A sequential design methodology for large-scale LBT systems

F. Claveau and Ph. Chevrel

Abstract—H₂ and H₁ based control design methodologies are widely used to deal with multivariable control problems. However, some special problems arise in the case of large-scale systems. On one hand, the complexity (due to the size of the models) of the related H₁ or H₂ standard problems may be too high to be solved in a classical manner. On the other hand, structure constraints on the controller often exist (use of decentralized processors) and complicate the optimization problem. This paper focuses on the problem of Lower Block Triangular (LBT) H₂ controller design for LBT systems. Based on the principle of sequential design, an algorithm is proposed, attempting to solve iteratively the global H₂ problem. It is finally applied to a particular problem, the control of a platoon of vehicles, and compared with local strategies.

I. INTRODUCTION

A physical system is called large-scale system when it presents a great number of inputs / outputs, and complex dynamics (great number of states), or / and if it is composed of distinct subsystems, more or less strongly interconnected. Some typical large-scale systems are: Power network [1], digital communication networks, manufacturing processes such as winding systems [2], automotive control [3].

Because of the structure constraints on the feedback or / and the complexity of the global problem, H₂ or H₁ design strategies can rarely be applied directly to control such systems. In fact, optimal control problems under structural constraints are most often non convex one. Since the 90th, some algorithms using bilinear matrices inequalities (BMI) were proposed to formulate and solve such problem (see e.g. [4]-[5] and references therein). More recently, the notion of quadratic invariance for particular structure of the system and the controller has been used to reformulate the optimization problem as a convex one in the term of the Youla-Kucera parameter [6],[7]. Although both of these strategies are interesting ways to deal with the problem, they require a too heavy computational effort to be applied to large-scale systems.

In order not to manipulate the global model, this paper revisits the concept of sequential design of hierarchical controller for LBT systems, in which local controllers are design one after the other [8],[9]. Based on the decomposition property of the H₂-norm [10],[11], the proposed sequential design strategy attempt to minimize a global H₂-problem by solving iteratively low complexity subproblems associated to each subsystems. Moreover, Lyapunov and Riccati equations are preferred over LMI.

The paper is organized as follows: the principal notations and the position of the problem are first presented in section 2. In section 3, the sequential LQ-LQG control design introduced by Özgüner and Perkins [8], and Šiljak [9] is revisited in the H₂ framework. In order to reduce the global H₂ criterion, an iterative algorithm is proposed in section 4. It is then tested on the platoon of vehicles control problem, considered as a benchmark and compared with the solution obtained using the BMI framework.

II. PROBLEM STATEMENT

A. Definitions and notations

Definition 1: A linear system S is said LBT if it can be represented by a state-space realization of the form (1), or equivalently a LBT transfer matrix.

\[ S(x(t)) = Ax(t) + Bu(t) \]

\[ y(t) = Cx(t) \]

where

\[ A = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} \\ \vdots \\ B_{N1} \cdots B_{NN} \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} \\ \vdots \\ C_{N1} \cdots C_{NN} \end{bmatrix} \]

\[ x(t) = \begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix} \in \mathbb{R}^n, \quad u(t) = \begin{bmatrix} u_1^T \\ \vdots \\ u_N^T \end{bmatrix} \in \mathbb{R}^p \]

\[ y(t) = \begin{bmatrix} y_1^T \\ \vdots \\ y_N^T \end{bmatrix} \in \mathbb{R}^p \] are the state, input, and output of S, \( x_i(t) \in \mathbb{R}^n, \) \( u_i(t) \in \mathbb{R}^m, \) and \( y_i(t) \in \mathbb{R}^p, \) the ones of subsystem \( S_i, \ i = 1, \ldots, N. \)

In what follows all subsystems \( S_i \) are supposed to be stabilizable and detectable.
B. The LBT $H_2$ control problem

Consider Fig. 1; $P(s)$ is the standard model associated to the global system (1), and $K(s)$ is the researched dynamical controller (3). $P(s)$ is defined by

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \begin{bmatrix} * \ A \ B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} u \end{bmatrix} x$$  \tag{2}

where $w(t) \in \mathbb{R}^m$, $w = [w_1^T \cdots w_N^T]^T$, $w_i(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^r$, $z = [z_1^T \cdots z_N^T]^T$, $z_i(t) \in \mathbb{R}^{n_i}$.

