A New Modeling and Control Framework for MEMS Characterization Utilizing Piezoresistive Microcantilever Sensors

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Abstract—This paper presents a comprehensive modeling and control framework for characterizing MEMS utilizing piezoresistive microcantilevers. These microcantilevers have recently received widespread attention due to their extreme sensitivity and simplicity in a variety of sensing applications. Most of the current studies; however, focus on a simple lumped-parameters representation rather than modeling the piezoresistive microcantilever itself. Due to the applications of the piezoresistive microcantilevers in nanoscale force sensing or non-contact atomic force microscopy with nano-Newton to pico-Newton range force measurement requirement, precise modeling of the piezoresistive microcantilevers is essential. For this, a distributed-parameters modeling is proposed and developed here to arrive at the most complete model of the piezoresistive microcantilever with tip-mass, tip-force and base movement considerations. In order to have online control and real-time sensor feedback, an inverse model of piezoresistive microcantilever is needed which utilizes the output voltage of the piezoresistive layer as well as the base motion information to predict the force acting on the microcantilever’s tip. Utilizing a novel approach, an inverse modeling framework and control algorithm are then proposed for the characterization of MEMS utilizing piezoresistive microcantilevers. Following the mathematical modeling and controller design, both numerical simulations and experimental results are presented to demonstrate the accuracy of the proposed distributed-parameters modeling when compared with the previously reported lumped-parameters approach. It is shown that by utilizing the distributed-parameters model rather than lumped-parameters approach and by predicting the exact motion of each point on the microcantilever, the precision of the piezoresistive microcantilever’s model is significantly enhanced.

I. INTRODUCTION

When microcantilevers were used as the force sensor in Atomic Force Microscopy (AFM), researchers discovered their tremendous sensitivity to different environmental factors such as acoustic noise, light, temperature, humidity and ambient pressure [1,2]. Due to the simplicity and availability, the AFM instrumentation has attracted widespread attention during the last decade. Increasing the number of researches in different disciplines such as biological, materials science, imaging and sensing indicate the importance of the microcantilever-based sensors and actuators [3,4].

Due to the recent advances in manufacturing of the microcantilevers with different shapes and material properties as well as embedded actuation and sensing capabilities, sensing based on the bending of microcantilever beams is known as a simple, cheap and accurate way when compared to other sensing methods [5]. Chen et al. [6] suggested a microcantilever model to predict the change of spring constant caused by surface stress; Fritz et al. [7] measured the DNA hybridization and receptor-ligand binding employing a microcantilever. Same studies based on the biochemical–mechanical transduction have recently been reported in adsorption of low-density lipoprotein, antigen–antibody binding, and an artificial nose. In all the above approaches, the surface stress is mainly measured by acquiring the microcantilever deflection utilizing an optical lever technique such as laser microscopy; however, alignment and calibration of the optical element for different testing species are not trivial, the optical lever technique is costly and even may fail when operating in non-transparent liquid [8].

Microcantilever’s deflection and surface stress measurement via piezoresistive layer has been recently proposed [9,10]. Piezoresistive cantilevers are usually used in force microscopy applications where difficulties in laser alignment make optical detection inconvenient. Some of the piezoresistive-based sensing applications are atomic data storage systems, cantilever arrays, high vacuum AFM measurements and portable cantilever-based sensors [11].

Even though numerous studies have recently focused on piezoresistive microcantilever sensors, almost in all of them, the piezoresistive microcantilever is replaced by a simple lumped-parameters model [9,10]. Due to the extreme precision of the piezoresistive microcantilever sensors, which is in the range of the nano-Newton, utilizing a more precise modeling approach is critical [12]. Here, a distributed-parameters modeling approach is proposed and developed to obtain the most accurate model of the microcantilever.

For the online feedback control purpose, finding the inverse model of the microcantilever is critical. This area in distributed-parameters piezoresistive microcantilever modeling approach has not received much attention. For this, a novel approach is proposed to obtain an accurate model of
the piezoresistive microcantilever which employs the output voltage of the microcantilever’s piezoresistive layer as well as information about the cantilever’s base motion to precisely predict the force acting on the cantilever’s tip. This model is then utilized in a novel control framework which controls the force acting on the microcantilever’s tip to characterize different MEMS.

