Distributed Tracking in Sensor Networks with Limited Sensing Range

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Abstract—In this paper, we address the problem of distributed tracking of a maneuvering target using sensor networks with nodes that possess limited sensing range (LSR). In such sensor networks, a target can only be observed by a small percentage of the sensors and is practically hidden to the remaining majority of the nodes. This feature is shared among most of today’s wireless sensor networks and differentiates them from their traditional counterparts involving data fusion for long-range sensors such as radars and sonars. Distributed Kalman filters have proven to be effective and scalable algorithms for distributed tracking in sensor networks. Our main contribution is to give a message-passing version of the Kalman-Consensus Filter (KCF)—introduced by the first author in CDC ‘07—that is capable of distributed tracking of a maneuvering target with a satisfactory performance. The architecture of this filter is a peer-to-peer (P2P) network of microfilters as extensions of local Kalman filters. The model proposed for the maneuvering target is a piece-wise linear switching system with two distinct modes of behavior that enables the target to stay inside a rectangular region in all time (for a bounded set of initial conditions). Simulation results are provided for a lattice-type sensor network with 100 LSR nodes tracking a target with switching modes of behavior which demonstrate the effectiveness of the proposed distributed data fusion and tracking algorithms.

Index Terms—sensor networks, Kalman-Consensus filtering, distributed data fusion, target tracking

I. INTRODUCTION

Sensor networks have emerged as ideal means of massive distributed sensing for scientific data gathering and tracking events/targets in a variety of home, health, industrial, and security applications. Distributed data fusion and filtering algorithms form the core component of any target tracking system [6]. Traditionally, such tracking systems heavily relied on Kalman filters (KFs) or Extended Kalman filters (EKF) [1] for nonlinear estimation and their decentralized forms [7], [21].

Consensus-based tracking [15], [17] and synchronization algorithms [24] in sensor networks are scalable and resilient have recently emerged as powerful tools for collaborative information processing. Distributed Kalman filtering (DKF) algorithms rely on consensus filters [20], [25]. A new generation of DKF algorithms with a peer-to-peer (P2P) architecture that rely on reaching a consensus on estimates of local Kalman filters have recently been introduced by Olfati-Saber in [16]. We refer to this class of distributed estimation algorithms as Kalman-Consensus Filters (KCF). An analytical derivation of the KCF algorithm is given in [16].

For nearly three decades, the main emphasis of multi-target tracking using multi-sensor platforms has been focused on fusion of information obtained from long-range sensors such as radars and sonars [22], [3], [4].

Today’s sensor networks are made out of limited sensing range (LSR) nodes with a relatively large communication range (or radio range). A wireless sensor network with nodes that have limited sensing range is depicted in Fig.1. The main challenge in tracking events using sensor networks with LSR is that a target cannot be observed by all sensors in the network. This property complicates the task of detection and tracking of all existing targets/events/processes.

Fig. 1. An ad hoc sensor network with n = 100 nodes with limited sensing range (LSR) tracking m = 5 targets. Each target induces a set of active sensors that can sense the target. The ad hoc network topology is omitted for the purpose of clarity.

In distributed tracking using sensor networks, the objective is to estimate and track the state of the targets using a distributed algorithm involving message-passing between a node i and all of its neighbors N_i over a network. Let us denote the communication range of the sensors by r_c and the sensing range by r_s. The communication range of the nodes determines the network topology G = (V, E) that we assume is an undirected graph.

Remark 1. Assuming that the ratio r_c/r_s is sufficiently large (i.e. r_c/r_s ≥ 2), regardless of the distribution of the location of the nodes, the set of active sensors that sense the...
target form a clique due to the triangular inequality.

In this paper, we focus on the case of tracking a single target, or \( m = 1 \). Due to limited sensing range of the nodes, all nodes in the network cannot sense the target. This “limited sensing capability” poses a challenge in implementation of the Kalman-Consensus filter in [16] as appropriate care should be taken in the interpretation of “lacking sensor measurements.”

