Fuzzy Control System Designs using Redundancy of Descriptor Representation: A Fuzzy Lyapunov Function Approach

Kazuo Tanaka, Takashi Nebuya, Hiroshi Ohtake and Hua O. Wang

Abstract—This paper presents fuzzy control system designs using redundancy of descriptor representation. A wider class of Takagi-Sugeno fuzzy controllers using the redundancy is employed to derive stabilization conditions for both common Lyapunov functions and fuzzy Lyapunov functions. We show that the fuzzy Lyapunov function approach is less conservative than the common Lyapunov function approach. A design example also illustrates the utility of the fuzzy Lyapunov function approach using redundancy of descriptor representation.

I. INTRODUCTION

Nonlinear control based on the Takagi-Sugeno fuzzy model [1] has received a lot of attention over the last decade (e.g., see [2]-[11]). An advantage of the fuzzy model-based control [12] is to provide a natural, simple and effective design approach although other nonlinear control techniques [13] require special and rather involved knowledge. In addition, it is known that any smooth nonlinear control systems can be approximated by the Takagi-Sugeno fuzzy models (with liner rule consequence) [14].

Recently, piecewise Lyapunov function approaches have received increasing attention as they attempt to relax the conservativeness of stability and stabilization problems. However, stabilization conditions for fuzzy Lyapunov functions [15] and piecewise Lyapunov functions [16] become BMIs in general. In [15], the well-known completing square technique was introduced to convert the BMIs into LMIs. The conversion causes conservative results in general. Hence, the converted LMIs do not completely contain the LMIs for the common quadratic Lyapunov function although the fuzzy Lyapunov function contains the common quadratic Lyapunov function as a special case. In this paper, we derive LMI design conditions (that contains the LMIs for the common quadratic Lyapunov function as a special case) using redundancy of descriptor representation. The redundancy also provides us the possibility of designing a fuzzy controller for systems with input nonlinearities. A fuzzy descriptor system design has been already discussed in [17]. The design in [17] did not fully take an advantages of redundancy of descriptor representation.

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II. FUZZY MODEL AND STABILITY CONDITIONS

Consider the following nonlinear systems:

\[
\dot{x}(t) = f(x(t), u(t)),
\]

where \(x(t) = [x_1(t) \ x_2(t) \ \cdots \ x_n(t)]^T\) is the state vector, \(u(t) = [u_1(t) \ u_2(t) \ \cdots \ u_m(t)]^T\) is the input vector. Based on the sector nonlinearity concept [12], we can exactly represent (1) with the Takagi-Sugeno fuzzy model (2) (globally or at least semi-globally).

Model Rule i: If \(z_1(t) = M_{i1} \) and \(\cdots\) and \(z_p(t) = M_{ip}\)

then \(\dot{x}(t) = A_i x(t) + B_i u(t) \quad i = 1, 2, \cdots, r,\)

where \(z_j(t) (j = 1, 2, \cdots, p)\) is the premise variable. The membership function associated with the \(i\)th Model Rule and \(j\)th premise variable component is denoted by \(M_{ij}\). \(r\) denotes the number of Model Rules. Each \(z_j(t)\) is a measurable time-varying quantity that may be states, inputs, measurable external variables and/or time. It has been tacitly assumed in fuzzy model-based design that each \(z_j(t)\) does not depend on the inputs \(u(t)\). However, the fuzzy control designs using redundancy of descriptor representation permit that each \(z_j(t)\) depends on the inputs \(u(t)\). This is an advantage of fuzzy control designs using redundancy of descriptor representation.

The defuzzification process of the model (2) can be represented as

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \{ A_i x(t) + B_i u(t) \},
\]

where \(z(t) = [z_1(t) \cdots z_p(t)]\). From the properties of membership functions, the following relations hold.

\[
h_i(z(t)) = \sum_{i=1}^{r} w_i(\sum_{i=1}^{r} w_i(z(t))) \geq 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1,
\]

This paper is organized as follows. Section II recalls the previous results with respect to fuzzy model and stability conditions. Section III introduces a wider class of Takagi-Sugeno fuzzy controllers and derives stabilization conditions based on common quadratic Lyapunov functions. Section IV presents stability conditions based on fuzzy Lyapunov functions. We show that the fuzzy Lyapunov function approach is less conservative than the common Lyapunov approach. Section V illustrates a design example to demonstrate the utility of the fuzzy Lyapunov function approach using redundancy of descriptor representation.
where \( w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \). The parallel distributed compensation provides the following control rules for the fuzzy model (2):

**Control Rule i:**

If \( z_1(t) \) is \( M_{i1} \) and \( \cdots \) and \( z_p(t) \) is \( M_{ip} \)

then \( u(t) = -F_i x(t) \quad i = 1, 2, \cdots, r \) \hfill (4)

The overall fuzzy controller can be calculated by

\[
u(t) = -\sum_{i=1}^{r} h_i(z(t))F_i x(t).
\]  \hfill (5)

