Special decentralized control problems and effectiveness of parameter-dependent Lyapunov function method

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Abstract—This paper is devoted to studying decentralized control problems from a special viewpoint and testing the effectiveness of parameter-dependent Lyapunov function method. First it is pointed out that in order to stabilize some given interconnected systems, some subsystems should be assigned to be unstable in some special cases. Then a special kind of decentralized control problem is studied. This kind of problem can be viewed as harmonic control among independent subsystems. Research results show that two unstable systems can generate a stable system through some effective cooperations. Linear matrix inequality (LMI)-based decentralized controller design method is also given for the special problems studied here by using parameter-dependent Lyapunov function method developed for robust stability.

I. INTRODUCTION

The domain of robust analysis and robust control synthesis has been thoroughly investigated in the last two decades. Since [14], Oliveira, Bernussou and Geromel have opened a new horizon for LMI-based robust stability study of systems with parametric uncertainty. The result in [14] has been generalized dramatically in recent years, see [1], [13], [2], [15], [16], [17], [19], [23], [25] and references therein. By introducing a new instrumental matrix variable, LMI corresponding to stability of linear continuous-time or discrete-time systems is relaxed to a new LMI in which Lyapunov matrix is independent of state matrix of systems. With this new extra degree of freedom, a parameter-dependent Lyapunov function can be built for robust stability and performance analysis and control. Generally this new method is less conservative than the traditional Lyapunov function method. And it is also pointed out in [15] that this method has less conservativeness than diagonal blocked Lyapunov function method on decentralized control. This paper is devoted to testing the effectiveness of parameter-dependent Lyapunov function method on decentralized control from a special viewpoint. Examples show that this method is also suitable for the unstable subsystem cases in interconnected systems.

Decentralized control of large scale systems has been studied extensively in the past four decades [3], [10], [11], [16], [20], [21], [22], [24]. The main difficulty of solving the decentralized control problem comes from the fact that the feedback gain is subject to structural constraints. Such constraints are of the same nature as the static output ones, which can be viewed as a full state feedback with structural constraints that select only the measured states. At the beginning study of large scale system theory, some people thought that a large scale system is decentrally stabilizable under controllability condition by strengthening the stability degree of subsystems. [24] showed that this idea is wrong by an example. And because of the existence of decentralized fixed modes, some large scale systems can not be decentrally stabilized at all. Generally, it is very conservative that closed-loop subsystems are all required to be stable. Under the stability of subsystems, the actions of interconnections are always ignored and even viewed as disadvantages. This kind of study is disadvantageous for the study of the actions of interconnections. Recently in [22], LMI method for decentralized control of nonlinear systems was presented and no stability assumptions were made for subsystems. Along the development of society, interconnections play more and more important roles in social systems, economic systems, power systems, etc. But the study of the effects of interconnections in large scale systems is still very little to the authors’ knowledge. Recently, some applications of small gain theorem was given in [5] to strengthen robust stability of interconnected systems. In fact, small gain theorem in decentralized control was first introduced and used in 1982, see Section 5.1 of [21]. And in [21], an example (Example 2.18) was given to show that in order to stabilize the whole system, some subsystems must be unstable. The effects of nonlinear input and output coupling was studied in [6], [7], [8]. We study decentralized control problems from a special viewpoint in this paper.

LMI methods have played leading roles during the last twenty years in linear systems theory [2], [9], [12]. We establish LMI-based decentralized controller design method for the special problems studied here by using parameter-dependent Lyapunov function method. This paper mainly focuses on interconnected systems composed of two subsystems. The results can be generalized to multiple subsystem cases. The rest of this paper is organized as follows. In section 2, by studying the structure of interconnections we point out that it is impossible to stabilize all subsystems and the whole system simultaneously by using decentralized controllers in some special cases, that is, in order to stabilize the whole system, some subsystems should be assigned to be unstable. This result shows that the stability of interconnected systems is not only dependent on the stability degree of subsystems in some cases, but is closely dependent...
on the interconnections. In addition, for sake of studying the effects of interconnections, we study a special kind of decentralized control problem which can be viewed as harmonic stability problem among independent subsystems. The results show that two unstable subsystems can generate a stable interconnected system. In section 3, we present LMI-based decentralized control design method. Some examples are given to illustrate the results in section 4. Examples show that parameter-dependent Lyapunov method is suitable for the cases of unstable subsystems. The last section concludes the paper.

Throughout this paper, det(.) denotes the determinant of the corresponding matrix. Stability of matrix and polynomial means Hurwitz stability. The superscript T means transpose for real matrices.

