Continuous-time Nonlinear System Identification Using Neural Network

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Abstract: In this paper, a continuous-time neural network nonlinear system identification algorithm using the system input/output signals is developed for a class of nonlinear systems. In control applications, the continuous-time nonlinear system model is more truthfully for the original nonlinear process compared to the widely used discrete-time neural network model. In the identification algorithm, a canonical form is selected to represent the identified system. The identification algorithm consists of two stages: (i) preprocessing the system input and output data to estimate the state variables in the chosen model coordinate; (ii) neural network parameter estimation. Discrete-time implementation of the developed algorithm is introduced. Identification examples are illustrated with a single-input-single-output benchmark model and a hardware-in-loop multi-input-multi-output 3 degrees-of-freedom differential thrust flight control testbed.

I. Introduction

Neural networks (NNs), with the ability to approximate a large class of nonlinear (NL) functions, provide a feasible uniform structure for NL system representations. In [1, 2] NN based system identification (ID) and control are systematically introduced. Since then, many different NN system ID approaches have been developed. An identified NN model can be used for system analysis and model-based controller design, as in [21].

In NL system ID algorithms, the process model is usually described as differential equations (continuous-time model) or difference equations (discrete-time model). In many NN system ID methods, discrete-time (DT) models are employed, such as NL ARMAX model [1], and the DT NL state space model [4][5].

For control applications, continuous-time (CT) dynamic models are more truthfully for the original NL systems since the physical systems are usually governed by CT differential equations. Many modern NL control design methods, such as feedback linearization, backstepping and trajectory linearization, utilize a CT state space model in the controller design and analysis. Moreover, the linear time-invariant (LTI) sampled-data control theory and design techniques are not readily applicable to NL systems.

The CT ID for linear system has been studied in [8, 23, 24, 25]. In [8, 23], concepts and techniques for CT system ID are introduced. In [24, 25], techniques for the ID of CT LTI system are discussed. To the authors’ best knowledge, it appears that there lacks research to extend these CT ID methods to NL system.

In this paper, techniques for CT LTI system ID are extended to a class of NL systems by using NNs. In the ID algorithm, a canonical form is selected to represent the identified system. In such a form, the state variables are derivatives of the output. Thus a linear filter, such as a pseudo-differentiator or state variable filter is able to estimate the state variables. The ID algorithm consists of two stages: (1) preprocessing the system I/O data to estimate state variables in the chosen model coordinate; (2) NN parameter estimation. Both the model structure and ID algorithms are CT. The discrete time implementation is also discussed in this paper.

One advantage of the developed ID method is that the identified model can be integrated with an existing analytical CT system model to improve the model fidelity. Another advantage is that the identified model lends itself to existing NL design methods. The DT model, in many cases, is an approximation of the CT model, which may result in a higher order model than the corresponding CT model. In DT system ID, a fixed sampling time interval is usually required. Such restriction is not necessary in CT system ID.

In [7], the CT dynamics are approximated by a NN model with difference quotients as part of the NN input. The overall model is actually not a CT model. In adaptive NN control, such as indirect NL adaptive model reference NNs control [9, 10], NN feedback linearization control [11-15], NN backstepping control [16-20], and adaptive NN trajectory linearization control [21], the implicit CT model identifier are used. In these implicit ID algorithms, the system state variables are all measurable or a non-model based state variable estimator is employed. The developed ID method in this paper is able to identify a CT NL system from merely input/output (I/O) data.

It should be noted that in [6], identifying and controlling NL CT systems using dynamic NNs are studied. Similar to the method in this paper, the identified system is transformed into a canonical form. In [6], dynamic NNs are employed in which the dynamic of the pseudo-differentiator is considered as part of the dynamic NN. The controller structure of [6] is in the adaptive control framework, which is similar to [11- 21]. In these NN adaptive structures, only the dynamics along the system trajectory is identified. In this paper, the developed method is capable of identifying the system dynamics in a manifold given properly designed system excitation input and the corresponding output.