II. MOTIVATION AND PROBLEM STATEMENT

Microcantilever-based sensors and actuators have recently attracted widespread attention due to their tremendous applications in biological and materials science technologies. Nanomanipulation, defining materials properties, fabricating electronic chipsets, testing of microelectronics circuits, assembly of MEMS, teleoperated surgeries, micro-injection and manipulation of chromosomes and genes serve as demonstrable examples of microcantilever applications. More specially, Fig.1 depicts a 3 DOF nanorobotic manipulator, namely MM3A, which is utilized for nanomanipulation, identification, sensing and imaging purposes. The piezoresistive microcantilever which can be appended to the nanomanipulator’s tip (see Fig. 2-a), combined with its base motion and alignment provided by the nanomanipulator, can be utilized in variety of applications such as force sensing, non-contact AFM imaging and nanomanipulation with nanoscale resolution requirement.

In previous publications of the authors [14-21], the problem of modeling and control of MM3A nanomanipulator has been addressed and a control framework was proposed to provide the most accurate movement of the nanomanipulator’s tip at nanoscale. In order to employ a combined task of nanomanipulation and sensing, there is a need to precisely model the piezoresistive microcantilever to arrive at a relationship between piezoresistive layer output and base motion and tip force of the microcantilever.

Due to the interaction force between nanomanipulator’s tip and nanoparticle, the piezoresistive microcantilever bends and because of the longitudinal deflection of the beam, the electrical resistance of the piezoresistive layer on the microcantilever changes (see Fig. 2). This change in piezoresistive layer’s electrical resistance can be converted into the electrical voltage signal by utilizing a simple electrical circuit.

III. MATHEMATICAL MODELING

Previously, lumped-parameters modeling scheme has been introduced in the literature and previous publications of the authors [9,10,14] to model the piezoresistive microcantilever. Here a distributed-parameters approach [14] is introduced and compared to the previously reported discrete modeling.
In modeling the piezoresistive microcantilever as a distributed-parameters system, the microcantilever in Fig. 2 is replaced by an Euler-Bernoulli beam depicted in Fig. 3. In Fig. 3, \( w(x,t) \) represents the deflection of the microcantilever at the position \( x \) on the cantilever and time \( t \). Note, \( w(L,t) \) represents the equivalent tip deflection of the beam, \( w_i(t) \), in the lumped-parameters representation. Utilizing the microcantilever model depicted in Fig. 3, the kinetic and potential energies of the piezoresistive microcantilever, Hamilton’s Principle and Assumed Mode Model (AMM) approach, the state-space representation of the system can be expressed as [14]:

\[
X_{ss} = A_{ss} X_{ss} + B_{ss} U_{ss}
\]

(1)

\[
Y_{ss} = C_{ss} X_{ss} + D_{ss} U
\]

(2)

where system states are:

\[
\begin{bmatrix}
\Omega
\end{bmatrix} = \begin{bmatrix}
\int_{0}^{t} f_s(t) \, dt, ds
\end{bmatrix}
\]

(3)

\[
\Omega = \begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_n
\end{bmatrix}
\]

(4)

with system inputs and outputs as:

\[
U = \begin{bmatrix}
\tilde{S}(t) \\
f(t) \\
\tilde{f}(t) \\
\ddot{f}(t)
\end{bmatrix}
\]

(5)

\[
V_o = Y_{ss} C_{pe}
\]

(6)

where all the variables and their definitions are listed in [14].

In the proposed state-space description of the piezoresistive microcantilever (1-12), the microcantilever is modeled with \( \mathbf{U} = [\tilde{S}(t), f(t), \tilde{f}(t), \ddot{f}(t)]^T \) as the input vector and \( Y_{ss} = V_o / C_{pe} \) as the output. The proposed formulation can be utilized to simulate the microcantilever’s output voltage as a function of input force acting on the microcantilever’s tip; however, in the actual experiment there is a need to determine the force acting on the microcantilever’s tip due to a certain base motion and the piezoresistive layer’s output voltage. For this, by replacing vector \( X_{ss} \) with vector \( x_1, \) force \( f \) with variable \( x_2, \ddot{f} \) with variable \( x_3 \) and \( \dddot{f} \) with variable \( x_4, \) equations (1-12) are rearranged as:

\[
\begin{align*}
\dot{x}_1 &= A_{ss} x_1 + B_{ss} \tilde{S} + B_{ss} x_2 + B_{ss} x_3 + B_{ss} x_4 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
V_o / C_{pe} &= C_{ss} x_1 + D_{ss} x_2
\end{align*}
\]