We introduce a model for a maneuvering target that is capable of staying inside a given rectangular region\(^1\). The dynamics of this target is a piece-wise linear switching system with two distinct modes of behavior. Intuitively, the target softly reflects upon hitting the walls. This is accomplished without using any collision-avoidance potential functions. Our objective is distributed tracking of the location of this maneuvering target (nonlinear process) using a lattice-type sensor network with limited sensing range.

Our main contribution is to provide a message-passing version of the Kalman-Consensus filter (adequate for packet-based networks) that achieves this task with a satisfactory performance. KCF is a P2P network of microfilters embedded in sensor nodes. A microfilter is a local estimator. To further improve the performance of tracking, we propose a hybrid architecture with a high-level fusion center that aggregates the estimate and covariance information of randomly selected set of nodes in the P2P microfilter network. We demonstrate that the performance of this hybrid architecture is virtually indistinguishable from that of a central Kalman filter on tracking this maneuvering target.

The outline of the paper is as follows. In Section II, the nonlinear model of a maneuvering target that remains in a rectangle is introduced. In Section III, the message-passing version of the KCF algorithm with application to LSR-type sensor networks is given. In Section IV, a hybrid P2P/Hierarchical tracking architecture is introduced. In Section V, simulation results are presented. Finally, concluding remarks are made in Section VI.

II. TARGET TRACKING PROBLEM

We consider a maneuvering target (or particle) with position \( q \in \mathbb{R}^2 \) and velocity \( p \in \mathbb{R}^2 \) represented by a nonlinear switching system

\[
x(k + 1) = A(x(k))x(k) + Bw(k)
\]

where \( A(x) = \begin{bmatrix} q_1(k), p_1(k), q_2(k), p_2(k) \end{bmatrix}^T \) denotes the state of the target at time \( k \). The target moves inside and outside of a square field \([-l, l]^2\] covered by a \( 10 \times 10 \) grid of \( n = 100 \) sensors. The matrix \( A(x) \) is designed so that the target is a linear process inside the region and as soon as the target leaves the region, a force orthogonal to the boundary of the region is applied to the target which eventually pushes the target back inside the region. The objective is to keep a mobile target in the sensing region of the sensor network with a lattice structure.

\[^1\text{This can be easily extended to convex polyhedra.}\]

Matrix \( A(x) \) is defined as

\[
A(x) = M(x) \otimes F_1 + (I_2 - M(x)) \otimes F_2
\]

\[
F_1 = \begin{bmatrix} 1 & \epsilon \\ 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & -\epsilon \\ -\epsilon c_1 & 1 - \epsilon c_2 \end{bmatrix},
\]

\[
M(x) = \begin{bmatrix} \mu(x_1) & 0 \\ 0 & \mu(x_3) \end{bmatrix}
\]

where \( F_1 \) and \( F_2 \) determine the dynamics of the target inside and outside of the region, respectively, and \( \mu(z) \) is a switching function taking 0-1 values defined by

\[
\mu(z) = \frac{\sigma(a + z) + \sigma(a - z)}{2}
\]

\[
\sigma(z) = \begin{cases} 1, & z \geq 0; \\ -1, & z < 0 \end{cases}
\]

In addition, matrix \( B \) is given by

\[
B = I_2 \otimes G, G = \begin{bmatrix} \epsilon^2 \sigma_0/2 \\ \epsilon \sigma_0 \end{bmatrix}
\]

where \( \epsilon \) is the step-size, \( c_1, c_2 > 0 \) are the parameters of a PD controller, the elements of \( w(k) \) are normal zero-mean Gaussian noise, and ‘\( \otimes \)’ denotes the Kronecker product of matrices. Inside, the square region, the target is a linear process

\[
x(k + 1) = A x(k) + B w(k)
\]

with \( A = I_2 \otimes F_1 \). This target model represents a constant velocity target \( \dot{q}_i = w_i(t) \) driven by the zero-mean white Gaussian noise \( w_i(t) \) with variance \( \sigma_0^2 \) [4].