In Section II we will present the proposed CT ID algorithm. Identification examples are presented in Section III with a single-input-single-output (SISO) benchmark model and a hardware-in-loop multi-input-multi-output (MIMO) 3 degrees-of-freedom (DOF) differential thrust flight control testbed.

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II. Neural Network Identification of Continuous-time Nonlinear System

A. Identification Model Structure

In this paper we consider identifying the NL affine system described as

\[ \dot{\xi} = f(\xi) + g(\xi)\mu \]

\[ \eta = h(\xi) \]

where \( \xi(t) \in \mathbb{R}^n \), \( \mu(t) \in \mathbb{R} \), \( \eta(t) \in \mathbb{R} \), \( f() \), \( g() \) are smooth vector fields defined on a domain \( D \in \mathbb{R}^n \), \( h() \) is a smooth and bounded function on \( D \). Confining our discussion on SISO systems is not a limitation of the proposed ID method. The proposed method, in principle, can be extended to MIMO systems.

In order to identify (1) from the input and output signal, the system (1) is required to be identifiable, which means that the entire dynamics in the domain \( D \) can be excited by feasible input signals, and the dynamics in the system can be observed from the output signals. This requirement can be satisfied if the plant dynamics (1) are controllable and observable.

Given that the plant dynamics are identifiable, an appropriate excitation signal is required to stimulate the dynamics of the system. In linear system ID, excitation signals with wide bandwidth, such as impulse and pseudo random signals are applied, such that all the modes in the system are stimulated. In NL ID, the complexity of the system prevents mechanically extending the linear system ID excitation signal design. Usually some a priori knowledge of the system dynamics is required and each ID process is designed individually. In this paper, we assume some appropriate excitation signals are available. Then the objective of ID is to find the appropriate vector fields \( f() \) and \( g() \) and smooth function \( h() \) which can reproduce the input-output behavior, given the system input and output signals.

Geometric theory of NL systems [22] shows that the minimal realization of a NL system is uniformly controllable and observable. The geometric theory also shows that two minimal realizations are locally diffeomorphic.

Based on the above discussions, a special form of NL state space model is chosen to represent the system dynamics, which is

\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_n &= f_n(x) + g_n(x)\mu \\
\eta &= [1 \ 0 \ 0 \ldots 0] x
\end{align*} \]

where \( x = [x_1, x_2, \ldots, x_n]^T, \mu \) and \( \eta \) are the state variable, input and output respectively, \( f_n(x) \) and \( g_n(x) \) are smooth functions. Define the Lie derivative of a scalar function \( h(\xi) \) with respect to a vector field \( f(\xi) \) as \( L_f h(\xi) = \frac{\partial h(\xi)}{\partial \xi} f(\xi). \)

Denote \( L^0_f h(\xi) = h(\xi), \) and \( L^0_f h(\xi) = L_f (L^0_f)^{-1} h(\xi). \) If the system (1) has a well defined relative degree \( r = n, \) then there exist a state coordinate transformation \( x = \Phi(\xi), \) such that the system (1) and (2) are diffeomorphic, and

\[ \begin{align*}
f_n(x) &= L_f^r h(\Phi^{-1}(x)) \\
g_n(x) &= L_g^r L_f^r^{-1} h(\Phi^{-1}(x)).
\end{align*} \]

System (2) is in an observability canonical form of SISO NL systems with the relative degree \( n. \) In such form, a pseudo-differentiator can be used to estimate the state variables from system output signals. The pseudo-differentiator actually functions as a non-model based observer for (2). The ID of the vector fields \( f() \) and \( g() \) and smooth function \( h() \) in (1) is transformed to ID of \( f_n(x) \) and \( g_n(x) \) in (2).