A is defined in (1), $B_2 = B$, $C_2 = C$, $D_{11} = 0$, $D_{22} = 0$, $B_i = \text{diag}(B_1^{i_1}, \cdots , B_N^{i_N})$, $C_i = \text{diag}(C_1^{i_1}, \cdots , C_N^{i_N})$,

$$D_{12} = \text{diag}(D_{12}^{i_1}, \cdots , D_{12}^{i_N}), D_{21} = \text{diag}(D_{21}^{i_1}, \cdots , D_{21}^{i_N})$$.

The controller $K(s)$ is structured in the same way than the global system $S$ (1), to be consistent with the information structure constraints.

$$K(s) = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} * \ A^{i_1} & B^{i_1} \\ C^{i_1} & D^{i_1} \end{bmatrix}$$  \tag{3}

where $D_k = 0$,

$$A_k = \begin{bmatrix} A_1^{i_1} & \cdots & A_N^{i_1} \\ \vdots & \ddots & \vdots \\ A_1^{i_N} & \cdots & A_N^{i_N} \end{bmatrix} , B_k = \begin{bmatrix} B_1^{i_1} & \cdots & B_N^{i_1} \\ B_1^{i_N} & \cdots & B_N^{i_N} \end{bmatrix} , C_k = \begin{bmatrix} C_1^{i_1} & \cdots & C_N^{i_1} \\ \vdots & \ddots & \vdots \\ C_1^{i_N} & \cdots & C_N^{i_N} \end{bmatrix}$$.

With $F(K,P)$ denoting the lower linear fractional transformation (LFT) on $P(s)$ and $K(s)$, we have

$$T_{zw} = F(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$  \tag{4}

**Problem 1:** LBT $H_2$ control problem

Under the classical assumptions [12], the LBT $H_2$ control problem considered here is defined as: “find the LBT controller given by (3) which $i$. internally stabilizes $T_{zw}$, $ii$. minimizes $J(K) = \|P_{zw}\|_2$”.

**Result 1:** The closed-loop system is LBT

$$J(K) = \|P_{zw}\|_2 = \begin{bmatrix} T_{zw_{i_1}} & 0 & 0 \\ \vdots & \ddots & \vdots \\ T_{zw_{i_N}} & \cdots & T_{zw_{i_N}} \end{bmatrix}.$$  \tag{5}

**Proof:** Obvious from (4), by taking into account the special structure of $P(s)$ (1) & (2) together with the LBT structure of the controller (3).

**Result 2:** The $H_2$ criterion associated to the LBT $H_2$ control problem may be written as

$$J(K_i, \ldots , K_N) = \|F_{zw_{i_1}}\|_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \|P_{zw_{ij}}\|_2^2.$$  \tag{6}

**Proof:** Obvious from result 1 and the $H_2$ norm properties.

Synthesize the structured controller $K(s)$ as a whole leads to an $H_2$ optimization problem under structural constraints. As mentioned in the introduction, even if the strategies using BMI (e.g. [4],[5]) or the quadratic invariance property [6],[7] are efficient to deal with the structural constraints inherent in the large-scale systems, they are not tractable solution on a numerical point of view. BMI will be used however in this paper as a reference, for comparison.

### III. THE LOCAL /SEQUENTIAL $H_2$ PROCEDURE

The most common way to deal with structurally constrained control for LBT systems is the sequential strategy; the $i^{th}$ controller is designed by taking into account the $1^{st}$ to the $(i-1)^{th}$ already implemented.