Each node has a linear sensing model that measures

\[
z_i(k) = H_i(k) x(k) + v_i(k)
\]

where \( H_i(k) \) is the output matrix and \( v_i(k) \) is the zero-mean Gaussian noise of the measurements of the \( i \)th node with covariance \( R_i \).

III. KALMAN-CONSENSUS FILTERING FOR SENSOR NETWORKS WITH LSR

Kalman-Consensus Filter, introduced by Olfati-Saber in [16], is an effective distributed estimation algorithm for sensor networks with a peer-to-peer (P2P) architecture. In this section, we present a message-passing version of the Kalman-Consensus Filter (KCF) for sensor networks with limited sensing range. The KCF algorithm (or Algorithm 1) relies on reaching a consensus on estimates obtained by local Kalman filters rather than distributed construction of the fused measurements and covariance information of a central Kalman filter as in [15].

Algorithm 1 is the discrete-time equivalent of the continuous-time Kalman-Consensus filter described in the following.

**Theorem 1.** (Kalman-Consensus Filter [16]) Consider a sensor network with a continuous-time linear sensing model

\[
z_i(t) = H_i(t) x + v_i(t)
\]

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Algorithm 1 Kalman-Consensus Filter (message-passing version during one cycle at time index $k$ for node $i$)

Given $P_i$, $\bar{x}_i$, and messages $m_j = \{u_j, U_j, \bar{x}_j\}$, $\forall j \in J_i = N_i \cup \{i\}$,

1. Obtain measurement $z_i$ with covariance $R_i$.
2. Compute information vector and matrix of node $i$
   \[
   u_i = H_i^T R_i^{-1} z_i \\
   U_i = H_i^T R_i^{-1} H_i
   \]
3. Broadcast message $m_i = (u_i, U_i, \bar{x}_i)$ to neighbors.
4. Receive messages from all neighbors.
5. Fuse information matrices and vectors
   \[
   y_i = \sum_{j \in J_i} u_j, S_i = \sum_{j \in J_i} U_j.
   \]
6. Compute the Kalman-Consensus state estimate
   \[
   \dot{\hat{x}}_i = \bar{x}_i + M_i (y_i - S_i \bar{x}_i) + \gamma M_i \sum_{j \in N_i} (\bar{x}_j - \bar{x}_i),
   \]
   \[
   \gamma = 1/\|M_i\| + 1, \quad \|X\| = \text{tr}(XX^T)^{\frac{1}{2}}
   \]
7. Update the state of the Microfilter
   \[
   P_i \leftarrow AM_i A^T + BQB^T \\
   \bar{x}_i \leftarrow A\bar{x}_i
   \]

where $x$ is the state of the target and $v_i(t)$ is the sensor noise. Suppose each node applies the following distributed estimation algorithm

\[
\dot{\bar{x}}_i = A\bar{x}_i + K_i (z_i - H_i \bar{x}_i) + \gamma P_i \sum_{j \in N_i} (\bar{x}_j - \bar{x}_i) \\
K_i = P_i H_i^T R_i^{-1} \\
P_i = A P_i + P_i A^T + BQB^T - K_i R_i K_i^T
   \]

with a Kalman-Consensus estimator and initial conditions $P_i(0) = P_0$ and $\dot{\bar{x}}_i(0) = x(0)$. Then, the collective dynamics of the estimation errors $\eta_i = x - \bar{x}_i$ (without noise) is a stable linear system with a Lyapunov function $V(\eta) = \sum_{i=1}^n \eta_i^T P_i^{-1} \eta_i$. Moreover, $\dot{V} \leq -2 \gamma \Psi_\gamma(\eta) \leq 0$ where

\[
\Psi_\gamma(\bar{x}) = \bar{x}^T L \bar{x} = \frac{1}{2} \sum_{(i,j) \in E} \|\bar{x}_j - \bar{x}_i\|^2
   \]

and $\hat{L} = L \otimes I_m$ is the $m$-dimensional Laplacian of the network. Furthermore, all estimators asymptotically reach a consensus, i.e. $\bar{x}_1 = \cdots = \bar{x}_n = x$.