A sufficient condition [12] for ensuring the stability of the feedback system consisting of (3) and (5) is given as follows;

\[
X > 0
\]  \hfill (6)

\[
-X A_i^T - A_i X + M_i^T B_i^T + B_i M_i > 0
\]  \hfill (7)

\[
-X A_i^T - A_i X - X A_j^T - A_j X + M_i^T B_i^T + B_i M_j + M_i^T B_j^T + B_j M_i \geq 0 \quad i < j
\]  \hfill (8)

where \( M_i = F_i X \). We can obtain feedback gains stabilizing (3) by solving the LMIs (6) - (8). However, it should be emphasized that the fuzzy controller (5) can not be applied in general when \( z(t) \) depend on \( u(t) \) since the premise variables \( z(t) \) depend on \( u(t) \), i.e., since the control inputs \( u(t) \) to be calculated are also contained in the right side hand of (5). In this case, even if we have a feasible solution for the LMIs (6)-(8), it is difficult to calculate the control input \( u(t) \) using (5). Section III will give an answer of the problem.

In [15], we defined a fuzzy Lyapunov function and derived stabilization conditions via the fuzzy Lyapunov function, where we required \( h_i(z) \) to be \( C^1 \) functions. It should be noted that the assumption is satisfied for fuzzy models constructed from smooth (at least \( C^1 \)) nonlinear systems by using a sector nonlinearity approach [12]. The sector nonlinearity approach can construct a global or semiglobal fuzzy model that exactly represent the dynamics of a nonlinear system. The candidate fuzzy Lyapunov function for the Takagi-Sugeno fuzzy system (2) is defined as

\[
V(x(t)) = \sum_{i=1}^{r} h_i(z(t)) x^T(t) P_i x(t),
\]  \hfill (9)

where \( P_i \) is a positive definite matrix. This candidate Lyapunov function satisfies (1) \( V \) is \( C^1 \), (2) \( V(0) = 0 \) and \( V(x) > 0 \) for \( x \neq 0 \) and (3) \( \|x\| \rightarrow \infty \Rightarrow V(x) \rightarrow \infty \). The fuzzy Lyapunov function shares the same membership functions with the Takagi-Sugeno fuzzy model of a system. Hence, the fuzzy Lyapunov function reduces to the common Lyapunov function when \( P = P_i \) for all \( i \). Unfortunately, stabilization conditions (for the control system consisting of (3) and (5)) based on the fuzzy Lyapunov function become BMIs. In [15], the well-known completing square technique was introduced to convert the BMIs into LMIs. The conversion causes conservative results in general. Hence, the converted LMIs do not completely contain the LMIs for the common quadratic Lyapunov function although the fuzzy Lyapunov function contains the common quadratic Lyapunov function as a special case. In this paper, we derive LMI design conditions (that contains the LMIs for the common quadratic Lyapunov function as a special case) using redundancy of descriptor representation.

### III. DESIGN BASED ON COMMON LYAPUNOV FUNCTION

#### A. New fuzzy controller using redundancy of descriptor representation

We propose a fuzzy controller (10) using redundancy of descriptor representation.

\[
Eu(t) = \sum_{i=1}^{r} h_i(z(t)) \{ K_i u(t) + F_i x(t) \}
\]  \hfill (10)

The controller (10) reduces to fuzzy dynamic state feedback controller [18] when \( E = I \). It reduces to the state feedback controller (5) when \( E = 0 \) and \( K_i = I \). Thus, the controller (10) is a more general form containing some types of controllers.

From (3) and (10), we have the following descriptor representation.

\[
E^+ \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i^x \dot{x}(t),
\]  \hfill (11)

where

\[
E^+ = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E \end{bmatrix},
A_i^x = \begin{bmatrix} 0 & A_i & B_i \\ I & -I & 0 \\ F_i & 0 & K_i \end{bmatrix},
\]

\[
\dot{x}(t) = [x^T(t) \quad x^T(t) \quad u^T(t)]^T.
\]

#### B. Stabilization conditions

Theorem 1 gives a sufficient stability condition for (11).

**Theorem 1:** If there exists matrix \( X^* \) satisfying (12) and (13), the control system (11) is stable.

\[
X^* E^+ = E^+ X^* \geq 0
\]  \hfill (12)

\[
A_i^x X^* + X^* A_i^x < 0 \quad i = 1, 2, \cdots, r
\]  \hfill (13)

(proof) The proof is omitted due to lack of space.

**Remark 1:** The number of stabilization conditions (6)-(8) to design the fuzzy controller (5) is \( r^2 + r \)/2 + 1. On the other hand, the number of stabilization conditions (12) and (13) to design the fuzzy controller (10) is \( r + 1 \). The reason why the number of conditions reduces is because the cross terms with respect to \( F_i \) and \( B_i \) do not exist, i.e., because
there exists and the conditions in Theorem 1 can be converted into LMIs. Note that the matrix $A^*$ contains the feedback gains $F_i$ and $K_i$. Therefore, the term $A^*X^*$ are not linear, that is, the condition (13) is not an LMI in general. To overcome the difficulty, we define $X^*$ as

$$X^* = \begin{bmatrix} X_{21} + X_{21}^T & A_i^T X - X_{21} + X_{22}^T \\ B_i^T X + M_i + X_{23}^T & \end{bmatrix}.$$

(14)

For the above $X^*$, the condition (13) becomes an LMI with respect to the feedback gains and the variables in $X^*$. Then, the conditions in Theorem 1 can be converted into LMIs.