Let us give a generalization in the $H_2$ framework of the classical sequential L.Q. – L.Q.G. control design strategy developed by Özgüner and Perkins [8], and Šiljak [9]. The principle of this sequential methodology is the following : Each subsystem $S_i$ has an associated cost function $J_i$ to be minimized, independent of lower (hierarchically) subsystems. At the same time, the effects of already closed-loop higher-level subsystems are treated locally as known disturbances. Let us consider Fig. 2 where $P$ is the standard model allowing to define the local $H_2$ criterion.

$$P_i \equiv \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{21} & P_{i_{22}} \end{bmatrix} \begin{bmatrix} P_{j_{00}} & P_{j_{01}} & P_{j_{02}} \\ P_{j_{10}} & P_{j_{11}} & P_{j_{12}} \\ P_{j_{20}} & P_{j_{21}} & P_{j_{22}} \end{bmatrix}$$  \tag{7}

$P_i$ and $P^N$ are specific in that they are obtained by suppressing respectively the first column and the last row.

Let us denote $u_i = [u_1^T \cdots u_N^T]^T$, $w_i = [w_1^T \cdots w_N^T]^T$,

$$y_i = [y_1^T \cdots y_N^T]^T, r_i = [r_1^T \cdots r_N^T]^T, G_i = \begin{bmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{21}} & P_{i_{22}} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix},$$  \tag{8}

and generically $T_{zw}$, the closed-loop transfer matrix from signal $\alpha$ to $\beta$; $T_{zw_{i_1}}$ is the $i^{th}$ row of $T_{zw}$.
The local / sequential design procedure based on local optimization H2 problems may now be stated.

**Algorithm 1:**
Starting from \( i = 1 \).
1. Find \( K^{i*} = \arg\min \left( \left\| F_i \left( G^i, K^i \right) \right\| \right) \).
2. \( i = i + 1 \)
3. Build \( \mathcal{G}^i = G^{i*} \left[ \begin{array}{cc} 0 & 0 \\ 0 & I \end{array} \right] \).
4. Find \( K^{i*} = \arg\min \left( \left\| F_i \left( \mathcal{G}^i, K^i \right) \right\| \right) \).
5. while \( i < N \) goto 2
6. End

**Remark:** To be applicable, some assumptions have naturally to be satisfied. Each \( \mathcal{G}^i \) for example must be such that \( \mathcal{G}_{12}^i \) and \( \mathcal{G}_{21}^i \) have no zeros on the imaginary axis. \( \mathcal{G}_{11}^i \) and \( \mathcal{G}_{22}^i \) are assumed to be strictly proper. Moreover, if the state-space realization is used, classical assumptions on stabilizability, detectability and non-singularity have to be made.

**Theorem 1:** Under the assumptions introduced in the previous remark, the local / sequential procedure leads to the global controller \( K^* (s) \) (given by (3) by replacing \( K^i \) by \( K^{i*} \) for each \( i \)) which has the following properties:

i. \( K^* \) stabilizes the global system.
ii. \( K^* \) minimizes \( J \left( K^{i, \ldots , K^{i*}}, K^i \right) \),
iii. but \( K^* \) do not optimize the global criterion \( J \left( K^1, \ldots , K^N \right) \).

**Proof:** i. The local stability reached by controllers \( K^{i*} \) imply the global stability by property of the LBT structure. ii. comes from the definition of \( J_i \), (12), assuming that \( K^j = K^{i*}, j \neq i \). iii. It is clear that solving sequentially the problems \( K^{i*} = \arg\min (J_i (K^i)) \), \( K^{i*} = \arg\min (J_i (K^{i*}, \ldots , K^{i-1*}, \ldots K^1, K^i)) \) does not lead to the solution of the problem \( (K^{i, \ldots , K^{i*}}) = \arg\min \left( \sum_{i=1}^{N} J_i (K^1, \ldots , K^i) \right) \).

The research of \( K^* (s) \) which minimize the global criterion \( J \left( K \right) = \left\| T_{zw} \right\| \) is considered in section 4.

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**Fig. 2.** local / sequential H2 procedure

Let us now introduce the following results.

**Result 3:**
\[
T_{z,w} = T_{z,w}^{ij}, \quad T_{z,w} = T_{z,w}^{ij}
\]
and both depend only of \( (K^i (s))_{s \in 0} \).