For sensor networks with limited sensing range, only a subset of nodes $V_a \subset V$ of the network called active sensors are able to sense the target. The remaining passive nodes $V_p \subset V$ do not obtain any meaningful measurement of a target that is not within their sensing range. Therefore, we define the information matrix of passive nodes as $U_i = 0, \forall i \in V_p$. The information vector of passive nodes is defined as $u_i = 0$ by assuming that their output matrices are zero, i.e. $H_i = 0$ for all $i \in V_p$. For mobile targets, the set of active and passive nodes are variable in time. The above procedure enables the application of Algorithm 1 to sensor networks with limited sensing range nodes without any ambiguity arising from the lack of measured data in passive nodes.

IV. HYBRID P2P/HIERARCHICAL ARCHITECTURE FOR DISTRIBUTED DATA FUSION

In this section, we propose a hierarchical architecture to obtain and fuse the estimates of the microfilters of a peer-to-peer (P2P) sensor network. The Kalman-Consensus filter can be viewed as a P2P network of interacting Microfilters (MFs) with input messages $m_j(k), j \in J_i$ (see Algorithm 1) and outputs $m_i(k+1)$ and $\bar{x}_i(k)$ which are the next message and the state estimate at time $k$, respectively. Each microfilter is a discrete-time dynamic system with internal state $(P_i, \bar{x}_i)$.

Here, we adopt a simplified higher level fusion center (FC) that randomly picks a subset $V_f(k) \subset V$ of the nodes of a P2P network of microfilters, as shown in Fig. 2 (a) and fuses their estimates using a linear data fusion rule:

\[
\hat{x}_f(k) = \frac{\sum_{i \in V_f} P_i^{-1}(k)\bar{x}_i(k)}{\sum_{i \in V_f} P_i^{-1}(k)}
   \]

More sophisticated data fusion algorithms such as covariance intersection [8], [12] can be utilized for sequential fusion of the information packets received at a fusion center via
multi-hop routing. A challenging problem in hierarchical architectures for data fusion is to deal with out-of-sequence data [2], [10].

In most existing tracking systems, either there is a central fusion center at the root of a spanning-tree that goes through all sensor nodes [13], [14], or a single or multiple sinks that fuse the information obtained by all the sensors [5]. Both approaches lead to purely hierarchical architectures for tracking.

A key contribution of this paper is to propose a hybrid architecture for distributed data fusion in sensor networks with limited sensing range shown in Fig. 2 (b). The heavy load of information processing in this tracking system is carried on by a P2P network of microfilters that are embedded in wireless sensors. Unlike the sink (or root) of traditional hierarchical systems, the fusion center in our proposed tracking architecture does not have to receive significant amount of data from many sensors that usually lead to congestion and high rates of packet-loss. Instead, the fusion center can talk to any small subset of the nodes of the P2P network to determine a fused estimate of the state of the target. Such a hybrid P2P/hierarchical network is robust to link/node failures and packet-loss effects [23] at its P2P level that leads to analysis of consensus problems in switching networks [19], [11], [18].

The term “microfilter” is chosen to indicate that millions of such local filters are needed to solve large-scale process tracking and data fusion problems in sensor networks as shown in Fig. 2 (b).

Due to the existence of loops in the P2P network of microfilters, double-counting of information arises that might become a challenge in certain applications as discussed in [9]. Based on our observations, this hybrid P2P/hierarchical architecture has a very satisfactory performance on tracking maneuvering targets that is comparable to a central Kalman filter.

V. SIMULATION RESULTS

We consider tracking of the position of the a maneuvering target with piece-wise linear switching dynamics given in (1) using a square grid of $n = 100$ that are evenly spaced in a region $[-l, l]^2$ with $l = 45$. The distance between each sensor and its nearest neighbor is $d_{min} = 10$. The sensing and communication range of each sensor is chosen uniformly to be $r_s = 1.5d_{min}$ and $r_c = 3d_{min} + 2$ (thus, $r_c/r_s > 2$).

