Abstract—In this paper, we consider the dynamic visual feedback control with the uncertainty of the camera coordinate frame based on the passivity. Firstly the brief summary of the nominal visual feedback systems with a fixed camera is given with the fundamental representation of a relative rigid body motion. Secondly we construct the visual feedback system with uncertainty which is not be limited to the orientation around the optical axis. Next, we derive the passivity of the dynamic visual feedback system by combining the manipulator dynamics and the visual feedback system. Based on the passivity, stability and $L_2$-gain performance analysis are discussed. Finally the validity of the proposed control law can be confirmed by comparing the simulation results.

I. INTRODUCTION

Robotics and intelligent machines need many information to behave autonomously under dynamical environments. One suitable way to recognize unknown surroundings is to use visual information. Vision based control of robotic systems involves the fusion of robot kinematics, dynamics, and computer vision to control the motion of the robot in an efficient manner. The combination of mechanical control with visual information, so-called visual feedback control or visual servoing, should become extremely important, when we consider a mechanical system working under dynamical environments [1], [2].

Classical visual servoing algorithms assume that the manipulator dynamics is negligible and do not interact with the visual feedback loop. However, this assumption is invalid for high speed tasks, while it holds for kinematic control problems [3]. Additionally, as mentioned in [1], camera calibration problems are long standing research issues in the visual feedback systems with a fixed camera as depicted in Fig. 1. Kelly [3] considered the set-point problems for the visual feedback systems with a fixed camera as depicted in Fig. 1. Visual feedback system with a fixed camera.

Camera Image

Robot Manipulator

Target Object

World Frame

Camera

Robot

Fig. 1. Visual feedback system with a fixed camera.

Secondly we construct the visual feedback system with the uncertainty of the camera coordinate frame is limited to the orientation around the optical axis.

In this paper, we consider the dynamic visual feedback control with the uncertainty of the camera coordinate frame. The uncertainty will not be limited to the orientation around the optical axis, although we use a simple camera model. In this work, our previous research [9] is extended to the case of uncertain visual feedback systems. Hence, we can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system by the same strategy in our previous works [6]-[9]. Stability and $L_2$-gain performance analysis for the dynamic visual feedback system will be discussed based on passivity with an energy function. Comparing the simulation results, the validity of the proposed control law can be confirmed.

Throughout this paper, we use the notation $e^{\hat{\xi}\theta_{ab}} \in \mathbb{R}^{3 \times 3}$ to represent the change of the principle axes of a frame $\Sigma_b$ relative to a frame $\Sigma_a$. The notation ‘$\wedge$’ (wedge) is the skew-symmetric operator such that $\hat{\xi}\theta = \xi \times \theta$ for the vector cross-product $\times$ and any vector $\theta \in \mathbb{R}^3$. The notation ‘$\vee$’ (vee) denotes the inverse operator to ‘$\wedge$’; i.e., $so(3) \to \mathbb{R}^3$. $\xi_{ab} \in \mathbb{R}^3$ specifies the direction of rotation and $\theta_{ab} \in \mathbb{R}$ is the angle of rotation. Here $\xi_{ab}$ denotes $\xi_{ab}\theta_{ab}$ for the simplicity of notation. We use the $4 \times 4$ matrix

$$g_{ab} = \begin{bmatrix} e^{\xi\theta_{ab}} & p_{ab} \\ 0 & 1 \end{bmatrix}$$

(1)

as the homogeneous representation of $g_{ab} = (p_{ab}, e^{\xi\theta_{ab}}) \in SE(3)$ which is the description of the configuration of a frame $\Sigma_b$ relative to a frame $\Sigma_a$. The adjoint transformation associated with $g_{ab}$ is denoted by $Ad(g_{ab})$ [10]. Let us define the vector form of the rotation matrix as $e_{R}(e^{\xi\theta_{ab}}) := sk(e^{\xi\theta_{ab}})\vee$ where $sk(e^{\xi\theta_{ab}})$ denotes $\frac{1}{2}(e^{\xi\theta_{ab}} - e^{-\xi\theta_{ab}})$. 

