Abstract—A new adaptive Unscented Kalman Filter (UKF) algorithm for actuator failure estimation is proposed. The novel filter method with adaptability to statistical characteristic of noise is presented to improve the estimation accuracy of traditional UKF. The algorithm with the adaptability to statistical characteristic of noise, named Kalman Filter (KF) -based adaptive UKF, is proposed to improve the UKF performance. Such an adaptive mechanism is intended to compensate the lack of a prior knowledge. The asymptotic property of the adaptive UKF is discussed. The Actuator Healthy Coefficients (AHCs) is introduced to denote the actuator failure model while the adaptive UKF is employed for on-line estimation of both the flight states and the AHCs parameters of rotorcraft UAV (RUAV). Simulations are conducted using the model of SIA-Heli-90 RUAV of Shenyang Institute of Automation, CAS. The results are compared with those obtained by normal UKF to demonstrate the effectiveness and improvements of the adaptive UKF algorithm. Besides, we also compare this algorithm with the MIT-based one which we propose in previous research.

I. INTRODUCTION

Fault detection (FD) techniques have been widely researched in many applications to detect faults in sensors and actuators such as process industry [1], Unmanned Ground Vehicle (UGV) [2], and fixed-wing aircrafts [3], but few FD applications to RUAV have been published [4]. The structure and the algorithm of the fault tolerant control can be changed to get the best possible response of the system, when a fault of the system is detected.

In recent years, the encouraging achievements from sequential estimation makes it becoming an important research direction for on-line modeling and model-reference control [5]. One of the most well-known sequential estimation methods used for nonlinear system is the extended Kalman Filter (EKF) [6]. The EKF applies the standard linear Kalman Filter to nonlinear system by simply linearizing all the nonlinear models. This linearization will introduce substantial errors in the estimates of the mean and covariance of the transformed distribution, which may lead to poor performance or even divergence of the filter. UKF is a novel estimation tool introduced by Julier and Uhlman [7]. In stead of truncating nonlinear dynamics for nonlinear estimation, the UKF approximates the distribution of the state with a finite set points.

Since the nonlinear models are used without linearization, it is much simpler to implement and more accurate results are expected. Its performance has been analytically shown to be similar to a truncated 2nd order EKF at an equal computational complexity with the EKF of $O(n^3)$ ($n$ is the dimension of state) [4].

However, since UKF is in the framework of the Kalman filter, it can only achieve good performance under certain assumptions about the system modeling. But in practice, the assumptions are usually not satisfied and the performance of the filter may be seriously degraded from the theoretical performance or even diverge. This situation is then worse while fault or damage occurs. In order to avoid the problems, an adaptive mechanism may be applied in order to automatically tune the filter parameters to adapt the real statistics that are insufficient known in a prior.

There have been many investigations in the area of adaptive filter. Maybeck [8] used a maximum-likelihood estimator to estimate the system errors covariance matrix. Lee and Alfriend [9] modified the above methods by introducing a window scale Factor. Loebis et al. [10] present an adaptive EKF method, which adjusts the measurement noise covariance matrix by fuzzy logic.

In recent research, we have proposed an adaptive UKF algorithm based on MIT rule [11]. Based on the MIT rule, an adaptive algorithm is developed to update the covariance of process noise by minimizing the cost function. The updated covariance is then fed back into the normal UKF. Such an adaptive mechanism intends to release the dependence of UKF on a prior knowledge of the noise environment and improve the convergence speed and estimation accuracy of normal UKF. We found that the CUP time consumption of the MIT-based algorithm is about 4 to 5 times more than normal UKF scheme. This may lead to the estimation performance significantly lower than before in real time fault detection.

In this paper, a novel adaptive UKF algorithm is proposed for actuator failure estimation of RUAV. In order to do this, the AHCs is introduced to describe the actuators’ failures, and the UKF is used to estimate both the states and the AHCs parameters in real time. Simulations with the Shenyang Institute of Automation RUAV test-bed SIA-Heli-90 model have been conducted. At last, comparisons with the normal UKF and MIT-based adaptive UKF are discussed.
II. Adaptive UKF based on Kalman Filter

A. Mechanism of KF-based Adaptive UKF

The KF-based adaptive UKF is composed of two parallel master-slave filters. The slave filter employs KF to estimate the noise covariance and the master UKF estimates the state, using the current noise covariance. If the slave filter is not added to the UKF algorithm, the master UKF will remain working well. In this way, the UKF degrades to normal UKF with fixed noise covariance. Such a master-slave filters architecture will not modify the master UKF algorithm, moreover, when the statistical characteristic of noise is almost fixed, stopping the slave filter will make the computational load lower. The structure of filter-estimation-based adaptive UKF is shown in the figure 1).

