tau}_{pe} = (1 - \hat{K}_{a} \hat{\mu}) \tau_{pd} - \hat{K}_{a} \hat{\nu} + L_{e} \dot{e} + (1 - \hat{\mu}) \hat{\nu}
\]

\[
\hat{K}_{a} = \text{ proj} (K_{a}, \nu \cdot \hat{\mu} \cdot \hat{\tau}_{pe}; \{\tau_{pd} + \hat{W}_{1}^{T} \Theta (h, \nu_{s}) + r_{p}\}
\]

\[
+ \hat{\nu} \cdot \hat{\tau}_{pe}
\]

\[
\hat{\nu} \quad \hat{\nu} = \text{ proj} (\hat{\mu}, \hat{\nu}; \{\tau_{pd} + \hat{W}_{1}^{T} \Theta (h, \nu_{s}) + r_{p}\}
\]

\[
+ \hat{\nu} \cdot \hat{\tau}_{pe}
\]

where \( \hat{\mu} = \frac{a_{min}}{a} \) is an estimated constant, which satisfies

\[
0 < \hat{\mu} < 1 \quad \text{with the known boundary of } a \in [a_{min}, a_{max}], \quad k_{h} \text{ is a positive constant}, \quad \hat{\nu} \text{ is the estimated values of } a, \eta \text{ is a positive constant}, \quad \Gamma \text{ is a positive adaptation gain diagonal matrix}, \quad k_{pd} \text{ is a positive constant}, \quad \beta \text{ is a constant gain, the definitions of } Pr(o)(\cdot) \text{ and } Pr(\beta)(\cdot) \text{ are provided in the Appendix.}
\]

Hence, the overall control system of integrated piezoelectric actuator is shown in Fig. 1.

![Fig. 1 Adaptive controller with hysteresis compensator and observer](image)

To this end, the stability and convergence of the overall control system of integrated piezoelectric actuator are summarized in the following theorem.

Theorem 1: For a piezoelectric actuator system (11) with unknown hysteresis (1) and a desired trajectory \( y_{d}(t) \), the observer based adaptive robust control law (29) can guarantee the tracking error \( e_{2}(t) \) between the output of actuator and the desired trajectory \( y_{d}(t) \) converge to a small neighborhood around zero by appropriately choosing suitable control gains \( k_{pd}, k_{h} \) and observer gains \( L_{e}, L_{2} \) and \( K_{pr} \).

Proof: Define a Lyapunov function

\[
V_{2} = \frac{1}{2} \frac{m}{k_{c}} r_{p}^{2} + \frac{1}{2} x_{p}^{2} + \frac{1}{2} x_{r}^{2} (\hat{W}_{1}^{T} \Gamma^{-1} \hat{W}_{1}) + \frac{1}{2} k_{pr} (1 - \hat{\mu} K_{a})^{2} + \frac{1}{2} \beta (\theta_{p} - \hat{\theta}_{p})^{2} (\theta_{p} - \hat{\theta}_{p}) + \frac{1}{2} e_{2}^{2} + \frac{1}{2} \varepsilon^{2}
\]

Noticing the control law (29), one obtains

\[
\dot{V}_{2} = -e_{2} r_{p} - L_{e} e_{2} r_{p} + K_{pr} e_{2} \dot{r}_{p} - L_{2} e_{2}^{2} - e_{2}^{2} + 21, \quad L_{2} e_{2}^{2} + K_{pr} e_{2} \dot{r}_{p}
\]

Using \( \dot{r}_{p} = \tau_{pd} - e_{2} \), \( \dot{r}_{p} \leq e_{1} \), and \( | \dot{r}_{p} | \leq 1 \), we may obtain

\[
\tau_{2} = \frac{1}{2} (a + h) \leq a \text{ and } b^{2}
\]

\[
\dot{V}_{2} \leq -e_{2}^{2} - L_{2} e_{2}^{2} + K_{pr} e_{2} \dot{r}_{p} - L_{2} e_{2}^{2} - e_{2}^{2} + e_{2}^{2} + K_{pr} \varepsilon^{2}
\]

From the Property 1 of Chapter 2 in [26], we know \( \tau_{2} \) is bounded (say, \( \tau_{2}^{2} \leq M^{2} \) where \( M \) is a constant), and then define a constant \( \delta = e_{2}^{2} + K_{pr} \varepsilon^{2} > e_{2}^{2} + K_{pr} \tau_{2}^{2} \) such that

\[
\dot{V}_{2} \leq -e_{2}^{2} - L_{2} e_{2}^{2} + K_{pr} e_{2} \dot{r}_{p} - L_{2} e_{2}^{2} - e_{2}^{2} + e_{2}^{2} + K_{pr} \varepsilon^{2} + \delta
\]

We select the control parameters \( k_{pd}, k_{h} \) and observer parameters \( L_{e}, L_{2} \) and \( K_{pr} \) satisfying the following inequalities:

\[
\frac{h}{k_{c}} + k_{pd} - \frac{1}{2} > 0, \quad K_{pr} > 2, \quad k_{h} \cdot a_{max} \cdot K_{a} - \frac{1}{2} K_{pr} - 1 > 0
\]
\[
L_1 > \frac{3}{2} L_2^2. \quad \text{Let } k_m = k_h \cdot a_{\text{max}} \cdot \dot{K}_a - \frac{1}{2} K_{pr}^2 - 1. \quad \text{If we have}
\]
\[
|P_r| > k_{p1} \frac{W_{1N}^2/4 + \varepsilon_{1N}}{k_m} \|p_r\| > W_{1N}/2 + \sqrt{\frac{W_{1N}^2/4 - \varepsilon_{1N} k_{p1}}{k_m}}
\]  
(32)
we can easily conclude that the closed-loop system is semi-globally bounded [27].