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II. PASSIVITY-BASED VISUAL FEEDBACK SYSTEM

A. Fundamental Representation for Visual Feedback System

Visual feedback systems typically use four coordinate frames which consist of a world frame $\Sigma_w$, a target object frame $\Sigma_o$, a camera frame $\Sigma_c$ and a hand (end-effector) frame $\Sigma_h$ as in Fig. 1. Then, $g_{wh}$, $g_{wc}$ and $g_{wo}$ denote the rigid body motions from $\Sigma_w$ to $\Sigma_h$, from $\Sigma_w$ to $\Sigma_c$ and from $\Sigma_w$ to $\Sigma_o$, respectively. Similarly, the relative rigid body motions from $\Sigma_c$ to $\Sigma_h$, from $\Sigma_c$ to $\Sigma_o$ and from $\Sigma_h$ to $\Sigma_o$ can be represented by $g_{ch}$, $g_{co}$ and $g_{ho}$, respectively, as shown in Fig. 1.

The relative rigid body motion from $\Sigma_c$ to $\Sigma_o$ can be led by using the composition rule for rigid body transformations ([10], Chap. 2, pp. 37, eq. (2.24)) as follows

$$ g_{co} = g_{wc}^{-1} g_{wo}. \tag{2} $$

The fundamental representation of the relative rigid body motion involves the velocity of each rigid body. To this aid, let us consider the velocity of a rigid body as described in [10]. Now, we define the body velocity of the camera relative to the world frame $\Sigma_w$ as

$$ \dot{V}_{wc} = \dot{g}_{wc}^{-1} \dot{g}_{wc} = \begin{bmatrix} \omega_{wc} & v_{wc} \\ 0 & 0 \end{bmatrix} \quad V_{wc} = \begin{bmatrix} v_{wc} \\ \omega_{wc} \end{bmatrix} \tag{3} $$

where $v_{wc}$ and $\omega_{wc}$ represent the velocity of the origin and the angular velocity from $\Sigma_w$ to $\Sigma_c$, respectively (10) Chap. 2, eq. (2.55)).

Then, the fundamental representation of the relative rigid body motion $g_{co}$ is described as follows [8],

$$ V_{co} = -\text{Ad}(g_{co}^{-1}) V_{wc} + V_{wo} \tag{4} $$

where $V_{wo}$ is the body velocity of the target object. The notation $\text{Ad}(g_{co})$ means the adjoint transformation associated with $g_{co}$ [10]. Roughly speaking, if both the camera and the target object move, then the relative rigid body motion $g_{co}$ will be derived from the difference between the camera velocity $V_{wc}$ and the target object velocity $V_{wo}$. In the case of the fixed camera configuration, the fundamental representation of the relative rigid body motion $g_{co}$ can be rewritten as

$$ V_{co} = V_{wo}, \tag{5} $$

because the camera is static, i.e. $V_{wc} = 0$ in the case of the fixed camera configuration.

B. Camera Model

To control the relative rigid body motion using visual information provided by a computer vision system, we derive the model of a pinhole camera with a perspective projection. Let $\lambda$ be a focal length, $p_{oi} \in \mathbb{R}^3$ and $p_{ci} \in \mathbb{R}^3$ be coordinates of the target object’s $i$-th feature point relative to $\Sigma_o$ and $\Sigma_c$, respectively. Using a transformation of the coordinates, we have

$$ p_{ci} = g_{co} p_{oi}, \tag{6} $$

where $p_{ci}$ and $p_{oi}$ should be regarded as $[p_{ci}^T, 1]^T$ and $[p_{oi}^T, 1]^T$ via the well-known representation in robotics, respectively (see, e.g. [10]).