(13-16)

where \( B_{ss} \) and \( D_{ss} \) represent the \( i^{th} \) column of \( B_{ss} \) and \( D_{ss} \) matrices, respectively. Eliminating variable \( x_4 \) in equations (13-16) yields the following equations:

\[
\begin{align*}
\dot{x}_1 &= \left( A_{ss} \frac{B_{ss} A_{ss}}{C_{ss} B_{ss}} \right) x_1 + \left( \frac{B_{ss}}{C_{ss} B_{ss}} \right) x_2 + \\
&\quad \left( \frac{B_{ss}}{C_{ss} B_{ss}} \right) x_3 + \left( \frac{B_{ss}}{C_{ss} B_{ss}} \right) \tilde{S} + \\
&\quad \left( \frac{B_{ss}}{C_{ss} B_{ss}} \right) V_o
\end{align*}
\]

(17)

\[
\begin{align*}
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{y} &= \frac{1}{C_{ss} B_{ss}} \left( \frac{B_{ss} \left( C_{ss} B_{ss} + D_{ss} \right)}{C_{ss} B_{ss}} \right) \tilde{S} + \\
&\quad \frac{B_{ss}}{C_{ss} B_{ss}} \left( \frac{B_{ss} \left( C_{ss} B_{ss} + D_{ss} \right)}{C_{ss} B_{ss}} \right) V_o
\end{align*}
\]

(18-20)

Utilizing the state-space modeling demonstrated in equations (17-20), the force acting on the microcantilever’s tip (i.e., output \( y = x_2 \)) can be obtained by having the microcantilever’s base acceleration \( (\tilde{S}) \) and piezoresistive layer’s output voltage’s derivative with respect to time \( (V_o') \).
IV. Controller Design

There is a variety of applications for the piezoresistive microcantilevers; however, in most of these applications there is a need to keep the force acting on the microcantilever’s tip at a certain constant value or track a predefined force trajectory. For instance, in contact AFM there is a need to keep the force acting on the microcantilever’s tip at a desired constant value; in nanomanipulation utilizing microcantilevers, there is need to have control on the applied force; and in nondestructive mechanical characterization of MEMS using cantilevers, the applied force should always track a predefined trajectory.

Hence, the control objective is to restrain the force acting at the microcantilever’s tip to a certain trajectory. The only control input is the microcantilever’s base motion which is applied through a piezoelectrically-driven micro/nano-stage. The microcantilever’s base motion can be monitored utilizing the built-in position sensors, and the microcantilever’s piezoresistive layer’s output voltage can be tracked employing a Wheatstone bridge. Fig. 4 depicts a schematic of the designed controller to track the desired force trajectory, employing the base motion.

As illustrated in Fig. 4, the desired force at the microcantilever’s tip  \( f_{\text{desired}}(t) \) is fed to the “Sample’s Mechanical Behavior” dynamics block. Sample’s Mechanical Behavior block relates the microcantilever’s tip motion  \( S(t) \) to the force acting on it. This relationship can be demonstrated by a simple spring constant \( k \) (i.e., \( f(t) = k(S(t) - w_L(t)) \)) or more complicated dynamics. In order to control the force acting on the microcantilever’s tip, there is a need to know at least a rough estimation of this relationship to determine how the microcantilever’s tip should move in order to direct the tip force to a desired value.

As seen in Fig. 4, piezoresistive microcantilever’s output voltage  \( V_o(t) \) is fed back into an observer to acquire the output voltage’s derivative \( \dot{V}_o(t) \). The observer utilized here is a variable structure observer, previously introduced in [14-21]. If \( q \) is the system parameter (here, \( S(t) \), \( \dot{S}(t) \) and \( V_o(t) \) ), in the variable structure observer the objective is to estimate \( \dot{q} \). If \( \dot{q} \) is the estimated value of \( \dot{q} \) by the observer, then the structure of the observer is selected [21]:

\[
\begin{align*}
\ddot{q} &= q - \dot{q} \quad (21) \\
\dot{\ddot{q}} &= \dot{q} - \dddot{q} \quad (22) \\
\dot{\dddot{q}} &= k_1 \text{sgn}(\dddot{q}) + k_2 \dddot{q} \quad (23)
\end{align*}
\]

Or in brief,

\[
\dot{\dddot{q}} = p + k_0 \dddot{q} 
\]

(24)

in which \( k_0, k_1 \) and \( k_2 \) are observer gains. As seen, the observer design does not depend on the controller or even system models and variables.

