Model-based Prediction of a Percutaneous Ventricular Assist Device Performance §

Yih-Choung Yu1, Marwan A Simaan2, Nicholas V. Zorn3, and Simon Mush1

1Department of Electrical and Computer Engineering, Lafayette College, Easton, PA 18042 USA
2Department of Electrical Engineering, University of Pittsburgh, Pittsburgh, PA 15261, USA
3Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, MA 02420, USA

§ This work was partially supported by NSF Grants BE-0420848 and ECS-0300097

Abstract – A percutaneous ventricular assist system is an external heart assist device that bypasses blood from the left atrium and returns it to the femoral artery to support patients who suffer from acute heart failure. The system consists of a centrifugal blood pump, an atrial drainage cannula, and various sizes of arterial perfusion cannula. Because the device allows cardiologists the freedom of choosing the arterial cannula based on a patient’s body size, it is extremely difficult but important to predict the level of support the device can provide to the patient before the device is up and running. In this paper, the TandemHeart pVAD (Cardiacassist Inc. Pittsburgh, PA) is modeled as a nonlinear electric circuit, including a speed dependent voltage source and current dependent resistors, to predict the performance of the system by specifying pump speed, mean arterial pressure (MAP), and mean left atrial pressure (LAP). The model structure is developed based upon the pipeline theory while the model parameters are identified by least-squares fit of the model to the experimental data. The flow rate is predicted by solving a quadratic equation while coefficients in the equation are scaled, depending on the arterial cannula configurations. The model can predict the flow rates accurately with error indices of all test conditions less than 6%, comparing the predicted flow from the model with the experimental data.

I. Introduction

A left Ventricular Assist Device (LVAD) is a controlled mechanical pump which is surgically implanted with the patient’s heart to assist the left ventricle in pumping oxygen-rich blood into the patient’s circulatory system [1-5]. A percutaneous left heart assist system, including a transeptal cannula, a blood pump, and a femoral arterial cannula, is a device that bypasses blood from left atrium to femoral artery through a blood pump [6,7]. The transeptal cannula, inserted from femoral vein by way of right atrium then into left atrium through the atrial septum, drains blood from left atrium into a blood pump. The blood pump then returns blood into patient’s femoral artery through an arterial cannula. The advantage of this type of ventricular support is that the whole procedure can be accomplished in a cardiac catheterization laboratory within a short period of time without a major open-heart surgery. Thus, it is beneficial to patients with emergency heart failure.

The use of this system requires selecting an appropriate size of arterial cannula to maximize the blood flow rate, and thus the cardiac support, provided by the system, while maintaining certain amount of distal flow through the patient’s leg to reduce the risk of distal leg ischemia. Since different sizes of arterial cannulae produce different fluid resistances in the assist circulation loop, this would cause a difference in the assist blood flow rate at the same MAP, LAP, and pump speed conditions. Determining the system performance based on the selection of arterial cannula usually relies on bench top experiments, which is time consuming. If a computer model were available for the evaluation, this would save the time requirement for bench testing. The computer model could also be a tool for cardiologists to choose an appropriate size of arterial cannula for patients.

Figure 1. Anatomical fitting of the TandemHeart pVAD

0-7803-9098-9/05/$25.00 ©2005 AACC 3835
This paper describes the use of a nonlinear electric circuit to model a TandemHeart percutaneous ventricular assist system shown in Figure 1, including a centrifugal blood pump as a speed dependent voltage source along with a current dependent resistor, a 21 Fr. atrial drainage cannula, and various sizes of femoral arterial cannulae as current dependent resistors. Model parameters were determined by least squares fit of the model to experimental data. By adding two constant voltage sources into the circuit to simulate MAP and LAP, this model can predict the bypass blood flow rate for a selected arterial cannula configuration for given MAP, LAP, and pump speed. An algorithm was developed previously [8] to determine the flow rate by minimizing an objective function through the SIMPLEX optimization algorithm [9] in MATLAB, which was time consuming and difficult to implement in real-time simulation. This paper presents an analytical solution for the flow rate using a non-linear circuit with model parameter scaling. Accuracy of this algorithm was tested using data from a bench-top experiment.