**Proof:** Obvious from the LBT structure of \( T_{zw} \) (see result 1) and the definition of \( w^i \).

**Result 4:** Let \( \mathcal{G}^i = G^i \left[ \begin{array}{cc} 0 & 0 \\ 0 & I \end{array} \right] \).

Then \( T_{z,w} = F_i (\mathcal{G}^i, K^i) \)

**Proof:** \( F_i (\mathcal{G}^i, K^i) = F_i (G^i, K^i) \left[ T_{z,w} 0 \\ 0 0 \right] \), from the LFT definition. By definition of \( G^i, F_i (G^i, K^i) = \left[ T_{z,w} 0 \\ 0 0 \right] \) holds. It is clear from result 3 that \( T_{z,w} = T_{z,w}^{ij} \) and \( T_{z,w} = T_{z,w}^{ij} \).

Moreover, \( T_{z,w} = T_{z,w}^{ij} \) results from the definition of \( w^i \).

Finally, \( T_{z,w}^{ij} \) results from the series connection of \( T_{z,w}^{ij} \) and \( T_{z,w}^{ij} \) which leads to result 4.

**Definition 2:** Let us define the local criterion:

\[
J \left( K^1, \ldots , K^i \right) = \left\| T_{z,w} \right\|_2 \left( \sum_{i=1}^{i} \right) \left\| T_{z,w} \right\|_2^2.
\]

**Result 5:** The global criterion \( J (K) = \left\| T_{zw} \right\| \) can be rewritten as

\[
J (K) = J \left( K^1, \ldots , K^N \right) = \sum_{i=1}^{N} J_i (K^1, \ldots , K^i).
\]

**Proof:** Obvious from def. 2 and the H2 norm properties.
IV. THE GLOBAL / SEQUENTIAL H₂ PROCEDURE

A. Algorithm

The main idea to be developed in this section consists, for a given set of initial subcontrollers, to modify sequentially each subcontroller in order to reduce the global H₂ criterion,

\[ J(K) = J(K^1, \ldots, K^n) = \sum_{i=1}^{N} J_i (K^1, \ldots, K^l) \]  \hspace{1cm} (14)

To do so, let us introduce some preliminary results. Starting from Fig. 2, let us denote:

\[ \hat{G}^i = \begin{bmatrix} \hat{G}_{11}^i & \hat{G}_{12}^i \\ \hat{G}_{21}^i & \hat{G}_{22}^i \end{bmatrix} = \begin{bmatrix} \hat{P}_{v1s} & \hat{P}_{v2s} \\ \hat{P}_{v1s} & \hat{P}_{v2s} \end{bmatrix}. \]  \hspace{1cm} (15)

Let also \( z^i_N = [z^i_N \cdots z^i_N]^T \).

\( \hat{G}^i \) and \( \hat{G}^N \) are specific in that they are obtained by suppressing respectively the first column and the second row. In the same way than for result 4, and referring to figure 3, we have the following result.

Result 6: Let

\[ \bar{G}^i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]  \hspace{1cm} (16)

Then \( T_{z_Nw}^i = F_i(\bar{G}^i, K') \) \hspace{1cm} (17)

Proof: \( F_i(\bar{G}^i, K') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \) \hspace{1cm} (18)

from the LFT definition. Furthermore, the equality \( F_i(\bar{G}^i, K') = \begin{bmatrix} T_{z_Nw}^i \\ T_{z_Nw}^i \end{bmatrix} \) holds. Series connection of the 3 terms in the right part of (18) leads then to \( T_{z_Nw}^i \).

Let us now assume that \( (K^1, \ldots, K^N) \) are admissible subcontrollers for the problem considered (i.e. they stabilize the global system). The question is: how to modify the \( i \)th subcontroller in order to obtain:

\[ J(K^1, \ldots, K^l, K^i, \ldots, K^N) < J(K^1, \ldots, K^l, K^i, \ldots, K^N) \]  \hspace{1cm} (19)

After (14), the choice of \( K_i \) affects all subcritierions \( J_i \) for \( l \geq i \). The way proposed here consists to modify \( K^i \) by \( K' \) solving the problem 2.