The perspective projection of the $i$-th feature point onto the image plane gives us the image plane coordinate $f_i := [f_{xi} f_{yi} f_{zi}^T] \in \mathbb{R}^2$ as follows

$$ f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \tag{7} $$

where $p_{ci} := [x_{ci} y_{ci} z_{ci}]^T$. It is straightforward to extend this model to the $m$ image points case by simply stacking the vectors of the image plane coordinate, i.e. $f := [f_1^T \ldots f_m^T]^T \in \mathbb{R}^{2m}$. We assume that multiple point features on a known object can be used.

C. Visual Feedback System with Fixed Camera

Here the brief summary of our prior work in [9] is given. In order to bring the actual relative rigid body motion $g_{ho}$ to a given reference $g_{dh}$ in Fig. 1, we consider the control and estimation problems in the visual feedback systems. The following dynamic model which just comes from the fundamental representation of the actual relative rigid body motion (5) is considered.

$$ \dot{V}_{co} = u_e \tag{8} $$

where $\dot{V}_{co} := [\dot{v}_{co}^T \dot{\omega}_{co}^T]^T$ and $\dot{V}_{co} := [\dot{v}_{co}^T \dot{\omega}_{co}^T]^T$ mean the estimated object velocity. Here, $\ddot{g}_{co} = (\ddot{p}_{co}, \ddot{\theta}_{\omega_{co}})$ denotes the estimated relative rigid body motion. Then, the estimation error of the relative rigid body motion from $\Sigma_c$ to $\Sigma_o$, i.e. the error between $g_{co}$ and $\ddot{g}_{co}$, is defined as

$$ e_{ee} := \ddot{g}_{co} - g_{co} \tag{9} $$

which is called the estimated object error. Using the notation $e_{R}(\ddot{\theta}_{\omega})$, the vector of the estimated object error is given by $e_{e} := [p_{ee}^T e_{R}(\ddot{\theta}_{\omega_{ee}})]^T$. The estimated object error system is represented by

$$ V_{ee} = (g_{ee}^{-1}) V_{ee} = -\text{Ad}(g_{ee}^{-1}) u_e + V_{wo} \tag{10} $$

where $u_e$ is the input in order to converge the estimated value to the actual relative rigid body motion.

Similarly, we define the error between $g_{dh}$ and $\ddot{g}_{ho}$, which is called the control error, as follows

$$ e_{gc} := g_{dh}^{-1} g_{ho} \tag{11} $$

where $\ddot{g}_{ho}$ is the estimated relative rigid body motion from $\Sigma_h$ to $\Sigma_o$ and obtained from

$$ \ddot{g}_{ho} = g_{ch}^{-1} \ddot{g}_{co}. \tag{12} $$

Here, we assume that $g_{ch}$ is calculated by using the known motion, i.e. $g_{wc}$ and $g_{wh}$, exactly. The vector of the control error is defined as $e_{c} := [p_{ec}^T e_{R}(\ddot{\theta}_{\omega_{ec}})]^T$. The control error system is described by

$$ V_{ee} = (g_{ee}^{-1}) V_{ee} = -\text{Ad}(g_{ee}^{-1}) V_{wh} + u_e \tag{13} $$

where $V_{wh}$ is the body velocity of the hand relative to $\Sigma_w$. 

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Let us take the output of the visual feedback system.

A. Uncertainty of Camera Coordinate Frame

In our approach, the uncertainty of the camera coordinate frame can be regarded as one of the camera calibration problems are long standing research issues [1]. It should be noted that the uncertainty of the camera coordinate frame, nevertheless, the estimated relative rigid body motion \( \hat{g}_{ho} \) equals the reference one \( g_d \) and the estimated one \( g_{co} \), respectively. Moreover, the error between \( \hat{g}_{ho} \) and \( g_{ho} \) can be also represented as \( g_{ce} \), while \( g_{ce} \) is defined as the error between \( \hat{g}_{co} \) and \( g_{co} \) in (9). Therefore, the actual relative rigid body motion \( g_{ho} \) tends to the reference one \( g_d \) when \( e_{ce} \rightarrow 0 \).

Now, we show the relation between the input and the output of the visual feedback system.