II. Model Description and Analysis

The electric analogue of the model is shown in Figure 2. The effects of fluid inertia and compliance were excluded from the model. Since the purpose of this model is to predict the average flow, the transient response in the system due to fluid inertia and compliance should be negligible. MAP and LAP were modeled as constant voltage sources. The effect of the native heart to the device was not explicitly modeled. However, as LAP and MAP are usually referred as the preload (the blood volume that the native heart needs to push out for circulation) and afterload (the pressure that is against the blood pumping out of the left ventricle) of the native heart, the effects of native heart to the TandemHeart pVAD system as well as the cardiovascular hemodynamics were implicitly included in LAP and MAP. Pressure drop, \( P \) (analogy to electric current), through the pipeline path, which can be generalized as [10]

\[
P = f(Q).
\]

Approximating (1) near a particular flow rate \( Q^* \) with a 2nd-order Taylor series expansion results in

\[
P = f(Q^*) + \frac{\partial f}{\partial Q}|_{Q^*} (Q-Q^*) + \frac{\partial^2 f}{\partial Q^2}|_{Q^*} \frac{(Q-Q^*)^2}{2}.
\]

By defining \( \Delta P = P - f(Q^*) \) and \( \Delta Q = Q - Q^* \), Eq (2) can be rearranged as

\[
\Delta P = R_0 \Delta Q + R_1 \Delta Q^2,
\]

where \( R_0 = \frac{\partial f}{\partial Q}|_{Q^*} \) and \( R_1 = \frac{1}{2} \frac{\partial^2 f}{\partial Q^2}|_{Q^*} \). Eq. (3) can be viewed as the pressure drop across a portion of the assist circulation, for example, the inflow or outflow cannula. Therefore, from Ohm’s Law, the fluid resistance, \( R \), along a conduit is an affine function of flow passing through the tube,

\[
R = R_0 + R_1 |\Delta Q|.
\]

The modulus operator in (4) represents the resistance is the same regardless of the flow direction.

In Figure 2, the 21 Fr. atrial drainage cannula was modeled by the flow dependent resistor \( R_I \), formulated as (4), and thus the pressure drop across \( R_I \) can be written as

\[
\Delta P_I = R_{I0} \cdot Q + R_{I1} |Q| \cdot Q
\]

where \( R_{I0} \) and \( R_{I1} \) were determined by least-squares fit to the experimental data. The centrifugal blood pump [5] was modeled by a voltage source, \( P_P \), along with a flow dependent internal resistance, \( R_P \). The pressure drop across the pump can be expressed as

\[
\Delta P_P = P_P(\omega) - R_{P0}(\omega) \cdot Q - R_{P1}(\omega) |Q| \cdot Q
\]

where \( P_P(\omega) \), \( R_{P0}(\omega) \), and \( R_{P1}(\omega) \) are pump speed dependent variables, determined by least-squares fit of pump pressure drop versus flow (H-Q) data. \( \omega \) is the pump speed in RPM.
\[ \Delta P_O = R_{O10} \cdot Q + R_{O11} \cdot Q, \]  

(7)

where \( R_{O10} \) and \( R_{O11} \) are the model parameters of a specific type of arterial cannula.

The pressure rise across the pVAD pump should overcome the pressure difference between left atrium and femoral artery, and the pressure losses across atrial drainage cannula and arterial cannula, and thus generate the blood flow through the system. From Kirchoff’s voltage law (KVL), this can be expressed as,

\[ \Delta P_p - \Delta P_I - \Delta P_O = MAP - LAP. \]  

(8)

Substituting (5) to (7) into (8) and moving MAP-LAP to the left-hand side yields to,

\[ A \cdot |Q| \cdot Q + B \cdot Q - C = 0, \]  

(9)

where \( A = R_{P1}(o) + R_{O11} + R_{I1} \), \( B = R_{P0}(o) + R_{O10} + R_{I0} \), and \( C = P_p(o) + LAP - MAP \). If \( Q \) always flows in forward direction, i.e. \( Q > 0 \), one can obtain \( Q \) by solving (9) for given pump speed, MAP, and LAP, as

\[ Q = \frac{-B + \sqrt{B^2 + 4AC}}{2A}. \]  