Problem 2: "Find

\[ K' = \text{argmin}_K \left( \sum_{i=1}^{N} J_i (K^1, \ldots, K^i, K' \ldots, K^N) \right). \]  \hspace{1cm} (20)

Fig. 3. The global / sequential H₂ procedure

Result 7: Let \( K^j = K'^j \) for all \( j \) except \( j = i \). Then problem 2 is equivalent to,

"Find \( K'^i = \text{argmin}_K \left( \parallel F_i (\bar{G}^i, K') \parallel \right) \)". \hspace{1cm} (21)

Proof: By construction of \( \bar{G}^i \). \hspace{1cm} □

Result 8: Let \( K'^i \) be defined by (21). Then the following inequality holds,

\[ J(K^1, \ldots, K'^i, \ldots, K^N) < J(K^1, \ldots, K'^i, \ldots, K^N). \]  \hspace{1cm} (22)

Proof: The result may be derived from (13) and (20). \hspace{1cm} □

The global / sequential design procedure based on an iterative optimization of the global H₂ criterion \( J(K) = \| T_{zw} \| \) may now be stated.

Algorithm 2:

Starting from \( i = 1, k = 0 \).

0. Initialization : Find a stabilizing controller \( K^{(0)} = (K^{(0)}_1, \ldots, K^{(0)}_N) \).

1. \( k = k + 1 \)

1.1 For \( i = 1 \) to \( N \)

1.1.1 Build \( \bar{G}^{(k)} = \begin{bmatrix} 0 & 0 \end{bmatrix} \)

\begin{bmatrix} T_{wz}^{(k-1)} \\ T_{wz}^{(k-1)} \end{bmatrix}, T_{wz}^{(k-1)} = \begin{bmatrix} 0 & 0 \end{bmatrix} \)

Find \( K^{(k)} = \text{argmin}_K \left( \parallel F_i (\bar{G}^{(k)}, K') \parallel \right) \).

1.2 End

2. If \( J(K^{(k)}) - J(K^{(k-1)}) < \varepsilon \) then stop, else goto 1

3. End

Remark 1: As for algorithm 1, classical H₂ control problem assumptions on \( \bar{G}^{(k)} \) must be satisfied.

Remark 2: The controller \( K'^i \) obtained thanks to algorithm 1 can be used to initialize algorithm 2.
Theorem 2: Under the classical assumptions on $\tilde{G}^{(l)}$, algorithm 2, associated to the global criterion $J(K) = \|F_zw\|_1$, converges monotonically.

Proof: It is clear from result 8 that the sequence of $\alpha_i = J(K^{(l)}, \ldots, K^i, \ldots, K^{N(l)-1})$ is decreasing as $i$ increases. It then follows from the Bolzano – Weierstrass theorem that the algorithm converges monotonically. □

Remark 1: There is no guaranty to reach the optimum controller $K_{opt}$ for the global criterion $J(K) = \|F_zw\|_1$. However (i) the global criterion is decreasing. (ii) if $K_{opt}$ is used as initialization point, then the algorithm will stay at this optimal point.

Remark 2: If only one of the local controllers is modified, the criterion will increase. However, it is not possible to conclude to the local optimality, regarding simultaneously all the $K^i$.

B. Synthesis of controller $K^{(l)}$

Synthesis of $K^{(l)}$ implies the manipulation of the global closed-loop standard model $\tilde{G}^{(l)}$ represented in Fig. 3 (replacing $T_{e,w_{l}}, T_{e,N}$, and $K^i$ by $T_{e,w_{l}}, T_{e,N}$, and $K^{(l)}$). This standard model may therefore be of a high order, leading to numerical difficulties.

Two ways to simplify the problem can be distinguished; model reduction and decomposition of the synthesis of the dynamic controller $K^{(l)}$. 1/ Model reduction will consist in reducing both the transfer matrices $T_{e,w_{l}}$ and $T_{e,N}$. It can be done thanks to methodologies using balanced truncations or singular perturbations [13]. One can expect a significant order reduction, without important degradation, as the hierarchically distant subsystems should act weakly on $S_e$.