**Result 1** [9]: If \( V_{wco}^b = 0 \), then the visual feedback system (14) satisfies

\[
\int_0^T u_{ce} \nu_{ce} \, dt \geq -\beta_{ce}, \quad \forall T > 0
\]

where \( \nu_{ce} \) is defined as

\[
\nu_{ce} := \begin{bmatrix} -Ad_{(g_{ho}^{-1})} & 0 \\ \quad & -I \end{bmatrix} e_{ce}
\]

and \( \beta_{ce} \) is a positive scalar.

Let us take \( u_{ce} \) as the input and \( \nu_{ce} \) as its output. Then, Result 1 would suggest that the visual feedback system (14) is passive from the input \( u_{ce} \) to the output \( \nu_{ce} \) just formally as in the definition in [12].

III. VISUAL FEEDBACK SYSTEM WITH UNCERTAIN COORDINATE FRAME

A. Uncertainty of Camera Coordinate Frame

Let the uncertainty of the camera coordinate frame be known a priori, while \( \Sigma_c \) denotes the actual one which is unknown. The uncertainty of the camera coordinate frame is denoted as

\[
g_\delta = g_{wco}^{-1} g_{wc}
\]

where \( g_{wco} \) is the rigid body motion from \( \Sigma_w \) to \( \Sigma_{co} \), \( g_{co} \) and \( g_{ch} \) are the relative rigid body motions from \( \Sigma_{co} \) to \( \Sigma_o \) and \( \Sigma_h \), respectively.

From (12), we notice that \( g_{ho} \) may not be estimated exactly as follow

\[
g_{ch}^{-1} g_{co} = (g_{ch} g_{ch}^{-1}) g_{co} = g_{ch}^{-1} g_{co} = g_{ho}
\]

even if the vector of the estimated object error \( e_c \) equals to zero. Consequently, \( g_{ho} \) will not converge to \( g_d \) owing to the uncertainty of the camera coordinate frame, nevertheless \( e_{ce} \) tends to zero in the visual feedback system (14) with the passivity-based control law proposed in [9].

B. Estimated Hand Error System

Here we will consider the observer for estimating the relative rigid body motion \( g_{ch}^{-1} \) in order to reduce the effect of uncertainty of the camera coordinate frame.

Based on the fundamental representation of the relative rigid body motion described as (4), the relation among \( \Sigma_w \), \( \Sigma_c \) and \( \Sigma_h \) can be expressed as

\[
V_{ch}^b = -Ad_{(g_{ch})} V_{wco}^b + V_{wh}^b = V_{wh}^b, \quad (20)
\]

because the camera is static in the fixed camera configuration. We shall consider the following model from (20)

\[
V_{ch}^b = u_h \quad (21)
\]

where \( u_h \) is the new input for reducing the estimation error between \( \hat{g}_{ch} \) and \( \bar{g}_{ch} \) which represents the estimated relative rigid body motion from \( \Sigma_c \) to \( \Sigma_h \).

Similarly to the camera model and the image information mentioned in II-B, we can discuss the image information concerned with \( \bar{g}_{ch} \) by replacing the target object frame \( \Sigma_o \) with the hand frame \( \Sigma_h \). Hence, we define the image information of the hand and the estimated one as \( f_h \) and \( \bar{f}_h \), respectively. It is assumed that both the image information of the target object and of the hand, i.e. \( f \) and \( f_h \), are available from a single camera.
In order to compose the estimation error system between \( \hat{\theta}_h \) and \( \hat{\theta}_e \), we call the estimated hand error system in this paper, the estimated hand error between \( \hat{\theta}_h \) and \( \hat{\theta}_e \) is defined as
\[
g_{eh} = \hat{g}_{eh}^{-1} \hat{g}_{eh}. \tag{22}
\]

Using the notation \( e^R(\hat{e}^\theta) \), the vector of the estimated hand error is given by \( e_h := [\hat{e}_h^T, e_h^T(\hat{e}^R_{\theta,eh})]^T \). Then, the relationship between \( f_h \) and \( \hat{f}_h \) can be given by
\[
f_h - \hat{f}_h = J(\hat{\theta}_h) e_h, \tag{23}
\]
where \( J(\hat{\theta}_h) \) plays the role of well-known image Jacobian and is defined in [8].