(10)

**Case 2. Dual arterial cannulae with the same size**

When two identical arterial cannulae are connected at the pump outlet (the switch is closed in Figure 1), the pressure drop across both cannulae should be the same,

\[ \Delta P_O \equiv \Delta P_{O1} = \Delta P_{O2}. \]  

(11)

Since both arterial cannulae are identical, the resistance should be the same. The identical pressure drops and flow resistances imply that \( Q_1 \) and \( Q_2 \) are equal to half of the total flow \( Q \) through the system,

\[ Q_1 = Q_2 = Q/2. \]  

(12)

Substituting (12) into (7), \( \Delta P_O \) in (7) can be rewritten as

\[ \Delta P_O = R_{O10} \cdot Q_1 + R_{O11} \cdot |Q_1| \cdot Q_1 = \left( \frac{R_{O10}}{2} \right) \cdot Q + \left( \frac{R_{O11}}{4} \right) \cdot |Q| \cdot Q. \]  

(13)

By changing \( R_{O10} \) to \( R_{O10}/2 \) and \( R_{O11} \) to \( R_{O11}/4 \) in the coefficients \( A \) and \( B \) in (9), the total flow, \( Q \), through the system can be solved using (10), if \( Q \) flows in forward direction.

**Case 3. Dual arterial cannulae with different sizes**

When the dual arterial cannulae are in different sizes, the pressure drops across both arterial cannulae are the same as described previously, i.e., equation (11) is still valid under this condition. However, the flow rate in one cannula is different from that in the other because of the geometric differences, and thus the resistances are different between two cannulae. Equation (7) can be expressed as

\[ \Delta P_O = R_{O10} \cdot Q_1 + R_{O11} \cdot |Q_1| \cdot Q_1 = R_{O20} \cdot Q_2 + R_{O21} \cdot |Q_2| \cdot Q_2, \]  

(14)

where \( R_{O10} \) and \( R_{O11} \) are the model parameters of the arterial cannula 1, and \( R_{O20} \) and \( R_{O21} \) are the parameters of the arterial cannula 2. \( Q_1 \) and \( Q_2 \) are flow rates through the arterial cannula 1 and 2 respectively. If \( Q_1 \) and \( Q_2 \) flow in the forward direction, they can be determined by solving (13). Then

\[ Q_1 = \frac{-R_{O10}}{2R_{O11}} + \sqrt{\left( \frac{R_{O10}}{2R_{O11}} \right)^2 + \left( \frac{\Delta P_O}{R_{O11}} \right)}, \]  

(15)

and

\[ Q_2 = \frac{-R_{O20}}{2R_{O21}} + \sqrt{\left( \frac{R_{O20}}{2R_{O21}} \right)^2 + \left( \frac{\Delta P_O}{R_{O21}} \right)}. \]  

(16)

Assuming that the resistance \( R_{O2} \) can be approximated by the resistance coefficients of \( R_{O1} \) [10],

\[ R_{O2} = \alpha \cdot R_{O10} + \alpha^2 \cdot R_{O11}, \]  

(17)

where the ratio \( \alpha \) can be either \( R_{O20}/R_{O10} \) or \( \sqrt{R_{O21}/R_{O11}} \). The total flow \( Q \) can be obtained by adding (15) with (16) while substituting (17) into (16),

\[ Q = (1 + \frac{1}{\alpha}) \left[ -\frac{R_{O10}}{2R_{O11}} + \sqrt{\left( \frac{R_{O10}}{2R_{O11}} \right)^2 + \left( \frac{\Delta P_O}{R_{O11}} \right)} \right]. \]  

(18)

\( \Delta P_O \) can be expressed as \( Q \) multiplied by an equivalent resistance, \( R_{EQ} \), by rearranging equation (18),

\[ \Delta P_O = Q \left[ \left( \frac{\alpha}{1+\alpha} \right) R_{O10} + \left( \frac{\alpha}{1+\alpha} \right)^2 R_{O11} \cdot Q \right], \]  