![Fig. 1. The structure of filter-estimation-based adaptive UKF](image)

B. The Slave Filter

Slave filter selection depends on the statistical characteristic of noise. We can choose the UKF as slave filter for nonlinear statistical characteristic of noise, as well as the KF for the linear one. Without loss of generality, the variety of the noise characteristic is unknown which we assumed as the irrelevant random bias driven by the noise. Obviously, here we can use the KF as the slave filter.

In the real system, the prior information cannot reflect the actual system state, because the change of the noise statistical characteristic result in the lower UKF performance. Here, we propose a slave filter to on-line estimate the statistical covariance matrix Qw. Usually, the process noise covariance Qw is a diagonal matrix. So the estimation of Qw can be simplified as the estimation of its diagonal elements.

Here, we assume the diagonal elements of master UKF's noise covariance matrix as q, we can get the state equation of the slave filter as:

\[ q_k = f_q(q_{k-1}) + w_{qk} \]  

(1)

where \( w_{qk} \) is the Gaussian white noise with zero mean. We can assume q as an irrelevant random vector for its unknown variety. The state equation of the slave filter changes to:

\[ q_k = q_{k-1} + w_{qk} \]  

(2)

The slave filters will get different observer equations for different noise covariance matrix estimations. The slave filter’s observer equation is as follows:

\[
\hat{S}_q = g(q_k) = vdiag \left( P_{y_k \mid y_k} \right) = vdiag \left( \left( K_k^T K_k \right)^{-1} K_k^T \left( P_{\hat{y}_k \mid k-1} - P_k \right) \right)
\]  

(3)

where \( vdiag(\cdot) \) is a main diagonal element vector. Refer to the UKF equations, we can get the state equation with \( K P_k = \left( K_k^T K_k \right)^{-1} K_k^T : \)

\[
\hat{S}_q = vdiag \left( KP_k \left( \sum_{i=0}^{2n} \left( \chi_{i,k \mid k-1} - \chi_{\hat{x}_k \mid k-1} \right)^T \right) K P_k^T \right) + vdiag \left( KP_k Q_w K P_k^T \right) = B p_k + vdiag \left( KP_k Q_w K P_k^T \right) = B p_k + H P_k q_k
\]  

(4)

where \( B p_k \) is a constant vector; \( H P_k \) is a constant matrix. If \( K P_k \in \mathbb{R}^{mq \times nq} \) and:

\[
K P_k = \begin{bmatrix}
kp_{11} & kp_{12} & \cdots & \cdots & kp_{1nq} \\
kp_{21} & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
kp_{mq1} & \cdots & \cdots & \cdots & kp_{mqnq}
\end{bmatrix}
\]  

(5)

\( Q_w \) is the diagonal matrix, thus \( H P_k \in \mathbb{R}^{mq \times nq} \), and:

\[
H P_k = \begin{bmatrix}
kp_{p11} & kp_{p12} & \cdots & \cdots & kp_{p1nq} \\
kp_{p21} & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
kp_{pqn1} & \cdots & \cdots & \cdots & kp_{pqnq}
\end{bmatrix}
\]  

(6)

The measurement of the slave filter is:

\[
S q_k = g(q_k) = vdiag \left( \frac{1}{N} \sum_{i=k-N+1}^{k} I n_k \cdot I n_k^T \right)
\]  

(7)

where the innovation \( I n_k \) is as the definition equation (8).

\[
I n_k = y_k - \hat{y}_{k \mid k-1}
\]  

(8)

C. KF-based Adaptive UKF algorithm

Based on the previous analysis, the state equations and measurement equations are linear when the measurement noise variety of the master UKF is unknown. We can choose the KF, with simplified computation, as the slave filter. The full slave KF algorithm is shown as follows:

Initialize with
\[
\begin{align*}
\bar{q}_0 &= E[q_0] \\
P_{q0} &= E[(q_0 - \bar{q}_0)(q_0 - \bar{q}_0)^T] \\
\end{align*}
\] (9)

**Time Update**

\[
\begin{align*}
\bar{q}_{k|k-1} &= \bar{q}_{k-1} \\
P_{qk|k-1} &= P_{qk-1} + Q^w_q \\
\end{align*}
\]

(10)

**Measurement Update**

\[
\begin{align*}
K_{qk} &= P_{qk|k-1} \cdot H P_k^T (H P_k \cdot P_{qk|k-1} \cdot H P_k^T + Q^w_q)^{-1} \\
P_{qk} &= (I - K_{qk} \cdot H P_k^T) P_{qk|k-1} \\
\tilde{q}_k &= \bar{q}_{k|k-1} + K_{qk} (S_{qk} - \tilde{S}_q[k|k-1]) \\
\end{align*}
\] (11)

where \(Q^w_q\) is the covariance parameters of the KF’s measurement noise, while \(Q^r_q\) is the covariance parameters of the KF’s process noise.

