Modeling, Estimation and Control of Cardiovascular Systems with A Left Ventricular Assist Device

Yi Wu\textsuperscript{1}, Paul Allaire\textsuperscript{2}, Gang Tao\textsuperscript{3} and Don Olsen\textsuperscript{4}

\textsuperscript{1}Department of Mechanical Engineering, Pennsylvania State University, Erie, PA 16563
\textsuperscript{2}Department of Mechanical and Aerospace Engineering, University of Virginia, Charlottesville, VA 22903
\textsuperscript{3}Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA 22903
\textsuperscript{4}Utah Artificial Heart Institute, Salt Lake City, UT 84103

Abstract—A dynamic model is developed through theoretical analysis and numerical solutions to approximate the response of human cardiovascular circulatory system. This system model has one critical time-varying parameter, the resistance of blood vessels. An parameter estimation scheme is derived to estimate this parameter, and the parameter estimate is used to implement an adaptive observer to estimate the aortic pressure for physiologic control. An optimal adaptive controller is proposed to control the estimated aortic pressure to track a reference signal updated by a nonlinear function of the pump head to meet the physiologic need.

A Matlab simulation model and an experimental mock human circulatory loop are employed as test environments for human cardiovascular circulatory systems with a left ventricular assist device and their physiologic controllers. Different physiologic conditions, such as the variation of left ventricular failures, variation of activities, and left ventricular suction, are evaluated to test the designed physiologic control system. Simulation and experimental results consistently show that the aortic pressure estimation error is small, and that the abnormal hemodynamic variables of a congestive heart failure patient are restored back to the normal physiologic range.

Keywords: Cardiovascular circulatory system, estimation, feedback control, left ventricular assist device, modeling.

1. INTRODUCTION

Heart disease is the leading cause of death in the Unite States, most of which is caused by predominant left ventricular failure: the left ventricle cannot pump blood at a rate which effectively supplies the body’s tissues and organs [2]. Left ventricular assist device (LVAD) is an artificial heart pump developed to aid the failed left ventricle by supplying additional blood flow to the body but does not replace the left ventricle. The researchers at Pennsylvania University, University of Virginia and Utah Artificial Heart Institute together, among other groups in the world, are developing a new magnetically levitated axial flow pump as an LVAD. The impeller of the pump is levitated by magnetic forces without any mechanical contact. There is no bearing wear or bearing seal so that: (i) there is no thrombosis factor, (ii) a mechanical durability for 10 to 20 years is expected. Therefore this type of pumps have the potential for permanent usage, a large improvement over present life of less than 2 years [1]. One key issue to be addressed in order to achieve this permanent use of LVADs is the development of an effective and efficient (robust and adaptive) physiologic control system.

In this paper, we give an overview of the recent research on the development of dynamic modeling of human cardiovascular circulatory systems and physiologic controllers for such systems with LVADs, and present some of our new analytical, simulation and experimental results on modeling and control of LVAD systems. Its full version is in http://www.personal.psu.edu/faculty/y/x/yxw22/ACC05.pdf.

2. HUMAN CIRCULATORY SYSTEM MODEL

A lumped parameter model of human cardiovascular circulatory systems can be analogously represented by a 11th-order electric circuit shown in Figure 1 [3]. It consists of analog blocks for left ventricle (\(LV\)), aorta (\(A\)), systemic artery (\(S\)), vein and right atrium (\(VA\)), right ventricle (\(RV\)), pulmonary artery (\(P\)), pulmonary vein and left atrium (\(L\)). Two parallel blocks for skeletal muscle (\(M\)) and non-muscular peripheral organs (\(O\)) respectively are placed between the blocks of systemic artery and systemic vein. Resistors represent the viscous property of blood flow, while inductances embody the inertia property of blood flow. Capacitors model the elastic property, or compliance of the vessel wall, and diodes are used to mimic the properties of one-directional valve. Muscular vascular resistance (\(R_{m}^{1}\)) and pulmonary vascular resistance (\(R_{m}^{3}\)) are modeled as variable resistors to accommodate their variations in different activity levels. With the muscle pump function represented by the intramuscular pressure \(P_{m}\) and venous valve \(D_{m}\), the sudden increase of central venous pressure at exercise onset was successfully simulated, in agreement with experimental results reported in literature [3].