(19)

where \( R_{EQ} = \left( \frac{\alpha}{1+\alpha} \right) R_{O10} + \left( \frac{\alpha}{1+\alpha} \right)^2 R_{O11} \cdot Q \). Therefore, the dual arterial cannulae in parallel can be viewed as a single cannula configuration in Case 1 by replacing \( R_{O10} \) with \( \left( \frac{\alpha}{1+\alpha} \right) R_{O10} \) and \( R_{O11} \) with \( \left( \frac{\alpha}{1+\alpha} \right)^2 R_{O11} \). As a result, the total flow \( Q \) can be solved analytically by (10).
III. Experiment

A test loop, shown in Figure 3, comprising a preload chamber, a 21 Fr. atrial drainage cannula as the inflow cannula, an AB-180 pVAD Pump, a test arterial cannula as the outflow cannula, and a tubing clamp, was used for experiment. A clamp-on style ultrasonic flow transducer (Transonic Systems, Inc., Model H9XL) was placed on a Tygon tubing (3/8” I.D.) between the arterial perfusion cannula and the pump outlet to measure flow through the pump. A tubing clamp was adjusted manually to achieve a desired set point of either flow rate measured by the ultrasonic flow transducer or mean arterial pressure measured by a Transpac IV Pressure Transducer and displayed on the patient monitor. The height of fluid in the preload chamber determines the pressure that filled the pump, adjusted to simulate left atrial pressure (LAP). The test fluid in the hydraulic loop was a blood analog, consisted of a solution of 35% glycerol and 65% saline (by volume) to simulate the viscosity of blood at 37°C. The saline is a 0.9% sodium chloride solution. The fluid is maintained at room temperature (21-23 °C).

Outflow Cannula

MAP Measurement

AB-180 pVAD Pump

Tubing Clamp

Transonic Flow Probe

Figure 3. Experiment setup

The test loop served in two purposes: (1) generating data to identify the model parameters, and (2) providing data to validate the accuracy of the model in predicting total bypass flow by changing pump speeds, MAP, LAP, as well as arterial cannula configuration.

Two cannulae with different sizes, Edwards Research Medical 12 Fr. arterial perfusion cannula and Medtronic dlp 17 Fr. arterial cannula, were arranged in the test loop individually as the outflow cannula in Figure 3. The arterial drainage cannula in Figure 3 was replaced by a 3/8” Tygon tubing for the test. The speed of the pVAD pump was set at 7500 RPM. The tubing clamp was used to adjust flow rate from 0 to the maximum flow rate with a 0.5 L/min increment. The resulting pressure drop was then recorded at each test point.

A Similar test was conducted to obtain the pressure drop and flow data for the AB-180 pVAD pump. Both cannulae in the test loop in Figure 3 were replaced by 3/8” Tygon tubings and the differential pressure transducer was placed across the pump inlet and outlet. Pump speed was set from 3000 to 7500 rpm with an increment of 1500 rpm. At each set speed, the tubing clamp was adjusted to obtain the flow set points from 0 to the maximum flow with a 0.5 L/min increment. The resulting pressure drop was then recorded at each test point.

In the validation test, the arterial perfusion cannulae tested in the experiment are listed in Table 1. In each test, a configuration of single or dual cannulae was arranged in the test loop, shown in Figure 3, as the outflow cannulae. Pump preload was set at 5 mmHg and 20 mmHg (mimicking LAP) by changing the fluid height in the preload chamber. In each preload setting, the pVAD pump was operated at speeds of 3000, 4500, 6000, and 7500 rpm. At each pump speed, the tubing clamp was adjusted to get the desired pump afterload of 60, 80, and 100 mmHg (mimicking MAP). These set points cover the whole range of operating conditions of the AB-180 pVAD pump. The resulting flow rate at each condition was recorded and compared with the prediction from the model to quantify the model accuracy.