Differentiating (22) and multiplying it by \( g_{eh}^{-1} \), we can obtain
\[
g_{eh}^{-1} g_{eh} = -g_{eh}^{-1} \dot{\hat{g}}_{eh} g_{eh} + g_{eh}^{-1} g_{eh} V_{eh}^b = -g_{eh}^{-1} \dot{u}_h g_{eh} + \dot{V}_{wh}. \tag{24}
\]
Furthermore, using the property concerning the adjoint transformation, the above equation can be transformed into the following
\[
V_{eh}^b = V_{wh}^b - \text{Ad}(g_{eh}^{-1}) u_h. \tag{25}
\]
Eq. (25) represents the estimated hand error system.

**C. Property of Visual Feedback System with Uncertain Coordinate Frame**

If \( g_{eo} \) and \( g_{eh} \) are measured, \( \theta_{ho} \) can be obtained without the effect of uncertainty of the camera coordinate frame. In order to reduce the effect of uncertainty, the estimated relative rigid body motion from \( \Sigma_h \) to \( \Sigma_o \) described in (12) should be replaced by
\[
\hat{\theta}_{ho} = \hat{g}_{eh}^{-1} \hat{g}_{eo}. \tag{26}
\]
Then, the control error system is transformed into
\[
V_{ec}^b = u_{ec} - \text{Ad}(\hat{g}_{eh}^{-1}) u_h. \tag{27}
\]
Moreover, we redefine the estimated object error as
\[
g_{eo} = \hat{g}_{eo}^{-1} g_{eo}. \tag{28}
\]
Where the estimated object error system is same as (10).

Combining (10), (25) and (27), the visual feedback system with a fixed camera is constructed as follows
\[
\begin{bmatrix}
V_{eh}^b \\
V_{ec}^b \\
V_{eh}^b
\end{bmatrix}
= \begin{bmatrix}
0 & I & -\text{Ad}(\hat{g}_{eh}^{-1}) \\
0 & -\text{Ad}(\hat{g}_{eh}^{-1}) & 0 \\
I & 0 & -\text{Ad}(\hat{g}_{eo}^{-1})
\end{bmatrix}
\begin{bmatrix}
u_{ec} \\
u_{eh} \\
u_{wh}
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix}, \tag{29}
\]
where
\[
u_{ec} := \begin{bmatrix} V_{eh}^b \\
u_{ec} \\
u_{eh}
\end{bmatrix} \tag{30}
\]
denotes the input for the visual feedback system.

Let us define the error vector of the visual feedback system (29) as \( e := [\hat{e}_c^T, e_c^T, \hat{e}_h^T]^T \). It should be noted that if the vectors of the control error, the estimated object error and the estimated hand error are equal to zero, then \( \theta_{ho}, \hat{\theta}_e \) and \( \hat{\theta}_h \) equal \( \theta_d, \hat{\theta}_e, \) and \( \hat{\theta}_h \), respectively. Therefore, \( \theta_{ho} \) tends to \( \theta_d \) when \( e \to 0 \), while the visual feedback system has the uncertainty of the camera coordinate frame.

**Lemma 1:** If \( V_{io}^b = 0 \), then the visual feedback system (29) satisfies
\[
\int_0^T u_{ec}^T \nu_{ec} \, dt \geq -\beta_{ec}, \quad \forall T > 0 \tag{31}
\]
where \( \nu_{ec} \) is defined as
\[
\nu_{ec} := \begin{bmatrix}
0 & 0 & \text{Ad}(\hat{g}_{eh}^{-1}) \\
\text{Ad}(\hat{g}_{eo}^{-1}) & 0 & 0 \\
0 & -\text{Ad}(\hat{g}_{eo}^{-1}) & 0
\end{bmatrix} e \tag{32}
\]
and \( \beta_{ec} \) is a positive scalar.