In this paper, the pressure and volume relation of ventricle is modeled as an exponential function during diastole and a scaled quadratic function during systole. Heart failure is modeled by the decrease of the scale of the
quadratic function, and the increase of the coefficients of the exponential function [3]. Simulation results reproduce the phenomena that commonly observed in LVAD candidate, such as decreased ejection fraction (EF), decreased cardiac output (CO), and elevated left ventricular end-diastolic pressure (LVEDP).

3. OBJECTIVES OF PHYSIOLOGIC CONTROL

Control in the physiologic sense of Continuous flow (CF) LVADs presents major challenges because they create suction when the impeller speed is too high, and thus LVAD flow exceeds the venous return to the left ventricle. The level of blood flow assistance can become inadequate when the speed is too low, for example, there may be a retrograde flow of blood through the rotating impeller, from the pressurized aorta into the left ventricle, during ventricular diastole [4]. As a destination therapy device, modulation of the degree of ventricular assist and overall cardiac output based on physiologic needs are critical for long term LVAD performance. The main objectives for physiologic control are: (i) to produce adequate cardiac output to ensure tissue perfusion, (ii) to support native cardiac function (i.e., antegrade flow across aortic valve), (iii) to adjust cardiac output to compensate for increased physiologic need due to activity, (iv) to avoid left ventricular collapse due to mechanical suction from the pump, and (v) to provide sufficient pump speed to avoid retrograde pump flow from aortic-left ventricular pressure differential.

All five objectives are related to either blood pressure or flow rate. To regulate these hemodynamic variables, they need to be detected first by corresponding sensors. In LVADs, the sensors for hemodynamic variables (blood flow, pressure) are not available, due to the constriction of volume, blood compatibility, and reliability issues. Long term available signals, such as pump head and motor signals, do not have direct physiologic meanings. To meet these objectives, a state estimator, which can derive the hemodynamic variable (blood flow or pressure) from available feedback signals, is needed for the design and implementation of a physiologic control system for an LVAD.

4. REVIEW OF PHYSIOLOGIC CONTROL DESIGNS

Many CF LVADs are operated at constant speed, such as the Jarvik 2000 [5] and the DeBakey pump. The DeBakey pump also allows the physician or trained personnel to manually adjust the pump speed until a perceived comfort level of perfusion is achieved. These constant speed setting or manual adjustment are adequate for short period usage of LVADs, but are insufficient for long-term application.

P/PI control. Parnis et al. used a P controller for the Jarvik 2000 VAD. The pump rotational speed was set as a linear function of the heart rate, which was obtained from the fundamental frequency of the motor current waveform [6]. Treadmill exercise and heart pacing studies were performed on a calf with the Jarvik 2000 VAD and no deleterious effects were detected within test durations in excess of 26 weeks [6]. Waters et al. designed a PI controller for an LVAD using the differential pressure over LVAD for feedback control [8].

Fuzzy logic control. A fuzzy controller was employed to regulate the LVAD flow to track a desired flow rate [9]. The LVAD flow rate was estimated from the motor current and rotational speed. Choi et al. also implemented a PI type fuzzy controller to realize an LVAD pump flow rate pulsatility tracking [10]. A reference pulsatility level of 15 mL/sec was selected to allow the natural heart to produce some stroke volume without introducing ventricular suction. The LVAD pump flow rate was estimated from the motor current and rotational speed. Simulation results showed that the control signal of this fuzzy controller produced a much smaller variation in the LVAD speed with regard to parameter variations than that of a PI controller.