2/ Simplification by decomposition of the synthesis of $K^{(l)}$ is made possible by using the state-space formulation. Application of the separation principle will permit to obtain $K^{(l)}$ in a state feedback – observer form. Referring to [8,9], the decomposition of the synthesis of state feedback and observer gains $K^{(l)}$ and $K^{(l)}_{ob}$ is possible considering the decomposition results in the theorem 1 in [8], or in [9] (L.Q. – L.Q.G. cases). The reformulation in the $H_2$ case is omitted by lack of place.

Briefly stated, considering for instance the state feedback gain $K^{(l)}_{st}$, it can be design as $[K^{(l)}_{st1}, K^{(l)}_{st2}]$ by solving sequentially reduced order Sylvester and Riccati equations.

V. CASE STUDY

A. Control of a platoon of vehicles

In order to appreciate the efficiency of the proposed algorithm, the speed control of a platoon of vehicles is now considered (e.g. [3],[8]). Briefly stated, the problem consists to keep the platoon of vehicles with a constant velocity and constant intra platoon separations. A platoon of vehicles is indeed a potential large-scale system being composed of interconnected subsystems. A structured controller is a natural solution to meet the technological constraints (local embedded controllers).

Consider a set of $N$ vehicles moving in a straight line as illustrated in fig. 4. The variables parameterizing the model of the $i^{th}$ vehicle are; $h_i(t)$, $v_i(t)$, $m_i$, $u_i(t)$, $-\alpha_i v_i(t)$ which are respectively the position of the $i^{th}$ vehicle at time $t$, its velocity, its masse, the force applied, and the drag force action. All the states are assumed to be measured. By considering the following deviations, $x_i = v_d - v_i$, $x_i^i = v_d - v_i$, and $\Delta x_i = e_{hi} - \Delta h_i$, $i = 2, \ldots, N$, where $v_d$ and $\Delta h_i$ are respectively the desired velocity and intra platoon separation, a state-space representation of a platoon of $N$ vehicles can be defined as (27) [3].

To focus on the methodology, it is assumed from now (as in [3]) that $\alpha_i = 1$, $m_i = 1$, and $N = 3$.

$$\begin{align*}
\mathbf{x}_1 &= \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} \\
\mathbf{y}_1 &= \begin{bmatrix} v_1^{(1)} \end{bmatrix} \\
\mathbf{u}_1 &= \begin{bmatrix} u_1^{(1)} \end{bmatrix} \\
\mathbf{h}_1 &= \begin{bmatrix} h_1 \end{bmatrix}
\end{align*}$$

$$\begin{align*}
\mathbf{x}_2 &= \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ \Delta x_2 \\ \Delta h_2 \\
\end{bmatrix} \\
\mathbf{y}_2 &= \begin{bmatrix} v_1^{(2)} \\ v_1^{(2)} \\ \Delta v_1^{(2)} \end{bmatrix} \\
\mathbf{u}_2 &= \begin{bmatrix} u_1^{(2)} \\ u_1^{(2)} \end{bmatrix} \\
\mathbf{h}_2 &= \begin{bmatrix} h_2 \end{bmatrix}
\end{align*}$$

$$\begin{align*}
\mathbf{x}_3 &= \begin{bmatrix} x_1^{(3)} \\ x_2^{(3)} \\ \Delta x_3 \\ \Delta h_3 \\
\end{bmatrix} \\
\mathbf{y}_3 &= \begin{bmatrix} v_1^{(3)} \\ v_1^{(3)} \\ \Delta v_1^{(3)} \end{bmatrix} \\
\mathbf{u}_3 &= \begin{bmatrix} u_1^{(3)} \\ u_1^{(3)} \end{bmatrix} \\
\mathbf{h}_3 &= \begin{bmatrix} h_3 \end{bmatrix}
\end{align*}$$

B. Global criterion

To construct the global criterion $J(K) = \|F_zw\|_1$, the standard model $P(s)$ (2) is define referring to [8] by $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $C_1 = \begin{bmatrix} C_1^s \\ 0 \end{bmatrix}$, $D_1 = D_2 = D_1^e = 0$ with $C_1 = diag(\sqrt{3}, I_2, I_2)$, $D_1^e = I_3$, $D_2 = 0$, and $D_3 = 0$.