B. Identification Algorithm

In the developed ID algorithm, the NL functions in the system (2) are approximated by NNs. To simplify the discussion below, linear parameter NNs are used to illustrate the algorithm. The algorithm can be extended to NNs with NL parameters. By using the linear parameter NN, system (2) is represented as

\[ \eta^{(n)} = W_f \Phi_f(x) + W_g \Phi_g(x)\mu + d(t) \]

where \( x = [\eta, \ldots, \eta^{(n-2)}, \eta^{(n-1)}], \) \( \Phi_f(x) \) and \( \Phi_g(x) \) are vectors of neuron stimulation functions, \( W_f \) and \( W_g \) are the NN weights and \( d(t) \) is the NN reconstruction error. The objective is to find the optimal \( W_f \) and \( W_g \) to minimize \( ||\eta^{(n)} - W_f \Phi_f(x) - W_g \Phi_g(x)\mu||_2. \)

The ID algorithm consists of two stages, which is similar to linear system ID methods in [24][8]. The primary stage is to estimate the input and output signal's derivatives. There are several ways to estimate the I/O signals' derivative, as introduced in [24] [25]. In the proposed ID algorithm, a series of pseudo-differentiators are used. The pseudo-differentiators are given in the transfer function form

\[ F_i(s) = \frac{a_is^i}{s^n + a_{m}s^{m-1} + \cdots + a_2s + a_1}, \ i = 1 \ldots n \]

where \( n \) is the system order and \( m \) is an integer \( m \geq i. \) A pseudo-differentiator is a band-pass filter which is capable of removing the system noise and selecting the desired bandwidth. The estimated derivatives are \( \hat{\eta}^{(i)}(s) = F_i(s)\eta(s), \) and the estimated state vector is

\[ \hat{x} = [\hat{\eta}, \ldots, \hat{\eta}^{(n-2)}, \hat{\eta}^{(n-1)}]. \]

Another approach to process the I/O data in the primary stage for linear systems is to integrate I/O data repeatedly. Such integration approach is not applicable for NL systems since there is no simple form for the integration of a NL function.

The second stage of the ID algorithm is the parameter estimation stage, in which the NN parameters are estimated from the data generated by the primary stage. CT optimization algorithms, such as recursive least square, Kalman filter and other gradient optimization methods, can be used in this stage. A simple gradient algorithm to update \( \hat{W}_f \) is expressed as
\[
\dot{\hat{W}}_f(t) = -\gamma \left( \tilde{W}_f \Phi_f(x) + \tilde{W}_g \Phi_g(x) \tilde{n} - \hat{n}^{(n)} \right) \Phi_f(v) - \beta \dot{\hat{W}}_f \\
\dot{\hat{W}}_g(t) = -\gamma \left( \tilde{W}_f \Phi_f(x) + \tilde{W}_g \Phi_g(x) \tilde{n} - \hat{n}^{(n)} \right) \Phi_f(v) \mu - \beta \dot{\hat{W}}_g
\]

where \( \gamma \) and \( \beta \) are positive constants. The parameter \( \gamma \) is the learning rate, and the parameter \( \beta \) is the robustification term to ensure the parameter convergence. In the actual application, other robustification techniques can also be applied.

The ID algorithm for system (2) is illustrated in Figure 1. It is a serial-parallel ID method.

![Figure 1 On-line System ID from I/O Data](image)

In the proposed ID method, the model structure and ID algorithm are both CT. The essential component in both the primary and secondary stages is a series of integrators. Thus such ID structure is suitable for implementation on hardware, such as an integrated circuit (IC) with programmable operational amplifiers (Op-Amps).

In practical applications, the ID algorithm can also be implemented in DT on a digital computer, while the identified model structure is still in CT form.

C. Discrete-time implementation

In the DT implementation, the ID problem of the system (2) is that given a sampled measurement of system I/O signal at the time stamp \( t_k (k = 1, 2, \ldots, N) \), how to estimate the NN parameter. First, the filters in the primary stage are approximated by a DT implementation. Numerically, the DT implementation is similar to DT ID method, though the underlying mathematical models are different. The DT approximation requires that the sampling rate be sufficiently high to capture the system dynamics, and the DT filters are executed at high accuracy.