Optimal control. Giridharan et al. designed an optimal controller with the structure of a PI controller [11]. The time varying coefficients of the PI controller were obtained offline from exhaustive, direct numerical searches to minimize a weighting function, which was the combination of the differential pressure across the pump and the variation rate of the rotational speed. A similar but more complex optimal controller was designed by Boston et al. to minimize a multi-objective penalty function $J(\omega) = \sum_{i=1}^{3} \mu_i J_i(\omega)$ [12], where $J_i(\omega)$ was the penalty function of cardiac output, arterial pressure, left atrial pressure respectively, and $\mu_i \geq 0$ with $\sum_{i=1}^{3} \mu_i = 1$, using the motor speed for feedback.

These control designs rely on some assumptions, such as: (i) the heart contractility and the peripheral circulation have no influence on the cardiac output [6,9], (ii) the beating

Fig. 1. Circuit analog of circulatory systems with an LVAD.
of the natural left ventricle is a sinusoidal disturbance to the LVAD [8], (iii) simple and accurate predetermined mathematical models of cardiac output, arterial pressure, and left atrial pressure with respect to the motor speed are available [12], (iv) Off-line condition duplicates exactly the physical cardiovascular condition [11]. The deviation of these assumptions from physical cardiovascular condition can destroy the performance of designed controller.

5. SIMPLIFIED SYSTEM MODEL

To develop an effective robust and adaptive controller to meet the desired physiologic need, we consider a simplified cardiovascular system model shown in Figure 2 (as compared with the 11th-order model shown in Figure 1).

In this model, the system elements are: \( C_{LV} \), left ventricle; \( C_A \), aorta; \( C_V \), systemic vein and right atrium; \( D_1 \), aortic valve; \( D_2 \), mitral valve; \( TPR \), total peripheral resistance; \( L \), blood inertia in peripheral circulation; \( R_L \), resistance of aortic valve; \( R_V \), resistance of vein and left atrium; \( \delta_m \), the mean pressure disturbance generated by muscle pump function in exercise; \( \delta_p \), pressure disturbance by pulmonary circulation. The system signals are: \( P_L \), left ventricular pressure; \( P_A \), aortic pressure; \( P_V \), central venous pressure; \( Q \), pump flow rate; \( TPF \), total peripheral flow rate.

As in [3], the dynamic equations of this model are

\[
\dot{x} = A^d x + B_1^d Q + B_2 \delta_p + B_3 \delta_m, \quad y = C x, \text{ in diastole (5.1)}
\]

\[
\dot{x} = A^s x + B_1^s Q + B_3 \delta_m + B_4 \delta_L, \quad y = C x, \text{ in systole (5.2)}
\]

where \( x = [x_1, x_2, x_3]^T \), \( x_1 = P_L - P_A \), \( x_2 = P_V - P_A \), \( x_3 = TPF \), \( y = \Delta P = P_A - P_L \).

![Fig. 2. A simplified system model.](image)

The response of the system (5.1)–(5.2), in the steady-state, is a stable oscillation. The typical waveform of the aortic pressure \( P_A \) over the time period of one heartbeat is shown in Figure 3, where the time interval for diastole is \([t_d, t_s]\) and the time interval for systole is \([t_s, t_d]\).
With the ensured stability, the system equation (5.1)–(5.2) can be approximated by a single-equation model

\[
\dot{x} = Ax + B_1 Q + B_2 \hat{y} + B_3 \delta_m, \quad y = Cx \quad (5.6)
\]

where \( A = \begin{bmatrix} -1/R_V & 1/C_V & 1/L \cr -1/L & 1/R \cr 0 & -1/R \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1/C_V \cr 1/L \cr 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1/C_V \cr -1/L \cr 0 \end{bmatrix},
\]

and \( B_1 = B_1^t, \) where \( R_V \) is an equivalent venous resistance. The circulatory system (5.1)–(5.2) and the system (5.6) have the same solution at \( t = t_s \) [3]. The output of (5.6) at \( t = t_s \) is equal to the pump head of the LVAD: \( \Delta P = P_A - P_{LV} \), whose maximum value in each heartbeat occurs at \( t = t_s \).