Hence, the following inequality holds
\[
- \frac{k_{p1} \cdot W_{1N}^2/4 + \varepsilon_{1N}}{k_m} < b_r
\]
where \(b_r > 0\) represents the radius of a ball inside the compact set \(C_r\) of the tracking error \(\bar{P}_{pe}(t)\).

Thus, any trajectory \(\bar{P}_{pe}(t)\) starting in compact set \(C_r = \{ \|x\| \leq b_r \}\) converges within \(C_r\) and is bounded. Then the filtered error of system \(r_p(t)\) and the tracking error of the hysteresis \(\bar{P}_{pe}(t)\) converge to a small neighborhood around zero. According to the standard Lyapunov theorem extension [20], this demonstrates the UUB (uniformly ultimately bounded) of \(r_p(t), \bar{P}_{pe}(t), W_1, \varepsilon_1\) and \(e_2\).

IV. SIMULATION STUDY

In this section, the effectiveness of the NN-based adaptive controller is demonstrated on a piezoelectric actuator described by (18) with unknown hysteresis. The coefficients of the dynamic system and hysteresis model for the simulation purpose are adopted from [28]: \(m = 0.016 \text{kg}, \quad b = 1.2 \text{Ns/\mu m}, \quad k = 4500 \text{N/\mu m}, \quad c = 0.9 \mu \text{N/V}, \quad a = 6, \quad \beta = 0.5, \quad v_s = 6 \mu \text{m/s}, \quad \beta = 0.1, \quad k_{pd} = 50\). The input reference signal is \(y_d = 3 \cdot \sin(0.2 \pi t)\).

The Neural Network has 10 hidden neurons for the first part of neural network and 5 hidden neurons for the rest parts of neural network with three jumping points \((0, v_s, -v_s)\). The gains for updating output weight matrix are all set as \(\Upsilon = \text{diag}\{10\}_{25 \times 25}\). The activation function \(\sigma()\) is a sigmoid basis function and activation function \(\phi()\) has the definition \(\phi() = \left(\frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}\right)_{x \geq 0}\), otherwise zero. The parameters for the observer are \(K_a = 20, k_h = 100, \eta = 0.1, \gamma = 0.1, \quad K_{pr} = 10, \quad L_1 = 100, \quad L_2 = 1\) and initial conditions are \(\dot{y}(0) = 0, \quad \dot{\tau}(0) = 0\). Fig. 3 shows that the tracking performance is much better than that of adaptive controlled piezoelectric actuator without hysteretic compensator shown in Fig. 2. The input and output maps of NN-based pre-inversion hysteresis compensator and hysteresis are given in Fig. 4. The desired control signal and real control signal map (Fig. 4c) shows that the curve is approximate to a line which means the relationship between two signals is approximately linear with some deviations. In order to show the effectiveness of the designed observer, we compare the observed hysteresis output \(\hat{\tau}_{pe}\) and the real hysteresis output \(\tau_{pr}\) in Fig. 5.

![Fig. 2 Performance of NN controller without hysteretic compensator](image)

![Fig. 3 Performance of NN controller with hysteresis, its compensator and observer](image)

![Fig. 4 (a) Hysteresis’s input and output map \(\tau_{pr} vs. v\); (b) Pre-inversion compensator’s input and output map \(v vs. \tau_{pd}\); (c) Desired control signal and Observed control signal curve \(\hat{\tau}_{pr} vs. \tau_{pd}\).](image)

![Fig. 5 Observed hysteresis output \(\hat{\tau}_{pr}\) and real hysteresis output \(\tau_{pr}\).](image)

V. CONCLUSION

In this paper, an observer-based controller for piezoelectric
actuator with unknown hysteresis is proposed. An augmented feed-forward MLP is used to approximate a complicated piecewise continuous unknown nonlinear function in the explicit solution to the differential equation of Duhem model. The adaptive compensation algorithm and the weight matrix update rules for NN are derived to cancel out the effect of hysteresis. An observer is designed to estimate the value of hysteresis output based on the input and output of the plant. With the designed pre-inversion compensator and observer, the stability of the integrated adaptive system and the boundedness of tracking error are proved. Future work includes the compensator design for the rate-dependent hysteresis.

REFERENCES


APPENDIX

The definitions of the projection operators for the equation (29) are given as:

\[ P \{ \rho \} (\Theta, Y, r) = \]

\[
\begin{cases}
0 & \text{if } \rho = 0 \\
\Theta & \text{if } \rho < 0 \\
Y & \text{if } \rho > 0 \\
\end{cases}
\]

\[
P \{ \rho \} (\Theta, Y, r) =
\begin{cases}
0 & \text{if } \rho = 0 \\
\Theta & \text{if } \rho < 0 \\
Y & \text{if } \rho > 0 \\
\end{cases}
\]


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