II. SPECIAL DECENTRALIZED CONTROL PROBLEMS

In this paper, we mainly consider the following interconnected system composed of two subsystems,

\[
\begin{cases}
\dot{x}_1 = A_1 x_1 + A_{12} x_2 + B_1 u_1 \\
\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u_2
\end{cases}
\]

\(u_1 = K_1 x_1, u_2 = K_2 x_2, A_{12}, A_{21}\) are matrices with compatible dimensions. We say system (1) is decentrally stabilizable, i.e., there exist \(K_1, K_2\) such that the state matrix of the closed-loop system

\[A_{cl} = \begin{bmatrix}
A_1 + B_1 K_1 & A_{12} \\
A_{21} & A_2 + B_2 K_2
\end{bmatrix}\]

is Hurwitz stable.

First, we consider a simple example. In system (1) if

\[A_1 = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}, A_{12} = \begin{bmatrix}
0 & \alpha \\
0 & 0
\end{bmatrix}, B_1 = \begin{bmatrix}
0 \\
1
\end{bmatrix},
\]

\[A_{21} = \begin{bmatrix}
0 & \beta \\
0 & 0
\end{bmatrix}, A_2 = \begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}, B_2 = \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

\(K_1 = -(k_1, k_2), K_2 = -(k_3, k_4), \) then

\[A_{cl} = \begin{bmatrix}
0 & 1 & 0 & \alpha \\
-\beta & -k_2 & 0 & 0 \\
0 & 0 & -k_3 & -k_4
\end{bmatrix} \]

Obviously at this time,

\[\text{det}(sI - A_{cl}) = (s^2 + k_2 s + k_1)(s^2 + k_3 s + k_4) - \alpha \beta k_1 k_3.\]

For this simple case, one can get the following results easily:

(i) When \(\alpha \beta = 0\), 1 is a fixed mode.

(ii) When \(\alpha \beta > 0\), for any \(k_1 > 0, i = 1, 2, 3, 4\), i.e., \(A_1 + B_1 K_1, A_2 + B_2 K_2\) are stable, \(A_{cl}\) can not be Hurwitz stable (the constant term of its characteristic polynomial is less than 0 at this time).

(iii) When \(\alpha \beta < 1\), it is possible that the interconnected system and two subsystems can be stabilized simultaneously.

In [21], an example(Example 2.18) was also given to show that in order to stabilize the whole system, some subsystems must be unstable. From the example given above, we can give a simple structural property for such systems.

**Theorem 1** If the interconnected system in (1) satisfies that

1) there exists \(A'_{12}\) such that \(A_{12} = A'_{12} A_2\) and \(A'_{12} B_2 = 0,\)

2) there exists \(A'_{21}\) such that \(A_{21} = A'_{21} A_1\) and \(A'_{21} B_1 = 0,\)

then there are not \(K_1, K_2\) such that \(A_1 + B_1 K_1, A_2 + B_2 K_2\) and \(A_{cl}\) are Hurwitz stable simultaneously when \(\text{det}(I - A'_{12} A'_{21}) < 0.\)

**Proof** Computing the determinant of \(A_{cl}\), one can get

\[\text{det}(A_{cl}) = \text{det}(A_1 + B_1 K_1) \text{det}(A_2 + B_2 K_2) - A_{21} A_1 (A_1 + B_1 K_1)^{-1} A_{12} A_2.\]

Noticing conditions 1), 2),

\[\text{det}(A_{cl}) = \text{det}(A_1 + B_1 K_1) \text{det}(A_2 + B_2 K_2 - A_{21} (A_1 + B_1 K_1) (A_1 + B_1 K_1)^{-1} A_{12} (A_2 + B_2 K_2)),\]

that is,

\[\text{det}(A_{cl}) = \text{det}(A_1 + B_1 K_1) \text{det}(A_2 + B_2 K_2) \text{det}(I - A_{21} A_{12}').\]

When \(A_1 + B_1 K_1, A_2 + B_2 K_2\) are Hurwitz stable and \(\text{det}(I - A_{21} A_{12}') < 0\), one gets that the constant term of the characteristic polynomial of \(A_{cl}\) is less than zero. Therefore, \(A_{cl}\) is unstable.