The proof that system (5.6) has the same solutions of states at \( t_s \) in each heartbeat as system (5.1)–(5.2) is very tedious [3], but can also be briefly explained. Systole and diastole are two opposing process. In diastole, state \( x_1 \) of human circulatory system tend to converge exponentially, while in systole, state \( x_1 \) tend to diverge towards the opposite direction because of the zero eigenvalues in matrix \( A \). The states of human circulatory system (5.1)–(5.2) thus exhibits periodic-like trajectory because of the alternation of systole and diastole. The bottom envelop of the periodic-like trajectory actually converge exponentially due to the proven exponential stability of the system (5.1)–(5.2) in previous paragraph, at a slower speed than the state trajectory in diastole, which is dominated solely by equation (5.1). Therefore system (5.6), which has the same structure as equation (5.1) but a different value for venous resistance, was proposed to match the bottom envelop. The equivalent venous resistance \( R'_V \) in system (5.6) is much bigger than the corresponding venous resistance \( R_V \) in equation (5.1), and matches the slow converging speed of bottom envelop. The value of \( R'_V \) can be determined by the measurements in LVAD implantation surgery by a formula given in [3].

System (5.6) provides a linear system whose parameter estimator and state estimators are easy to design. Since states of human circulatory system (5.1)–(5.2) are equal to the states of system (5.6) at \( t = t_s \), we can obtain the states of human circulatory system at \( t_s \) in each heartbeat. The average value of aortic pressure, which is determined more by the bottom envelop of states than by the instantaneous value of states, is of more interest than the instantaneous value of aortic pressure. The approximation of states of system (5.1)–(5.2) with the state estimate of system (5.6) can be extended to the whole heartbeat besides at \( t = t_s \).

With an accurate value for \( R'_V \) in system (5.6), the approximation error can be very small. However, big fluctuation of input, variation of systole vs. diastole ratio, heart rate will increase the approximation error because the perfectly matched value for \( R'_V \) may change in these conditions [3].

Based on this equation of the circulatory system, a state observer, a feedback controller, and a parameter estimator can be developed for physiologic control. From this system expression (the presence of the parameter \( C_{LV} \)), we see that the diastolic behavior is more dominant in the circulatory system response in a mean sense.

### 6. An Adaptive Optimal Controller

We now present the detailed results of our adaptive optimal physiologic controller whose design consists of three parts: an adaptive parameter estimation scheme to estimate the total peripheral resistance \( TPR \), an adaptive state observer using the estimate of \( TPR \), and an optimal PI controller also using the estimate of \( TPR \) for controller parameter selection. This physiologic control system is shown in Figure 4.

**Parameter estimation.** From (6.6), it follows that

\[
(3s + a_2 s^2 + a_1 s + a_0) y(s) = (b_2 s^2 + b_1 s + b_0) Q(s) + (b_2'' s^2 + b_1'' s + b_0'') \delta_m(s) + (b_2''' s^2 + b_1''' s + b_0''') \delta_m(s)
\]

for some constants \( a_i \) and \( b_i \) (which depend on \( TPR \), \( b_i'' \) and \( b_i''' \), \( i = 0, 1, 2 \). The nominal value of \( \delta_m \) can be set to 5 mmHg, and the low frequency component of \( \delta_m \) is less than 5 mmHg [3]. A low-pass filter

\[
H(s) = \frac{s + 125}{s + 125}
\]

with \( \Lambda(s) = s^3 + 15 s^2 + 75 s + 125 \), can be used to remove the high frequency components of \( \delta_m \), so that \( \delta_m \) can be ignored for parameter estimation of \( TPR \), using such a filter.