**Proof:** Consider the following positive definite function
\[
V_{ec} = E(g_{oe}) + E(\hat{\theta}_{eo}) + E(g_{eh}) \tag{33}
\]
where \( E(g) := \frac{1}{2} \| p \|^2 + \phi(e^R \hat{\theta}) \) and \( \phi(e^R \hat{\theta}) := \frac{1}{2} \text{tr}(I - e^R \hat{\theta}) \) which is the error function of the rotation matrix (see e.g. [11]). Differentiating (33) with respect to time yields
\[
\dot{V}_{ec} = e^T \begin{bmatrix}
\text{Ad}(e^R \hat{\theta}_{eo}) & 0 & 0 \\
0 & \text{Ad}(\hat{\theta}_{eo}) & 0 \\
0 & 0 & \text{Ad}(\hat{\theta}_{eh})
\end{bmatrix}
\begin{bmatrix} V_{ec} \\
V_{ce} \\
V_{eh}
\end{bmatrix} \tag{34}
\]
where we use the property \( \dot{\phi}(e^R \hat{\theta}) = e^R \dot{\theta} \). Observing the skew-symmetry of the matrices \( \dot{\theta}_{ec}, \dot{\theta}_{oe} \) and \( \dot{\theta}_{eh} \), the above equation along the trajectories of the system (29) can be transformed into
\[
\dot{V}_{ec} = e^T \begin{bmatrix}
0 & \text{Ad}(\hat{\theta}_{eo}) & 0 \\
0 & -I & 0 \\
\text{Ad}(\hat{\theta}_{eh}) & 0 & -I
\end{bmatrix}
\begin{bmatrix} u_{ec} \\
u_{eh} \\
u_{wh}
\end{bmatrix} \tag{35}
\]
Integrating (35) from 0 to \( T \), we can obtain
\[
\int_0^T u_{ec}^T \nu_{ec} \, dt \geq -V_{ec}(0) := -\beta_{ec} \tag{36}
\]
where \( \beta_{ec} \) is the positive scalar which only depends on the initial states of \( \theta_{ec}, \hat{\theta}_{eo} \) and \( \hat{\theta}_{eh} \).

**Remark 1:** In the visual feedback system, \( \dot{\theta}_{ec}^T \dot{\omega}_{we} \theta_{ee} = 0, \dot{\theta}_{eo}^T \dot{\omega}_{we} \theta_{ee} = 0, \dot{\theta}_{eh}^T \dot{\omega}_{wh} \theta_{eh} = 0 \) holds. This skew-symmetric property is analogous to the one of the robot dynamics, i.e. \( x^T (M - 2C)x = 0, \forall x \in \mathbb{R}^n \) (where \( M \in \mathbb{R}^{n \times n} \) is the manipulator inertia matrix and \( C \in \mathbb{R}^{n \times n} \) is the Coriolis matrix [10]). Thus, Lemma 1 suggests that the visual feedback system (29) is passive from the input \( u_{ec} \) to the output \( \nu_{ec} \), as in the definition in [12].
IV. DYNAMIC VISUAL FEEDBACK CONTROL

A. Dynamic Visual Feedback System

The manipulator dynamics can be written as

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau + \tau_d \]  

(37)

where \( q, \dot{q} \) and \( \ddot{q} \) are the joint angles, velocities and accelerations, respectively. \( \tau \) is the vector of the input torques and \( \tau_d \) represents a disturbance input.

The body velocity of the hand \( V_{wh}^b \) is given by

\[ V_{wh}^b = J_b(q) \dot{q} \]  

(38)

where \( J_b(q) \) is the manipulator body Jacobian \([10]\). We define the reference of the joint velocities as \( \dot{q}_d := J_b^T(q) u_d \) where \( u_d \) represents the desired body velocity of the hand. Thus, \( V_{wh}^b \) in (30) should be replaced by \( u_d \).