Once the derivatives are obtained, the parameter estimation in the second stage becomes a regression problem. Assuming there are total \( N \) samples of I/O measurement at \( \{ t_k \}_{k=1, \ldots, N} \), at each sampling time, we have

\[
\hat{\eta}^{(n)}(t_k) = W_f \Phi_f(\tilde{x}(t_k)) + W_g \Phi_g(\tilde{x}(t_k)) \tilde{n}(t_k) + d(t_k)
\]

where \( \tilde{x}(t_k) = \left[ \hat{\eta}^{(n-1)}(t_k), \hat{\eta}^{(n-2)}(t_k), \ldots, \hat{\eta}(t_k) \right] \). Assuming \( d(t_k) \) is Gaussian, the NN parameters can be estimated as

\[
\hat{W}_f, \hat{W}_g = X M^T (M M^T)^{-1}
\]

where

\[
X = \left[ \hat{\eta}^{(n)}(t_1), \hat{\eta}^{(n)}(t_2), \ldots, \hat{\eta}^{(n)}(t_N) \right]
\]

and

\[
M = \begin{bmatrix}
\Phi_f(x(t_1)) & \Phi_f(x(t_2)) & \cdots & \Phi_f(x(t_N)) \\
\Phi_g(x(t_1)) & \Phi_g(x(t_2)) & \cdots & \Phi_g(x(x(t_N))
\end{bmatrix}
\]

It is noted that the algorithms described in this section are a framework. Thus, more complex algorithms for filter design and parameter regression can be applied.

III. Identification Example

In this section, two NL system ID examples using the proposed method are presented. The first example is a 2nd-order NL SISO benchmark system. The second example is a NL MIMO 3DOF differential thrust flight control testbed.

A. A 2nd-order nonlinear SISO benchmark system

The NL system to be identified is

\[
\begin{align*}
\dot{\xi} &= f(\xi) + g(\xi) \mu \\
\eta &= h(\xi)
\end{align*}
\]

where \( \xi = [\xi_1, \xi_2]^T, f(\xi) = [\sin \xi_2 - \xi_1]^T, g(\xi) = [0, 1]^T, \)

\( h(\xi) = \xi_1 \) and \( a = 1 \). The objective is to identify the system dynamics on the domain \( D = \{ \xi \in \mathbb{R}^2 | \xi_1 > 0, -\frac{\pi}{2} < \xi_2 < \frac{\pi}{2} \} \) and \( 0 < u < 1 \).

The Lie derivatives of the system are \( L_1^1 h(\xi) = \sin \xi_2, \)

\( L_1^2 h(\xi) = -a \xi_2 \cos \xi_2, \)

\( L_2^1 h(\xi) = 0 \quad \text{and} \quad L_2^2 h(\xi) = \cos \xi_2. \)

The system has relative degree \( r = 2 \) on \( D \). The coordinate transformation is

\[
x = \Phi(\xi) = \left[ \begin{array}{c}
\xi_1 \\
\sin \xi_2
\end{array} \right]
\]

In the normal form, \( f_2(x) = -ax_1^2 \cos(\arcsin \left( \frac{x_2}{a} \right)), \)

\( g_2(x) = a \cos(\arcsin \left( \frac{x_2}{a} \right)). \)

In the NN ID, system input signal is selected as \( \mu(t) = 0.6 + 0.5 \sin(2t + \frac{\pi}{2}) \), and the system initial condition is \( \xi_0 = [0, 0] \). Figure 2 shows the phase plot of the system response, which covers a large portion of \( D \). In the on-line ID, the pseudo-differentiators' bandwidths are set as \( 40 \) (rad/s), \( \gamma = 5 \) and \( \beta = 0.5 \).

Figure 3 shows the estimated \( \hat{\eta}^{(2)} \) and the NN approximation. Figure 4 illustrates functions of \( f_2(x) \) and \( g_2(x) \), which are retrieved from the simulation, and NN approximation. Figure 5 shows the time history of \( ||\hat{W}_f||_2 \) and \( ||\hat{W}_g||_2 \). Figure 3 and Figure 4 show that the NN is able to approximate the NL functions in the normal form.