Filtering both sides of (6.1) by \( H(s) \) and arranging the resulting terms, we can derive the parametric equation

\[
\dot{\theta} \phi(s) = z(s)
\]

for \( \theta^* = TPR \) and some functions \( \phi(s) \) and \( z(s) \) whose denominators are \( \Lambda(s) = s^3 + 15 s^2 + 75 s + 125 \) and numerators are combinations of \( y(s) \), \( Q(s) \) and \( \delta_m(s) \) (with \( \delta_m \) ignored).

In the time-domain, we define the estimation error \( \epsilon(t) = \theta(t) \phi(t) - z(t) \), where \( \theta(t) \) is the estimator of \( \theta^* \), and use
the following adaptive law to update the estimate $\theta(t)$:

$$\dot{\theta}(t) = -\frac{\gamma e(t)\phi(t)}{1+\phi^2(t)}, \gamma > 0. \quad (6.3)$$

**State observer.** The state observer structure for the estimate $\hat{x}$ of $x = [x_1, x_2, x_3]^T$ in (5.1)–(5.2) is a standard one, based on the equivalent model (5.6) with all parameters but $TPR$ known and $\delta_m$ ignored. The adaptive estimate of $TPR$ obtained on-line from the above parameter estimation procedure is used, leading to an adaptive observer. The estimate of $P_A$ is given as $\hat{P}_A = H\hat{x} + N$ (see (5.5)).

**Optimal PI controller.** The objective of physiologic control is to raise the aortic pressure to a certain level (95 mmHg as the nominal value). It is achieved by a feedback control design and a chosen reference signal $r(t)$.

The motor equation of an LVAD can be described by

$$J\dot{\omega} = K_I - T_{load}, \quad LI + RI + K_r\omega = V \quad (6.4)$$

where $V$ is the applied voltage, $\omega$ is the pump rotational speed, $I$ is the motor current, and $T_{load} = \frac{\Delta PQR}{\eta}$ is the blood hydraulic torque on the pump impeller, with $\eta$ being the pump efficiency. Since the motor inductance and the pump moment of inertia $J$ are small, the motor equation is simplified as

$$V = \frac{\Delta PQR}{\eta\delta_m} + K_r\omega. \quad (6.5)$$

This, together with that in (5.3), gives an expression of $Q$ in terms of $\Delta P$ and $V$, which can be linearized as $Q = \beta_1\Delta P + \beta_2V$ for some parameters $\beta_1$ and $\beta_2$ [3].

The implication of such an expression is that the equation (5.6) can be re-expressed with $V$ as the input signal [3].

The control objective now is to find a feedback control signal (voltage) $V$ for the re-expressed (5.6) such that the aortic pressure $P_A$ (through its estimate $\hat{P}_A$ from the adaptive state observer) tracks a reference signal $r(t)$ chosen to meet certain physiologic need.

For the control system shown in Figure 4, $K_I$ is an integral gain, $K$ is a feedback gain vector calculated from an optimal control design procedure [3] in which the estimate of $TPR$ is used to replace the unknown $TPR$ in optimal control and state observation, leading to an adaptive control scheme. To derive such a control scheme, we started with the linearized expression $Q = \beta_1\Delta P + \beta_2V$, to express the system (5.6) as

$$\dot{x} = (A + B_1\beta_1 C)x + B_1\beta_2 V + B_3\delta_P + B_5\delta_m, \Delta P = Cx. \quad (6.6)$$

An adaptive state observer using system parameters and an adaptive estimate of $TPR$ is constructed to generate an estimate $\hat{x}$ of $x$ and an estimate $\hat{P}_A$ of $P_A$. The adaptive optimal controller uses $\hat{P}_A$ for feedback control.

In this physiologic control scheme, the reference signal $r$ is generated from a design function $f$ and an auxiliary signal $r_m$, using a precompensation algorithm: $r = r_m + f(\Delta P)$. The function $f$ is a nonlinear function of $\Delta P$, which is inversely proportional to $\Delta P$. If the LVAD flow rate is lower than the venous return, $\Delta P$ is decreased, and in turn the reference signal $r$ is elevated. If the pump flow rate is higher than the venous return, the reference signal $r$ is lowered.