<table>
<thead>
<tr>
<th>Test</th>
<th>Arterial Cannula at the Outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single 12 Fr.</td>
</tr>
<tr>
<td>2</td>
<td>Single 17 Fr.</td>
</tr>
<tr>
<td>3</td>
<td>12 Fr. + 17 Fr.</td>
</tr>
</tbody>
</table>

IV. Results

1. Model parameter identification

The pressure drop and flow data obtained for the 12 Fr. and 17Fr. arterial cannulae in the experiment were used in least-squares fit in Microsoft Excel to determine the model parameters R_{O10} and R_{O11} in (7) for each individual cannula. Table 2 summarizes the identified model parameters of each cannula. R’s close to 1 imply a good fit of the models to the experimental data. The model parameters of the arterial drainage cannula in (5) were determined by using the same data fitting procedure using pressure drop and flow data collected in the arterial drainage cannula testing. The results are listed in Table 2.

The parameters P_{Pr}, R_{P0}, and R_{P1} in (6) are speed dependent parameters, which can be obtained from least-squares fit of experimental data. Table 3 lists the model parameters in (6) identified at pump speeds of 3000, 4500, 6000, and 7500 RPM.
Table 2. Identified model parameters of the arterial and atrial drainage cannulae

<table>
<thead>
<tr>
<th>Arterial cannula</th>
<th>R_{00}</th>
<th>R_{01}</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 Fr.</td>
<td>17.469</td>
<td>29.819</td>
<td>0.9998</td>
</tr>
<tr>
<td>17 Fr.</td>
<td>5.8022</td>
<td>6.8518</td>
<td>0.9998</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Atrial drainage cannula</th>
<th>R_{10}</th>
<th>R_{11}</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.1567</td>
<td>12.683</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Table 3. List of pump model parameters identified from least-squares fit to the experimental data

<table>
<thead>
<tr>
<th>RPM</th>
<th>P_p</th>
<th>P_{p0}</th>
<th>R_p1</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>64.688</td>
<td>5.5595</td>
<td>0.6464</td>
<td>0.9999</td>
</tr>
<tr>
<td>4500</td>
<td>148.03</td>
<td>6.6184</td>
<td>0.7285</td>
<td>0.9945</td>
</tr>
<tr>
<td>6000</td>
<td>274.2</td>
<td>11.328</td>
<td>0.1026</td>
<td>0.9978</td>
</tr>
<tr>
<td>7500</td>
<td>431.77</td>
<td>12.252</td>
<td>0.3694</td>
<td>0.9986</td>
</tr>
</tbody>
</table>

2. Evaluation of the prediction accuracy

Given pump speed, MAP, and LAP, the total flow $Q$ from the device with a single 12 Fr or 17 Fr arterial cannula can be carried out by substituting the appropriate arterial cannula model parameters, $R_{00}$ and $R_{01}$, to determine the coefficients, $A$, $B$, and $C$ in (9) and then solving for $Q$ in (10). An error index, defined by

$$E_I = \frac{100}{\sqrt{\sum_{k=1}^{n} [Q_{\text{measured}}(k) - Q_{\text{estimated}}(k)]^2}} \times 100\%$$

was used to quantify the overall accuracy. Panels (a) and (b) in Figure 6 show the predicted flow versus measured flow with 12 Fr and 17 Fr arterial cannulae.

Comparing the predicted results from 12 Fr and 17 Fr cannulae in parallel with the measured data was evaluated in four possible values of $\alpha$, depending on which cannula was used as the reference ($R_{01}$) and how $\alpha$ was obtained as described in Table 4. The equivalent arterial cannula resistance was calculated by (19) and then used to determine the flow rate in (10). Predictions versus measurements in these four cases are shown in panel (c), (d), (e), and (f).

Table 4. Test conditions for dual arterial cannulae (12 Fr + 17 Fr) configuration

<table>
<thead>
<tr>
<th>Panel</th>
<th>Reference Resistance ($R_{01}$)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>12 Fr. Arterial Cannula</td>
<td>$R_{6(17)}/R_{6(12)}$</td>
</tr>
<tr>
<td>(d)</td>
<td>12 Fr. Arterial Cannula</td>
<td>$\sqrt{R_{1(17)}/R_{1(12)}}$</td>
</tr>
<tr>
<td>(e)</td>
<td>17 Fr. Arterial Cannula</td>
<td>$R_{6(12)}/R_{6(17)}$</td>
</tr>
<tr>
<td>(f)</td>
<td>17 Fr. Arterial Cannula</td>
<td>$\sqrt{R_{1(12)}/R_{1(17)}}$</td>
</tr>
</tbody>
</table>