The identified model has two integrators, which accumulate the NN model errors. In order to test the NN identified model, a serial-parallel form based on the identified model is used. An input signal that is different from the training signals, is fed into the plant and the NN model

\[
\begin{align*}
\hat{x}_1 &= \tilde{x}_2 - k_1(\hat{\eta} - \eta) \\
\hat{x}_2 &= \tilde{W}_f \Phi_f(\tilde{x}) + \tilde{W}_g \Phi_g(\tilde{x}) \mu - k_2(\hat{\eta} - \eta) \\
\hat{\eta} &= \tilde{x}_1
\end{align*}
\]
where $k_1$ and $k_2$ are selected such that the characteristic equation $\lambda^2 + k_1 \lambda + k_2 = 0$ has stable eigenvalues.

In the first test, the input is $\mu = 0.4$. Figure 6 shows plant output and the NN system output. Figure 7 shows the comparison of the plant $\eta^{(2)}$ and NN approximation. Tests using sinusoidal input signals with different frequencies from the training input signal are shown in Figure 8 to Figure 11.

From this example, it can be concluded that the proposed CT NN NL system ID is feasible for capturing a NL system dynamics. However, similar to other ID methods, the performance of the proposed ID method depended on the quality of the training data. The training data should be designed to stimulate all the system dynamic modes.
\[ \dot{\phi} = p + q \sin(\phi)\tan(\theta) + r \cos(\phi)\tan(\theta) \]
\[ \dot{\theta} = q \cos(\phi) - r \sin(\theta) \]
\[ \dot{\psi} = q \sin(\phi)\sec(\theta) + r \cos(\phi)\sec(\theta) \]
\[ \dot{p} = I_{pp}^q p q + I_{pr}^q q r + g_0^p T_i + g_0^p T_n \]
\[ \dot{q} = I_{pp}^q p q + I_{pr}^q r^2 + I_{p^2}^q p r + g_0^q T_m \]
\[ \dot{r} = I_{pq}^q p q + I_{qr}^q q r + g_0^q T_i + g_0^q T_n \]

where \( \phi, \theta \) and \( \psi \) are the Euler roll, pitch and yaw angle respectively; \( p, q \) and \( r \) are the body rate of the UFO in the body frame; and \( T_i, T_m \) and \( T_n \) are rolling, pitching and yawing moment generated by the propeller in the body frame. The relationship between the propeller moment and the DC motor voltage is described by the following equation
\[ [T_i, T_m, T_n]^T = T_A[V_1, V_2, V_3]^T \]
where \( V_1, V_2 \) and \( V_3 \) are the applied voltage to each motor, and \( T_A \) is the control allocation matrix. More details of the modeling of UFO and the trajectory linearization control (TLC) design were summarized in [26]. In this paper, NNs are used to identify bodyrate \((p, q, r)\) dynamics from the simulation data.

**B. MIMO 3DOF Flight Test Bed Dynamics**

The second ID example is to identify the dynamics of a MIMO NL 3DOF differential thrust flight control testbed — the Quanser UFO, which is installed with three propellers driven by DC motors. It is able to rotate freely about three axes. The attitude angles are measured by optical encoders installed at the rotation axes. Figure 12 shows the UFO’s setup.

The simplified dynamic model of the UFO is a MIMO NL system, which is described as

**Figure 12 Quanser’s UFO**

Figure 13 and Figure 14 show the UFO’s response to a predefined command trajectory using TLC. It should be noted that the plant itself is unstable without the closed-loop controller. Similar to the first example, a state variable filter was used to estimate the body rate’s derivatives. The identified NN model was tested in the serial-parallel configuration with the plant simulation model. Figure 15 shows the NN model simulation result compared to the actual plant response. It can be seen that the NN identified model is close to the plant.

**Figure 13. UFO attitude tracking response**

**Figure 14. UFO attitude tracking response**

**Figure 15. UFO attitude tracking response**
In this paper, a continuous-time nonlinear system identification method using neural network for a class of nonlinear systems is developed. Identification examples are illustrated to demonstrate the effectiveness of the proposed method.

**Reference**