**D. Asymptotic Behavior**

In this section, we attempt to analyze the asymptotic behavior of the adaptive UKF algorithm.

The proposed filter-based adaptive UKF comprises master and slave filter. The whole system is stable if the two filters are stable separately.

We consider a system with state equation as (2) and measurement equation as (17). The KF algorithm is shown in the equation (9) to (11). Then we can get the stable condition of this KF algorithm as [12]:

1) \((I, HP)\) is uniformly completely observable;

2) \((I, G)\) is uniformly completely controllable, where \(GG^T = E [w_q w_q^T]\).

Obviously, the full adaptive UKF algorithm is stable when the proposed two filters full fill the previous conditions.

**III. AHCs and Active Estimation**

**A. RUAV Dynamics Modeling**

![ RUAV dynamics obey the Newton-Euler equation for rigid body in translational and rotational motion. Here we consider a typical rigid RUAV in/near hover flight and the dynamic equation is conveniently described with respect to the body coordinate system, which is written as: ](image)

\[
\begin{bmatrix}
\dot{V}_B^B \\
\dot{\Omega}_B^B \\
\end{bmatrix} \begin{bmatrix}
\Omega_B^B \times mV_B^B \\
\Omega_B^B \times I\Omega_B^B \\
\end{bmatrix} = \begin{bmatrix}
F_B^B \\
M_{ext}^B \\
\end{bmatrix}
\] (12)

The free body diagram of helicopter with respect to body coordinate system is shown in figure 2. By employing the lumped-parameter approach, which considers the RUAV as the composition of the main rotor, tail rotor, fuselage, horizontal stabilizer, and vertical stabilizer. These components are considered as the source of forces and moments.

The external force and moment in hovering can be written as:

\[
F_{ext}^B = \begin{bmatrix}
X_M \\
Y_M + Y_T \\
Z_M \\
\end{bmatrix} + R_{TP\rightarrow B} \begin{bmatrix}
0 \\
mg \\
\end{bmatrix}
\] (13)

\[
M_{ext}^B = \begin{bmatrix}
R_M + Y_M h_M + Z_M Y_M + Y_T h_T \\
M_M + M_T - X_M h_M + Z_M l_M \\
N_M - Y_M l_M - Y_T l_T \\
\end{bmatrix}
\] (14)

The forces and torques generated by the main rotor are controlled by \(T_M, a_1\) and \(b_1\). The tail rotor is considered as a source of pure lateral force \(Y_T\) and anti-torque \(Q_T\), which are controlled by \(T_T\). Thus, the forces and moments can be expressed as:

\[
\begin{align*}
X_M &= -T_M \sin a_1 \\
Y_M &= -T_M \sin b_1 \\
Z_M &= -T_M \cos a_1 \cos b_1 \\
R_M &= -b_1 (dR_1/d\theta_1) - Q_M \sin a_1 \\
M_M &= -a_1 (dM_1/d\alpha_1) - Q_M \sin b_1 \\
N_M &= -Q_M \cos a_1 \cos b_1 \\
Y_T &= -T_T \\
M_T &= -Q_T \\
\end{align*}
\] (15)

where \(T_M, T_T, a_1, b_1\) are main rotor torque, tail rotor torque, longitudinal and lateral flapping angle. We assume the four parameters are linearly to the control surfaces of the actuators by omitting nonlinear characteristics.

\[
\begin{bmatrix}
T_M & T_T & a_1 & b_1 \\
\end{bmatrix} = A^{4 \times 5} \cdot U_{out}^{5 \times 1}
\] (16)

**B. Actuator Failure Model with AHCs**

The actuator failures of the RUAV including the control surface stick, control surface bias and partial loss in the actuator effectiveness [18].

Define \(U_{in}\) as the inputs (servo pulse width) and \(U_{out}\) as the outputs (control surface) of the actuators. The fault tolerant architecture assumes the following actuator model:

\[
U_{out} = \Gamma_f U_{in} + \Delta_f
\] (17)
where $\gamma_i$ and $\delta_i$ are the proportional effectiveness and failure bias of $i$th actuator’s AHCs.