**Remark 1** Obviously, under the conditions in Theorem 1, when \(\text{det}(I - A_{21} A_{12}') = 0\), 0 is a fixed mode; when \(\text{det}(I - A_{21} A_{12}') > 0\), it is possible that there exist \(K_1, K_2\) such that \(A_1 + B_1 K_1, A_2 + B_2 K_2, A_{cl}\) are Hurwitz stable simultaneously. And if \(A_{cl}\) is stable, there must be one of \(A_1 + B_1 K_1, A_2 + B_2 K_2\) is unstable when \(\text{det}(I - A_{21} A_{12}') < 0\). For the study of the effects of interconnections in large scale systems, it is important to design decentralized controllers when some subsystems must be unstable. At this time, the interconnections play real roles for the stability of large scale systems.

**Corollary 1** For any interconnected matrix \(A = \begin{bmatrix}
A_1 & A_{12} \\
A_{21} & A_2
\end{bmatrix}\), if \(A_{12}\) and \(A_{21}\) can be written as \(A_{12} = A_{12}' A_2, A_{21} = A_{21}' A_1\), and \(\text{det}(I - A_{21} A_{12}') < 0\), then \(A_1, A_2\) and \(A\) can not be stable simultaneously.

Obviously, the results above can be generalized to cases of multiple subsystems. For example, for an interconnected system composed of three subsystems, its closed-loop system matrix is given by

\[A_{cl} = \begin{bmatrix}
A_1 + B_1 K_1 & A_{12} & A_{13} \\
A_{21} & A_2 + B_2 K_2 & A_{23} \\
A_{31} & A_{32} & A_3 + B_3 K_3
\end{bmatrix}.\]

Let \(\mathbf{A}_{1,13} = \begin{bmatrix}
A_1 \\
A_{21} \\
A_{31}
\end{bmatrix} \), \(\mathbf{A}_{1,2} = \text{diag}(B_1, B_2)\), \(\mathbf{A}_{1,3} = \begin{bmatrix}
A_{13} \\
A_{23} \\
A_{33}
\end{bmatrix} \), \(\mathbf{A}_{31} = \begin{bmatrix}
A_{31} \\
A_{32}
\end{bmatrix} \). If the following conditions are satisfied,

1) there exists \(A'_{13}\) such that \(\mathbf{A}_{13} = A'_{13} \mathbf{A}_{31}\) and \(A'_{13} B_3 = 0,\)

2) there exists \(A'_{31}\) such that \(\mathbf{A}_{31} = A'_{31} \mathbf{A}_{13}\) and \(A'_{31} \mathbf{B}_1 = 0.\)

then there are not \(K_1, K_2, K_3\) such that \(\mathbf{A}_{1} + \)}
$\mathcal{B}_i \text{diag}(K_1, K_2)$, $A_3 + B_3 K_3$ and $A_{cl}$ are stable simultaneously when $\text{det}(I - A'_{31} A'_{31}) < 0$.

The above results show that in some cases the stability of interconnected systems is closely dependent on the interconnections. In order to study the actions of the interconnections between subsystems further, we study a special kind of decentralized control problem which can be viewed as harmonic stability problem of subsystems. In this paper we study the following simple cases.

Consider the following interconnected system

$$
\begin{align*}
\dot{x}_1 &= A_1 x_1 + b_{12} u_{12}, \\
\dot{x}_2 &= A_2 x_2 + b_{21} u_{21},
\end{align*}
$$

where $u_{12} = k_{12} x_2$, $u_{21} = k_{21} x_1$, $b_{12}, b_{21}$ are given real vectors. $k_{12}, k_{21}$ are real row vectors to be determined. There is information interchange between two subsystems. It means that two systems are cooperating. For this special decentralized control problem, one can get some simple result for its stabilizability with the following lemmas.

**Lemma 1** Given a real monic polynomial $f(\lambda)$ with degree $n$, $f(\lambda)$ has no real root if and only if $f(x) > 0$, $\forall x \in \mathbb{R}$.

One can prove this lemma easily by writing $f(\lambda)$ as

$$
f(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n)
$$

where $\lambda_1, \lambda_2, \cdots, \lambda_n$ are roots of $f(\lambda)$.

**Lemma 2** Given a real monic polynomial

$$
f(\lambda) = \lambda^n + a_{n-1} \lambda^{n-1} + a_{n-2} \lambda^{n-2} + \cdots + a_1 \lambda + a_0
$$

with $a_{n-1} > 0$, there exists a real stable polynomial

$$
g(\lambda) = \lambda^n + a_{n-1} \lambda^{n-1} + b_{n-2} \lambda^{n-2} + \cdots + b_1 \lambda + b_0
$$

such that $g(\lambda) - f(\lambda)$ has at least one real root.