Let us define the error vector with respect to the joint velocities of the manipulator dynamics as \( \xi := \dot{q} - \dot{q}_d \). Now, we consider the passivity-based dynamic visual feedback control law as follows.

\[
\tau = M(q) \ddot{q}_d + C(q, \dot{q}) \dot{q}_d + g(q) + u_\xi + J_b^T(q) \left( A_d J_b(q) \ddot{q}_e + A_d \dot{q}_e \right) \].

(39)

The new input \( u_\xi \) is to be determined in order to achieve the control objectives.

Using (29), (37) and (39), the visual feedback system with manipulator dynamics (we call the dynamic visual feedback system) can be derived as follows:

\[
\begin{bmatrix}
\dot{\xi} \\
\dot{V}_{wh}^b \\
\dot{V}_{ec}^b \\
\dot{V}_{eh}^b
\end{bmatrix} =
\begin{bmatrix}
-M^{-1} C \xi + M^{-1} J_b^T A_d J_b(q) \dot{q}_e + A_d (q_e \dot{q}_e) \dot{q}_h \\
0 & -A_d (q_e \dot{q}_e) & 0 & 0 \\
J_b(q) & 0 & 0 & -A_d (q_e \dot{q}_e) \\
0 & I & -A_d & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\xi} \\
\dot{V}_{wh}^b \\
\dot{V}_{ec}^b \\
\dot{V}_{eh}^b
\end{bmatrix} +
\begin{bmatrix}
M^{-1} 0 0 0 \\
0 0 I 0 \\
0 0 -A_d & 0 \\
I 0 -A_d & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\dot{u}_d \\
\dot{u}_e \\
\dot{u}_h
\end{bmatrix} \]  

(40)

where \( x := \begin{bmatrix} \xi^T & e^T \end{bmatrix}^T \) and \( u := \begin{bmatrix} u_\xi^T & u_d^T & u_e^T & u_h^T \end{bmatrix}^T \). We define the disturbance of dynamic visual feedback system as \( w := \begin{bmatrix} 0 & \begin{bmatrix} \tau_d^T & (V_{wh})^T \end{bmatrix} \end{bmatrix}^T \). Before constructing the dynamic visual feedback control law, we derive an important lemma.

**Lemma 2:** If \( w = 0 \), then the dynamic visual feedback system (40) satisfies

\[ \int_0^T \dot{\nu} d\tau \geq -\beta, \ \forall T > 0 \]  

(41)

where

\[
\nu := Nx, \ N := \begin{bmatrix}
I & 0 & 0 & 0 \\
0 & 0 & 0 & \text{Ad} (e^{-q_e \dot{q}_h}) \\
0 & \text{Ad} (e^{-q_e \dot{q}_h}) & -I & 0 \\
0 & -\text{Ad} (q_e \dot{q}_e) & 0 & -I
\end{bmatrix}.
\]

Due to space limitations, the proof is only sketched. By using the following positive definite function, the proof can be completed by invoking Lemma 1

\[ V(x) = \frac{1}{2} \xi^T M \xi + E(g_ec) + E(g_ee) + E(g_eh). \]  

(42)

**Remark 2:** Similarly to Lemma 1, Lemma 2 would suggest that the dynamic visual feedback system is passive from the input \( u \) to the output \( \nu \) just formally. From Lemma 2, we can state that the dynamic visual feedback system (40) preserves the passivity of the visual feedback system (29). This is one of main contributions of this work.