C. States and AHCs Parameters Joint Estimation

To estimate the AHCs parameters, we use the KF-based adaptive UKF to obtain the coefficients. The parameter estimation follows a similar framework as that of the state-estimation AUKF. In AUKF-based parameter estimation, the AHCs and state vectors are concatenated into a single augmented state vector: $x^a_k = [P, \Phi, V^B, \Omega^B, \Gamma_f, \Delta_f]^T$. Estimation done recursively by writing the dynamics for the joint state as:

$$\begin{align*}
x^a_k &= \hat{f}(x^a_{k-1}, u_k) + w^a_k \\
y_k &= h(x^a_k) + v_k
\end{align*}$$

The AHCs active modeling has the advantages over some existing modeling techniques where only the proportional loss in the effectiveness has been considered [2].

IV. SIMULATIONS AND DISCUSSIONS

A. The SIA-Heli-90 RUAV Platform

The SIA-Heli-90 RUAV test-bed [17] is designed to be a common experimental platform for control and fault-tolerant related study. The hardware components are selected with considerations of weight, availability and performance.

The SIA-Heli-90 aerial vehicle is a high quality helicopter that is changed by us using a remote control (RC) hobby helicopter that is operated with a remote controller. The modified system allows the payload of more than 5 kilograms, and onboard sensors. The photograph of the implemented RUAV control system is presented in figure 3 and the primary parameters are shown in table I.

B. The Simulations of Actuator Failures Estimation

The proposed failure estimation scheme tested using the SIA-Heli-90 mathematical model which identified with the real flight data from the hovering experiments.

The use of the mathematical model makes easier to test the actuators’ failure estimation scheme, because real flight experiments with a failure actuator can be potentially dangerous for the helicopter because it can take the RUAV out of control and it may crash. Thus, we planed to simulate a real faulty condition in an actuator while away form the security problems of the RUAV. Here, we combine the fault detection algorithm with feedback linearization control scheme to perform the simulations.

It is obviously that the yaw, longitudinal and latitudinal controls, ones near zero at trim of hovering, might be more difficult to estimate - since a change in effectiveness only would be less immediately apparent. Here we assumed that the tail collective pitch angle actuator has the failure while others are remain well. Here we consider the actuator failure as a parameter. Then the UKF is employed for online estimation of both motion states and parameters of helicopter AHCs. In the following, we compare the performance of the adaptive-UKF-based and the conventional UKF-based failure estimation schemes. Additionally, we refer our recent research on MIT-based adaptive UKF and compare it with KF-based algorithm in CPU time and estimated accuracy.

Set the state vector as:

$$X = [p_X, p_Y, p_Z, \Phi, \Psi, v_X, v_Y, v_Z, \omega^B_X, \omega^B_Y, \omega^B_Z, \gamma_2, \delta_2]$$

The experiment starts out with the initial augmented state:

$$X_0 = [0, 0, 0, -0.0456, -0.0177, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

The measurement and control interval is $T = 0.02s$. The UKF parameters are listed here:

$$\hat{X}_0 = X_0$$

$$\hat{P}_0 = diag\{10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-9}\}$$

### TABLE I

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMU</td>
<td>Angular Rate Range: ±100°/sec</td>
</tr>
<tr>
<td></td>
<td>Acceleration Range: ±4g</td>
</tr>
<tr>
<td></td>
<td>Digital Output Format: RS-232</td>
</tr>
<tr>
<td></td>
<td>Update Rate: 100 Hz</td>
</tr>
<tr>
<td></td>
<td>Size: 7.62 × 9.53 × 8.13 cm</td>
</tr>
<tr>
<td></td>
<td>Weight: 6.4kg</td>
</tr>
<tr>
<td>GPS</td>
<td>Position Accuracy (CEP): 0.2m</td>
</tr>
<tr>
<td></td>
<td>Digital Output Format: RS-232</td>
</tr>
<tr>
<td></td>
<td>Update Rate: 10 Hz</td>
</tr>
<tr>
<td></td>
<td>Size: 7.11 × 4.96 × 0.12 cm</td>
</tr>
<tr>
<td>Compass</td>
<td>Pitch, Roll Angular Range: ±80°</td>
</tr>
<tr>
<td></td>
<td>Digital Output Format: RS-232</td>
</tr>
<tr>
<td></td>
<td>Update Rate: 20 Hz</td>
</tr>
<tr>
<td></td>
<td>Size: 1.5 × 4.2 × 0.88 cm</td>
</tr>
<tr>
<td></td>
<td>Weight: 92g</td>
</tr>
</tbody>
</table>
\[ Q^w = \text{diag}\{10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, 10^{-7}, \ldots, 10^{-9}, 10^{-9}\} \]
\[ Q^v = \text{diag}\{10^{-8}, 10^{-8}, \ldots\} \]

\[ \alpha = 0.5 \]
\[ \beta = 2 \]

1) The Changes of Process Noise: The covariance matrix \( Q^w \) as a prior knowledge is most important to the performance and stability of the UKF. If we can not get the accurate matrix or if it changed as a result of the AHCs modified, the UKF will have a bad performance or even instability.