**Proof** When the degree of $f(\lambda)$ $n$ is odd, it holds obviously. When $n$ is even, we state that there are a sable polynomial $g(\lambda)$ given as above and two points $x_1, x_2 \in (-a_{n-1}, 0)$ such that

$$
g(x_1) - f(x_1) < 0, \quad g(x_2) - f(x_2) > 0.
$$

In fact, obviously there are a sable polynomial $g(\lambda)$ given as above and two points $x_1, x_2 \in (-a_{n-1}, 0)$ such that $g(x_1) < 0$ and $g(x_2) > 0$. At this time, $f(x_1)$ and $f(x_2)$ are two fixed numbers. Then $|g(x_i)|, i = 1, 2$ can be enlarged arbitrarily by enlarging the imaginary parts of the roots of $g(\lambda)$, so (3) can be guaranteed easily. By Lemma 1, we know that $g(\lambda) - f(\lambda)$ has at least one real root with such $g(\lambda)$.

**Theorem 2** If $(A_1, b_{12}), (A_2, b_{21})$ are controllable, and $a = \text{tr}(A_1) + \text{tr}(A_2) < 0$, where $\text{tr}(\cdot)$ denotes the trace of the corresponding matrix, then there are $k_{12}$ and $k_{21}$ such that $A_{cl} = \begin{pmatrix} A_1 & b_{12} k_{12} \\ b_{21} k_{21} & A_2 \end{pmatrix}$ is Hurwitz stable.

**Proof:** Suppose $(A_1, b_{12}), (A_2, b_{21})$ are with the standard controllable model. Let the orders of $A_1$ and $A_2$ be $n$ and $m$, respectively. Set $H_1(s) = k_{21}(sI - A_1)^{-1} b_{12}$, $H_2(s) = k_{12}(sI - A_2)^{-1} b_{21}$, $k_{12} = (\beta_0, \beta_1, \cdots, \beta_{m-1})$, $k_{21} = (\alpha_0, \alpha_1, \cdots, \alpha_{n-1})$, $d_1(s) = \text{det}(sI - A_1)$, $d_2(s) = \text{det}(sI - A_2)$, $k_1(s) = \alpha_0 + \alpha_1 s + \cdots + \alpha_{n-1} s^{n-1}$, $k_2(s) = \beta_0 + \beta_1 s + \cdots + \beta_{m-1} s^{m-1}$, then $A_{cl}$ is stable if and only if the feedback system shown in Fig. 1 is stable, i.e., the polynomial $d_{cl}(s) = d_1(s) d_2(s) - k_1(s) k_2(s)$ is stable. Obviously, $d_{cl}(s)$ is a monic polynomial and the coefficient of $s^{n+m-1}$ in $d_{cl}(s)$ is $-a = -(\text{tr}(A_1) + \text{tr}(A_2)) > 0$. Let $d_1(s) d_2(s) = s^{n+m} - a s^{n+m-1} + d_0(s)$, then the stability of $d_{cl}(s)$ is completely determined by $d(s) = d_0(s) - k_1(s) k_2(s)$. When at least one of $n, m$ is odd, one can choose $d(s)$ arbitrarily such that $d_{cl}(s)$ is stable, and decompose $d(s) - d_0(s)$ into the product of real polynomials $k_1(s)$ and $k_2(s)$. This means that we find real vectors $k_{12}, k_{21}$ such that $A_{cl}$ is stable. When all $n, m$ are even, the degrees of $k_1(s)$ and $k_2(s)$ are odd, but the degree of $k_1(s) k_2(s)$ is even. At this time, one needs choose $d(s)$ such that $d_{cl}(s)$ is stable and $d(s) - d_0(s)$ has a real root in order to decompose $d(s) - d_0(s)$ into the product of two real polynomials. Obviously, this can be completed using Lemma 2. By Lemma 2, there is a stable polynomial $d_{cl}(x)$ with the sum of all its roots equal to $a$, and $d_1(s) d_2(s) - d_{cl}(s)$ has at least one real root. This implies that $d_1(s) d_2(s) - d_{cl}(s)$ can be decomposed into the product of two real polynomials $k_1(s)$ and $k_2(s)$. This completes the proof.