This is important because the body need for blood flow may vary a lot in the presence of physiologic state variation. In these variations, some parameter of the human circulatory system, like total peripheral resistance, may change a lot, and cause the variation of the venous return, in turn left ventricular pressure $P_LV$. As a result, the pump head of LVAD $\Delta P$ will exhibit a variation too [3]. The update of reference signal by the designed $f$ function of $\Delta P$ can guarantee that the body need can always be matched by the LVAD flow rate, and prevent the underperfusion and overperfusion of human body that may happen if constant reference value is used in the variation of physiologic states.

7. SIMULATION STUDY

The control system shown in Figure 4 was simulated using Matlab/Simulink, with the controller designed based on the reduced-order model (5.6) and applied the full-order system shown in Figure 1. Simulation results for various healthy and CHF cases were obtained, which validated our analytical work in modeling, estimation and control of cardiovascular circulatory systems with a LVAD [3].

The simulation results from rest to exercise with an LVAD is now summarized. The resistance of blood vessel ($TPR$) decreases dramatically in exercise, and the total peripheral flow ($TPF$) increases significantly. The average estimation error for aortic pressure is maintained less than 1 mmHg in this activity variation. The value for reference signal $r$ changes from 100 mmHg to 105.4 mmHg in exercise. The blood flow rate, the LVPED, and the average aortic pressure, are restored by the LVAD to: 5.6 L/min in rest and 9.8 L/min in exercise, 1.1 mmHg in rest and 2.3 mmHg in exercise, and 99.6 mmHg and 103.2 mmHg respectively. These values are close to the simulation results for a healthy person in rest and exercise conditions [3].

8. EXPERIMENTAL STUDY

A mock human circulatory loop was set up as an in vitro test rig for different versions of prototype LVADs, as shown in Figure 5, whose components mimic the key components of the human cardiovascular system, and can simulate different normal or pathologic states and activities of a cardiovascular system [13] (a small centrifugal pump MY2 was used in the place of an LVAD).

Figures 6 illustrate the experimental results of a mock human loop with the MY2 pump in pathologic states I, II, and III respectively. MY2 pump is controlled by a real-time controller board DS 1104 (dSPACE Inc., MA.). Another experiment of the mock human loop without a LVAD is run in the exactly same pathologic states for the purpose of comparison. The blood flow rate, LVPED, and average aortic pressure without a LVAD were found to be: 3.52 L/min, 16.2 mmHg, and 47.2 mmHg respectively in State I; 2.36 L/min, 17.4 mmHg, and 28.8 mmHg respectively in State II; 4.58 L/min, 20 mmHg, and 68.7 mmHg respectively in State III.
mmHg respectively in State III [3]. The average estimation error for aortic pressure in the experiment with a LVAD was maintained less than 2 mmHg. The abnormal hemodynamic variables, such as the blood flow rate, the LV PED, and the average aortic pressure, are all restored to the normal physiologic range, 5-6 L/min, <15 mmHg, and ~95 mmHg respectively, by the designed physiologic controller in the presence of pathologic state variations. The reference signal is set constant with the value of 95 mmHg without an online update in the experiment because the mock circulatory loop was unable to reproduce the relation between the left ventricular pressure $P_{LV}$ and the venous return [3], thus invalidate the use of the $f$ function to update the reference signal derived upon that relation.

9. CONCLUDING REMARKS

The design of a physiologic controller for a permanent LVAD is described in this paper. With the single-equation model (5.6) derived for the human circulatory system, the adaptive estimation and control methods has been applied in the controller design. Computer simulation and mock circulatory loop test consistently show good controller performance in the variation of physiologic states, in terms of aortic pressure estimation error, restoring abnormal hemodynamic variables back to normal range, etc.

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