B. Stability Analysis for Dynamic Visual Feedback System

It is well known that there is a direct link between passivity and Lyapunov stability. Thus, we propose the following control input.

\[ u = -K \nu = -K Nx, \ K := \begin{bmatrix} K_e & 0 & 0 & 0 \\
0 & K_e & 0 & 0 \\
0 & 0 & K_h & 0 \\
0 & 0 & 0 & K_h \end{bmatrix} \]  

(43)

where \( K_e := \text{diag}\{k_{e1}, \ldots, k_{en}\} \) denotes the positive gain matrix for each joint axis. \( K_e := \text{diag}\{k_{e1}, \ldots, k_{e6}\} \) and \( K_h := \text{diag}\{k_{h1}, \ldots, k_{h6}\} \) are the positive gain matrices of \( x, y \) and \( z \) axes of the translation and the rotation for the control error, the estimated object one and the estimated hand one, respectively. The result with respect to asymptotic stability of the proposed control input (43) can be established as follows.

**Theorem 1:** If \( w = 0 \), then the equilibrium point \( x = 0 \) for the closed-loop system (40) and (43) is asymptotic stable.

It can be proved by the energy function (42) as a Lyapunov function. We omit the proof due to space limitations. Considering the manipulator dynamics, Theorem 1 shows the stability via Lyapunov method for the full 3D dynamic visual feedback system. It is interesting to note that stability analysis is based on the passivity as described in (41).

C. \( L_2 \)-gain Performance Analysis for Dynamic Visual Feedback System

Based on the dissipative systems theory, we consider \( L_2 \)-gain performance analysis for the dynamic visual feedback system (40) in one of the typical problems, i.e. the disturbance attenuation problem. Now, let us define

\[ P := N^T K N - \frac{1}{2 \gamma^2} W - \frac{1}{2} I \]  

(44)

where \( \gamma \in \mathbb{R} \) is positive and \( W := \text{diag}\{I, 0, I, 0\} \). Then we have the following theorem.

**Theorem 2:** Given a positive scalar \( \gamma \) and consider the control input (43) with the gains \( K_e, K_e, K_e \) and \( K_h \) such that the matrix \( P \) is positive semi-definite, then the closed-loop system (40) and (43) has \( L_2 \)-gain \( \leq \gamma \).

The proof is omitted due to space limitations. Theorem 2 can be proved using the energy function (42) as a
storage function for $L_2$-gain performance analysis. The $L_2$-gain performance analysis of the dynamic visual feedback system is discussed via the dissipative systems theory. In $H_\infty$-type control, we can consider some problems by establishing the adequate generalized plant. This paper has discussed $L_2$-gain performance analysis for the disturbance attenuation problem. The proposed strategy can be extended for the other type of generalized plants of the dynamic visual feedback systems.

V. SIMULATION

The simulation results on the two degree-of-freedom manipulator as depicted in Fig. 3 are shown in order to understand our proposed method simply, though it is valid for 3D visual feedback systems. We use the reference of the relative rigid body motion as a constant value, i.e. $p_d = [0 \ 0 \ -0.81]^T$ and $e^{g_d} = I$. Specifically, we consider the set point problem, i.e. the target object is static, in order to compare the performance of the proposed control law and the previous one discussed in [9] clearly. The uncertainty of the camera coordinate frame is chosen as $g_d = (0.05 \ -0.02 \ 0.04)^T e^{[-\pi/24 \ \pi/25 \ \pi/18]^T}$. Fig. 5 shows the error between $g_{ho}$ and $g_d$ which is defined as $e_{er} := g_d^{-1} g_{ho}$. The errors for the control objective with the proposed control law tend to zero in both cases, while the previous control law with the uncertainty cannot achieve the control objective. Hence, the proposed control law is valid for the uncertainty of the camera coordinate frame. In the case of the tracking problem with the moving target object, it can be confirmed by the same simulation in [9].

VI. CONCLUSIONS

This paper dealt with the dynamic visual feedback control with the uncertainty of the camera coordinate frame, which is not limited to the orientation around the optical axis. We can derive that the dynamic visual feedback system preserves the passivity of the visual feedback system by the same strategy as in our previous works [6]-[9]. Stability and $L_2$-gain performance analysis for the dynamic visual feedback system are discussed based on passivity with the energy function. The validity of the proposed control law is confirmed by comparing the simulation results.

REFERENCES