Each sensor measures the position of the target in a plane, i.e.

$$H_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \forall i \in V$$

The measurement and process noise statistics are chosen as follows:

$$R_i = k_v^2 I_2, Q = k_w^2 I_4$$

with $k_v = 3$, $k_w = \sigma_0 = 5$. The initial state of the target is $x_0 = (-5, 7, 0, 20)^T$, $P_0 = 10k_w^2 I_4$.

The step-size of all simulations is $T = 0.04$ (sec) equivalent to a sampling rate of $f_0 = 25Hz$. The parameters of the target are chosen as $c_1 = 0.75, c_2 = 1, a = 40$ (a is chosen slightly less than $l = 45$ to guarantee that the target always stays inside the sensor field $[-l, l]^2$).

According to the choices of the sensing and communication range $r_s$ and $r_c$, at any time, a minimum of 4 and a maximum of 9 sensors (less than 10%) can sense a moving target. Despite such challenging sensing restrictions, in a peer-to-peer distributed tracking framework, every sensor has to have a “quality” estimate of the state of the target using local interactions with neighboring sensors.

To explore the tracking performance of the Kalman-Consensus filter (KCF) (Algorithm 1), we use the performance of a central Kalman filter as a base performance. Implementation of the central KF for sensor networks with limited sensing range leads to a filter with time-varying collective output matrix $H_i(k) = H \otimes \alpha(k)$ where $\alpha(k)$ is a column vector with 0 and 1 elements indicating passivity and activity of the $i$th sensor at time $k$, respectively (or $\alpha_i(k) = 1$ for all $i \in V_a$ and $\alpha_i(k) = 0$, otherwise). In contrast, for sensor networks with long-range sensors such as radars and sonars, the output matrix $H_i$ is fixed in time.

Fig. 3 demonstrates the performance of the central KF. Given the fact that target is a nonlinear system, the performance of this extended Kalman filter is very satisfactory. This performance can be used as a base level to judge the performance of the distributed Kalman filtering algorithm.

Fig. 4 illustrates the performance of the fusion center of the P2P network of microfilters (Kalman-Consensus filter) for sensor networks with limited sensing range. In comparison with the central KF, the performance is almost the same if not better. Note that single runs and different trajectories are used, but the maneuvering target behaves the same way in each run by reflecting upon hitting the horizontal/vertical limits of $\pm a$ around the origin.

Fig. 5 shows the collective tracking performance of the Kalman-Consensus filter (without any data fusion at the higher-level fusion center) for sensor networks with limited sensing range. In this case, the mean-squared error (MSE) is approximately 3 times of the MSE of a central KF in Fig. 3. Based on Fig. 5 (c), all nodes reach a consensus on their position estimates throughout the tracking period after a relatively short initial transient period. Fig. 6 illustrates the consecutive snapshots of the estimates of all nodes in the P2P network of microfilter (or KCF).

VI. CONCLUSIONS

The problem of distributed tracking of a maneuvering target using sensor networks with limited sensing range is addressed. A novel switching model is introduced for a target that is able to remain inside a rectangle in all time. We presented a message-passing version of the Kalman-Consensus filter in [16] and demonstrated its effectiveness in tracking this maneuvering target using a sensor network where less than 10% of the nodes sense the target at any given time. A hybrid P2P/Hierarchical tracking architecture was also introduced with a superior performance compared to the P2P architecture.
Fig. 3. Tracking performance of the central Kalman filter for sensor networks with LSR: (a) the trajectory of the target and position estimate and (b) position estimation error (MSE=0.54) after moving average filtering using a window of length 30 samples.

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Fig. 5. Collective tracking performance of the P2P network of microfilters (KCF) without data fusion at the higher-level fusion for sensor networks with LSR and the target trajectory in Fig. 4: (a) collective position estimation error (MSE=1.50) after moving average filtering, (b) the trajectory of estimates of all nodes. All nodes are in approximate agreement regarding their estimates.


Fig. 6. Snapshots of the estimates of all nodes in P2P network of microfilters (KCF) (without any fusion centers).