**Remark 2** From the proof of the theorem, we know that the eigenvalues of $A_{cl}$ can be assigned arbitrarily with the only constraint $a = \lambda_1 + \cdots + \lambda_{n+m}$ when one of $n$ and $m$ is odd. When $n$ and $m$ are even simultaneously, the eigenvalues of $A_{cl}$ can also be assigned properly with the constraints $a = \lambda_1 + \cdots + \lambda_{n+m}$ and $k_{12}, k_{21}$ being real vectors.
Remark 3 System (2) can be viewed as cooperative behavior between two subsystems. Two subsystems can be disabled themselves (they can be unstable), but they can realize a stable system through intercrossed feedback. Subsystem does not use the information itself, but it use the other subsystem’s information. That is, they can help with each other to attain some target. Of course, there may be self-feedback in subsystems themselves. We can imagine that under cooperations subsystems need not to be controllable or stabilizable themselves. See the following system.

If there is self-feedback in subsystems, system (2) can be stated as follows.

$$\dot{x}_1 = A_1 x_1 + b_1 u_1 + b_{12} u_{12},$$
$$\dot{x}_2 = A_2 x_2 + b_2 u_2 + b_{21} u_{21},$$
(4)

where \(u_{12}, u_{21}, b_{12}, b_{21}\) are given as in system (2), \(b_1, b_2\) are real vectors with compatible dimensions, \(u_1 = k_1 x_1, u_2 = k_2 x_2\). By using Theorem 2, one can get the following result easily.

Theorem 3 If \((A_1, [b_1 b_{12}]), (A_2, [b_2, b_{21}])\) are controllable and \(b_1, b_2\) are not zero vectors simultaneously, then there are real vectors \(k_1, k_{12}, k_2, k_{21}\) such that system (4) is stable, i.e., \(A_{cl} = \begin{pmatrix} A_1 + b_1 k_1 & b_{12} k_{12} \\ b_{21} k_{21} & A_2 + b_2 k_2 \end{pmatrix}\) is Hurwitz stable.

Remark 4 One can see clearly in Theorem 3, \((A_1, b_1)\) and \((A_2, b_2)\) need not to be controllable or stabilizable. Theorem 3 shows that two subsystems with effective control can cooperate easily for sake of stability. The actions of interconnections are shown here to some degree. And obviously, the framework in Theorems 1, 2, 3 can be generalized to multi-subsystem cases. In addition, refer to [4] for the case of \(b_1, b_2, b_{12}, b_{21}\) being matrices in which it is a little tedious for constructing intercrossed feedback.

III. PARAMETER-DEPENDENT LYAPUNOV METHOD

Although we analyzed some special decentralized control problems in the section above, it is still hard to design decentralized controllers. Fortunately, [14] presented parameter-dependent Lyapunov method. In what follows, by using this method we present LMI-based design method for the problems discussed above. First we introduce the following lemma to begin this section.

Lemma 3 Given a real matrix \(A \in \mathbb{R}^{n \times n}\), \(A\) is Hurwitz stable if, and only if, there exist a matrix \(P = P^T > 0\) and any matrix \(V\) such that

$$\begin{pmatrix} -V - V^T & V^T A^T + P \\
AV + P & -P \end{pmatrix} < 0.$$  
(5)

One can turn (5) into Lyapunov inequality \(PA^T + AP < 0\) easily by using the well known projection lemma in LMI method. By introducing a new variable \(V\), the products of \(PA\) and \(A^T P\) are relaxed to new products \(AV\) and \(V^T A^T\). \(V\) needs not be symmetric and positive definite. In this way Lyapunov matrix \(P\) can be parameter-dependent for the study of robust stability and robust performances[14], [15], [23], [25]. The case of diagonal blocked matrix \(V\) for decentralized control of discrete-time systems was considered in [15]. Here we discuss upper trigonal constraint of \(V\) for system (1) as follows, lower trigonal constraint can be considered similarly. Sometimes, upper trigonal constraint is less conservative than diagonal constraint. Corresponding to system (1), we suppose

$$V = \begin{pmatrix} V_1 & V_{12} \\ V_{12} & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & X_{12} \\ X_{12} & X_2 \end{pmatrix},$$
(6)

Remark 5 For simplicity, we assumed that \(V\) and \(X\) acquire the upper triangular structure as in (6). In fact, it is only fit for the case of \(\text{order}(A_1) = \text{order}(A_2)\). If \(\text{order}(A_1) \neq \text{order}(A_2)\), for example \(\text{order}(A_1) < \text{order}(A_2)\), we can add zero blocks in \(V\) and \(X\) as follows to meet such cases,

$$V = \begin{pmatrix} V_1 & V_{12} \\ V_{12} & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & X_{12} \\ X_{12} & X_2 \end{pmatrix},$$

where \(V_{12} = (V_1 0), X_{12} = (X_1 0)\). When \(\text{order}(A_1) > \text{order}(A_2)\), we assume \(V\) and \(X\) acquire the following lower triangular structure

$$V = \begin{pmatrix} V_1 & 0 \\ V_{21} & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & 0 \\ X_{21} & X_2 \end{pmatrix},$$

where \(V_{21} = (V_2 0), X_{21} = (X_2 0)\), then we can get the similar result as in Theorem 4.