The output voltage’s derivative \( \dot{V}_o(t) \) and the base acceleration \( \dddot{S}(t) \) are fed back into the “Inverse Distributed Parameters Modeling” block, which is introduced in (17-20), to estimate the force acting on the microcantilever’s tip, \( f(t) \). Utilizing the lumped-parameters modeling approach and equations [14], the microcantilever’s tip deflection \( w_L(t) \) can be computed. Employing the microcantilever’s tip deflection \( w_L(t) \) and the microcantilever’s tip desire motion \( S(t) - w_L(t) \) desired the desired microcantilever base motion \( S_{\text{desired}}(t) \) is calculated. Comparing the desired microcantilever’s base motion \( S_{\text{desired}}(t) \) with the microcantilever’s actual base motion \( S(t) \), the microcantilever’s base position error is obtained and fed back into the “Modified Robust Controller wiht Perturbation Estimation (MRC-PE)” block, previously introduced by the authors [21]. The controller command is then sent to the nanostage, which moves the microcantilever’s base respectively in order to regulate the force acting on the microcantilever’s tip.
V. NUMERICAL SIMULATIONS AND EXPERIMENTAL VERIFICATION

The self-sensing microcantilever named here PRC-400 which is utilized as the piezoresistive microcantilever for force sensing applications here, is fully discussed with the numerical values representing this microcantilever in previous publications of the authors [14].

A set of simulation results and experimental tests are utilized here to demonstrate the accuracy and effectiveness of the proposed modeling approach. The experimental setup, consists of Polytec MSA-400 Micro System Analyzer, Physik Instrumente (PI) P-753.11c PZT-driven nanostage, PI E-500 Modular Piezo Control System, PI M-126.DG translation microstager, PI C-809.40 4-channel servo-amplifier Motion I/O interface, dSPACE DS1104 controller board, Pulnix TM-1400 camera with 50x Mitutoyo lens, Kleindiek FMS-EM force measurement system, and Seiko Instruments Institute PRC-400 self-sensing microcantilever (see Fig. 5).

In order to calibrate and verify the derived distributed-parameters modeling, given in (17-20), there is a need to apply predefined forces to the piezoresistive cantilever’s tip and monitor the output voltage and then compare it with the simulation results.

The simulation results for the lumped-parameters modeling and distributed-parameters models with different number of modes and experimental results are depicted in Fig. 6. When a range of step force is applied at the tip of the microcantilever, output voltage of the piezoresistive layer of the microcantilever changes from zero to a final steady-state value. Fig. 6 depicts the final steady-state voltage of the piezoresistive layer’s voltage output with respect to a range of forces. As seen from Fig. 6, employing the lumped-parameters modeling approach to model the piezoresistive microcantilever results in a significant error in the output voltage to the input force ratio \( V_o/f \), the slope of the lines in Fig. 6. As is illustrated in Fig. 6, by employing more than two modes, the distributed-parameters modeling derived in (17-20) can precisely predict the piezoresistive microcantilever’s output voltage due to an applied tip force. However, utilizing the commonly used lumped-parameters model, results in a 20% error in the predicted output voltage.

In recent publication of the authors [22], the proposed controller scheme has been fully discussed and its performance and effectiveness has been proven through the comprehensive simulation and experimental results; a set of force tracking problems was defined and it was shown utilizing the proposed controller algorithm here and the distributed-parameters modeling approach can significantly improve performance of the force tracking [22].

VI. CONCLUSION

This paper presented a comprehensive modeling and control approach in characterizing MEMS utilizing a piezoresistive microcantilever sensor. Due to the applications of the piezoresistive microcantilevers in nanoscale force sensing or non-contact atomic force microscopy with nano-Newton to pico-Newton range force measurement requirement, precise modeling of the piezoresistive microcantilevers is essential. Instead of the previously used...
lumped-parameters representation, a distributed-parameters modeling was proposed and developed here to arrive at the most complete model of the piezoresistive microcantilever with tip mass, boundary force and base movement consideration. In order to have online control and real-time sensor feedback, an inverse model of piezoresistive microcantilever is needed in order to utilize the output voltage of the piezoresistive layer as well as the base motion information to predict the force acting on the microcantilever’s tip. For this, utilizing a novel approach, an inverse modeling framework and control algorithm for the characterization utilizing the piezoresistive microcantilever was presented. Following the mathematical modeling and controller design, both numerical simulations and experimental results were presented to demonstrate the accuracy of the proposed distributed-parameters modeling when compared with the previously reported lumped-parameters modeling approach. It was shown that for the microcantilever used here, when using only a 2-mode representation of distributed-parameters model, significant enhancement in piezoresistive output precision occur due to an applied force. Such modeling enhancement could be utilized in a variety of MEMS characterization tools utilizing microcantilevers.

REFERENCES