Remark: Some local $H_2$ problems to be solved are singulars. However, these problems have been solved using
the results of [14],[15].

C. Results

The following structured controllers have been compared; \( K^* \), the static controller obtained by the local / sequential H\(_2\) procedure, \( K_{opt} \), the static controller optimized thanks to a BMI algorithm (heuristic in [5] was used), and lastly \( K^*(s) \), a dynamic controller obtained thanks to algorithm 2. The global criterion \( \| P_{zw} \| \) obtained with each of them are compared. The results are summed up in Table I. Table II demonstrates the evolution of the criterion through the iterations of algorithm 2.

As expected, the criterion decreases monotonically through the iterations. More iterations would have been necessary to exhibit definitely the convergence. The differences observed between \( K_{opt} \) and \( K^*(s) \) comes from the fact that if \( K_{opt} \) is probably the best static controller, \( K^*(s) \) is a dynamic controller, hence with more degree of freedom.

Table I. Criterions for several structured controllers

<table>
<thead>
<tr>
<th>Controllers</th>
<th>( K^* )</th>
<th>( K_{opt} )</th>
<th>( K^*(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( | P_{zw} | )</td>
<td>12.135</td>
<td>11.676</td>
<td>11.589</td>
</tr>
</tbody>
</table>

Table II. Evolutions of the criterion with iterations

<table>
<thead>
<tr>
<th>( K^*(s) = [K^{(1)}(s), K^{(2)}(s), K^{(3)}(s)] )</th>
<th>( | P_{zw} | )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( [K^{(0)}(s), K^{(2)}(s), K^{(3)}(s)] = K^* )</td>
<td>12.135</td>
</tr>
<tr>
<td>( [K^{(1)}(s), K^{(0)}(s), K^{(0)}(s)] )</td>
<td>11.676</td>
</tr>
<tr>
<td>( [K^{(0)}(s), K^{(0)}(s), K^{(0)}(s)] )</td>
<td>11.650</td>
</tr>
<tr>
<td>( [K^{(1)}(s), K^{(1)}(s), K^{(1)}(s)] )</td>
<td>11.624</td>
</tr>
<tr>
<td>( [K^{(2)}(s), K^{(2)}(s), K^{(2)}(s)] )</td>
<td>11.589</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

The H\(_2\) norm has many appealing properties [10]-[11]. In particular, the square of the H\(_2\)-norm of a block transfer matrix is equal to the sum of the square of the H\(_2\)-norm of each block. Relying on this property, a sequential strategy to solve a global H\(_2\)-problem has been considered.

Structured H\(_2\) controller design requires to solve an optimization problem involving a linear objective criterion under matrices inequalities constraints [5],[7]. This optimization problem is not tractable in the general case of large-scale systems.

This paper has considered the particular case of LBT large-scale systems. In that case, the subsystems may be considered one after another through a sequential design. Each subproblem consists then to minimize a local H\(_2\) problem of low complexity. If this strategy insures the stability of the global LBT system, it is however suboptimal with regard to the global criterion.

The work in this paper highlights the comparison between the “classical” sequential H\(_2\) and the structured H\(_2\) design. Moreover, it extends the applicability of the sequential strategy to the case where a global H\(_2\) criterion has to be minimized. A state observer feedback form may be used at each step, allowing to reduce the numerical complexity of each elementary problem. The convergence of the algorithm proposed has been proved, and illustrated on the platoon of vehicles control problem. Model reduction is included as a part of the algorithm to prevent a rapid increase of the controller degree.

Based on the results of this paper, future works will look for an adaptation of existing H\(_2\)-based control design methodologies [10] to the case of large-scale LBT systems. Its extension to non-LBT system will be also considered to be finally applied to the problem of winding systems [2].

REFERENCES

[1] G. Quazza, "Large scale control problems in electric power systems", IFAC Symposium on large scale systems, pp. 1 – 18, 1976