Table 5. Comparison of predicted flow vs. measured flow

<table>
<thead>
<tr>
<th>Panel</th>
<th>Arterial cannula configuration</th>
<th>Slope ($y = a \cdot x$)</th>
<th>R^2</th>
<th>E_I (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Single 12 Fr</td>
<td>1.0233</td>
<td>0.9993</td>
<td>2.63</td>
</tr>
<tr>
<td>(b)</td>
<td>Single 17 Fr</td>
<td>1.0341</td>
<td>0.9991</td>
<td>3.66</td>
</tr>
<tr>
<td>(c)</td>
<td>12 Fr. + 17 Fr</td>
<td>1.0583</td>
<td>0.9996</td>
<td>5.91</td>
</tr>
<tr>
<td>(d)</td>
<td>12 Fr. + 17 Fr</td>
<td>1.0123</td>
<td>0.9996</td>
<td>1.51</td>
</tr>
<tr>
<td>(e)</td>
<td>12 Fr. + 17 Fr</td>
<td>1.0041</td>
<td>0.9995</td>
<td>1.07</td>
</tr>
<tr>
<td>(f)</td>
<td>12 Fr. + 17 Fr</td>
<td>1.0123</td>
<td>0.9995</td>
<td>2.82</td>
</tr>
</tbody>
</table>

V. Discussion and Conclusion

The flow prediction versus measurement plots in Figure 6 show strong linear relationship ($r^2 \approx 1$). The train lines with slopes close to one in all cases imply that the model predicted the flow rate accurately. However, the slopes in all cases were greater than 1, which indicates the model produced over-estimates most of the time. This might be due to un-modeled errors in experiment setup, where extra fluid resistance could be produced by additional tubing connections in the test loop. The prediction error indices, described by (20), for all arterial cannula configurations were less than 6% as shown in Table 5. In dual cannula configuration, the ratio, $\alpha$, can be determined in different ways, depending on the selection of reference cannula. However, different selections of $\alpha$ did not produce any significant difference in terms of accuracy in the test.

Table 6. Comparison of predicted flow vs. measured flow

In conclusion, a nonlinear circuit model was developed to predict the performance of a percutaneous ventricular assist system, including a centrifugal blood pump, an atrial drainage cannula, and various sizes of arterial perfusion cannulae, based on the selections of different configurations of arterial cannulation. The predicted flow rate was obtained by solving a quadratic equation with resistance coefficient scaling when dual arterial cannula configuration is used. It is shown that the model can predict the bypass flow rate through the system with an error index less than 6% within a specified LAP, MAP, and pump speeds. This model could be a useful tool to assist cardiologists to select an appropriate configuration of arterial cannulation for a specific patient. It can be integrated with a computer model of the cardiovascular system as a tool to train the technical personnel who operates the device in clinical facilities. In addition, the same modeling concept described herein could be applicable to evaluate or design micro fluidic systems in which conducting bench top experiments might be too difficult or extremely time consuming.

References


---

Figure 6. Flow prediction vs. measurement with various outflow cannula configurations

(a) Single 12 Fr

\[ y = 1.0233x \]

\[ R^2 = 0.9993 \]

(b) Single 17 Fr

\[ y = 1.0341x \]

\[ R^2 = 0.9991 \]

(c) \( R_{O1}=R_{12}; \alpha=R_0(17)/R_0(12) \)

\[ y = 1.0583x \]

\[ R^2 = 0.9996 \]

(d) \( R_{O1}=R_{12}; \alpha=\sqrt{R_1(17)/R_1(12)} \)

\[ y = 1.0123x \]

\[ R^2 = 0.9996 \]

(e) \( R_{O1}=R_{17}; \alpha=R_0(12)/R_0(17) \)

\[ y = 1.0041x \]

\[ R^2 = 0.9995 \]

(f) \( R_{O1}=R_{17}; \alpha=\sqrt{R_1(12)/R_1(17)} \)

\[ y = 1.0123x \]

\[ R^2 = 0.9996 \]