Let

$$A = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix}, \quad B = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}, \quad K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$$
(7)

in system (1). By Lemma 3, one can get the following result for stability of system (1).

Theorem 4 If there are \(P = P^T\) and \(V, X\) with form (6) such that

$$\begin{pmatrix} -V - V^T & V^T A^T + X^T B^T + P & V^T \\ AV + B X + P & -P & 0 \\ V & 0 & -P \end{pmatrix} < 0,$$

then there exist diagonal blocked matrix \(K\) as in (7) such that \(A_{cl} = A + BK\) is stable. At this time, decentralized controllers can be obtained as \(K_1 = X_1 V_1^{-1}, K_2 = X_2 V_2^{-1}\).

Remark 6 From Theorem 4, one can see that \(P\) is not blocked, and \(V_1, V_2\) are generally not symmetric, of course not positive definite. Intuitively, one can imagine that at this time, \(A_1 + B_1 K_1\) or \(A_2 + B_2 K_2\) can be unstable under stability of \(A_{cl}\). One can also see this from the forthcoming examples. In addition, one can establish some similar results for systems (2) and (4).

IV. EXAMPLES

Example 1 In the simple example studied in section 2, let \(\alpha = \beta = 2\). By Theorem 4, one can get a \(K = \ldots\)
(−2.62 −5.69 0 0 0
0 0 1.13 −11.48). Obviously \( A_2 + B_2 K_2 \)
is not stable, but \( A_{cl} \) is stable.

**Example 2** Consider system (2) defined by matrices

\[
A_1 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
−3 & −3 & −8 & −5
\end{pmatrix},
b_{12} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix},
\]

\[
A_2 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
−3 & −2 & −5 & 0
\end{pmatrix},
b_{21} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix}.
\]

Obviously, the conditions in Theorem 2 are satisfied. \( A_{cl} \) is not stable here. Using the method in Theorem 4, one can get decentralized controllers

\[
k_{21} = (−1.7976 −3.0015 −5.4645 −4.6861),
\]

\[
k_{12} = (0.1507 1.0410 0.8293 1.3124)
\]

such that \( A_{cl} \) is stable in (2).

**Example 3** Consider system (4) defined by matrices

\[
A_1 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 2 & 0 \\
0 & 2 & 0 & −1
\end{pmatrix},
b_{12} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix},
\]

\[
A_2 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1
\end{pmatrix},
b_{21} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix},
\]

and \( b_1 = b_2 = (0 0 1 0)^T \). Obviously, the conditions in Theorem 3 are satisfied. Using the method in Theorem 4, one can get decentralized controllers

\[
k_1 = (−2.7658 −6.3430 −6.2113 −1.0951),
\]

\[
k_2 = (−3.4075 −4.8896 −4.6361 −1.5468),
\]

\[
k_{21} = (−0.5882 0.7342 −0.2848 −1.1162),
\]

\[
k_{12} = (1.4464 0.6239 0.0819 1.9946)
\]

such that \( A_{cl} \) is stable in (4).

From the examples and discussions above one can see that subsystems need not to be all stable, even in some special cases some subsystems must be unstable. This shows the special effects of interconnections.

\[\text{V. CONCLUSIONS}\]

This paper is devoted to studying decentralized control of some special interconnected systems. Some simple interconnected structures are established in which subsystems need not be stable. The results here can be generalized to cases of multiple subsystems. An LMI-based decentralized controller design method is also given by using parameter-dependent Lyapunov function method. Although this method is suitable for the problems here in some cases. It is still very conservative. It is an interesting topic to develop more effective decentralized controller design method for the problems here. We hope that these results can be helpful for understanding the actions of interconnections in large scale systems. Modern economic development with high-speed is always along with the bankruptcy of small enterprises. This means that the benefits of small enterprises are sacrificed for the privilege of high-speed development. For large scale systems, this paper indicates that sometimes the stability of subsystems should also be sacrificed for the stability of the whole system.

\[\text{REFERENCES}\]