Here we change the true process noise intensity as:

\[
\begin{cases}
Q^w_{\text{ukf}0} = Q^w_{\text{ukf}} & t < 5s \\
Q^w_{\text{ukf}} = Q^w_{\text{ukf}} \times 10^2 & t \geq 5s
\end{cases}
\]

(20)

The estimation accuracy of the two different adaptive UKF algorithms with respect to changes of the process noise statistics is tested.

The estimation errors of the UKF and the KF-based adaptive UKF under the same condition of the process noise intensity change are illustrated in figure 4. The UKF cannot produce optimal estimates due to the violation of the optimality conditions when the noise information changed at 5s. On the other hand, the estimation errors in adaptive case is quickly overcome and almost the same as its previous size.

Fig. 4. State Estimation Errors with the Time-Varying Process Noise using KF-based UKF

2) The Change of AHCs Parameters: To demonstrate the effectiveness of the failure estimation scheme of the RUAV actuators, the failure scenario of abrupt proportional reduction and bias in tail collective pitch actuator is assumed:

\[
\begin{cases}
\gamma_4 = 100\%, \delta_4 = 0 & 0 \leq t < 6s \\
\gamma_4 = 50\%, \delta_4 = 10 & t \geq 6s
\end{cases}
\]

(21)

In this section, we compare the performance of the adaptive UKF-based and the conventional UKF-based failure estimation algorithms. The state vector is subject to zero mean additive white noise with covariance:

\[ Q^w_{\text{ukf}0} = Q^w_{\text{ukf}} \]

(22)

and other conditions of the system are the same as that in the previous section.

As is shown in the figure 5, an example actuator failure experiment is presented. At 5s, the actuator gets AHCs parameters of proportional of 50% and bias of 10. The estimation of the proportional effectiveness and the failure bias AHC parameters can follow the true parameters in less than 4 seconds while the offsets are less than 0.4% with the adaptive UKF scheme. However, conventional UKF-based algorithm cannot estimate the true value actually in 15 seconds.

Fig. 5. The KF-based adaptive UKF AHCs estimation

3) Comparisons Between MIT-based and KF-based Adaptive UKF algorithms: We have compared the KF-based adaptive UKF with normal UKF with the help of previous simulations. In this section, we refer our recent research results on MIT-based adaptive UKF and get the comparisons between MIT-based and KF-based adaptive UKF algorithms focus on the estimated accuracy and estimated CPU times based on the simulations.

The cost function of the estimated accuracy defined as follows:

\[ J = \sqrt{\frac{1}{N_k} \sum_{k=1}^{N_k} (x_k - \bar{x}_k)^2} \]

(23)

where \( N_k \) is the sample times, \( x_k \) are the states or parameters of the system, and \( \bar{x}_k \) are the estimated state or parameters.

For comparison purposes between the two algorithms, we did an additional simulation. The figure 6 illustrates the Euler angles of the RUAV, including roll angle \( \Phi \), pitch angle \( \Theta \), and yaw angle \( \Psi \) in hovering.

The comparison results in two respects of estimated accuracy and CPU time between the two algorithms are listed in

Fig. 6. The tracking performance comparison between the two adaptive UKFs in hovering while a tail collective pitch actuator failure at 6s
Table II

<table>
<thead>
<tr>
<th>S #1</th>
<th>Normal UKF</th>
<th>M-AUKF</th>
<th>K-AUKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Time</td>
<td>5.54</td>
<td>26.23</td>
<td>9.13</td>
</tr>
<tr>
<td>(\Psi) EA</td>
<td>0.00033</td>
<td>0.00179</td>
<td>0.00135</td>
</tr>
<tr>
<td>(\Phi) EA</td>
<td>0.00005</td>
<td>0.00015</td>
<td>0.00020</td>
</tr>
<tr>
<td>(\Theta) EA</td>
<td>0.00055</td>
<td>0.00014</td>
<td>0.00017</td>
</tr>
</tbody>
</table>

As can be seen in the table and figures, two adaptive algorithms obtained much better performance than normal UKF obviously, when the process noise or parameters changed. Compared with two different adaptive methods, the MIT-based scheme get the higher estimated accuracy, especially in the situation of parameter estimation.

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References