**Corollary 1:** If we use (14) as a common $X^*$, then the conditions (13) and (12) reduce to

$$\begin{align*}
-X_{22} - X_{22}^T & + N_i^T \\
-X_{23}^T & < 0 \\
X = X^T > 0 & \text{ and } X_{33}^T E = E^T X_{33} > 0, \text{ respectively, where } M_i = X_{33}^T F_i \text{ and } N_i = X_{33}^T K_i. \text{ The symbol } "\ast" \text{ denotes the transposed elements (matrices) for symmetric positions.}
\end{align*}$$

(15)

When $K_i = I$ and $E = 0$, the LMI conditions in Corollary 1 can be simplified as follows.

**Corollary 2:** Assume that we use (14) as a common $X^*$. When $K_i = I$ and $E = 0$, i.e., when (10) reduces to (5), the conditions (13) and (12) reduce to

$$\begin{align*}
-X_{22} - X_{22}^T & + N_i^T \\
-X_{23}^T & < 0
\end{align*}$$

(16)

and $X = X^T > 0$, respectively, where $M_i = X_{33}^T F_i$.

**IV. DESIGN BASED ON FUZZY LYAPUNOV FUNCTION**

We have already proposed a fuzzy Lyapunov function [15] for the ordinary fuzzy system. This paper extends the result to fuzzy descriptor systems. The fuzzy Lyapunov function provides us with relaxed stability results. As in [15], we require $h_i(z)$ to be $C^1$ functions.

**Theorem 2:** Assume that $h_k(z(t)) \leq \phi_k$ for $k = 1, 2, \cdots, r-1$. Then, the control system (11) is stable if there exists $X^*$ satisfying

$$X^* E^* = E^* X^* \geq 0 \quad i = 1, 2, \cdots, r$$

(17)

and

$$E^*(X^*_k - X^*_i) \geq 0 \quad k = 1, 2, \cdots, r - 1$$

(18)

$$\frac{1}{2}(A_i^T X_i^* + X_i^* + A_i^T X_i^* + X_i^* + X_i^* T A_i^*)$$

$$\geq \sum_{k=1}^{r-1} \phi_k E^*(X_i^* - X_f^*) < 0 \quad i \leq j.$$ (19)

(proof) The proof is omitted due to lack of space.

**Remark 2:** The condition (13) implies the condition (19). In other word, the condition (19) reduces to the condition (13) when $X^* = X_i^*$ for all $i$. The condition (18) always holds when $X^* = X_i^*$ for all $i$. Therefore, Theorem 2 is less conservative than Theorem 1.

**Remark 3:** In Theorem 2, we assumed that $\hat{h}_k(z(t)) \leq \phi_k$. However, in real system designs, it is not easy to find $\phi_k$ satisfying the assumption. A way of solving the problem was addressed in [15].

The conditions in Theorem 2 can be converted into the LMIs if we use

$$X_i^* = \begin{bmatrix} X_i & 0 & 0 \\ X_{21i} & X_{22i} & X_{23i} \\ 0 & 0 & X_{33i} \end{bmatrix}$$

(20)

as $X_i^*$.

**Corollary 3:** If we use the matrix (20), then the conditions (19), (17) and (18) reduce to

$$\begin{align*}
1 & \begin{bmatrix} X_{21i} + X_{21i}^T + X_{21j}^T + X_{21j} \\ A_i^T X_j + A_j^T X_i - X_{21i} + X_{22i}^T + X_{22i} - X_{21j} + X_{22j} \\ B_i^T X_j + B_j^T X_i + M_i + M_j + X_{23i}^T + X_{23j}^T \end{bmatrix} \\
& \geq \begin{bmatrix} -X_{22i} - X_{22i}^T - X_{22j} - X_{22j}^T \\ -X_{23i} - X_{23j}^T \end{bmatrix} \begin{bmatrix} N_i + N_i^T \end{bmatrix} \\
& + \sum_{k=1}^{r-1} \phi_k \begin{bmatrix} X_k - X_r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0 \quad i \leq j,
\end{align*}$$

(21)

$$X_i^T E^T X_{33} \geq 0$$

(22)

$$X_i^T E^T X_{33} \geq 0$$

(23)

and

$$X_k - X_r \geq 0 \quad k = 1, 2, \cdots, r - 1,$$ (24)

respectively, where $M_i = X_{33i}^T F_i$ and $N_i = X_{33i}^T K_i$.

**Remark 4:** If we use $X_{33i}$ instead of $X_{33i}$, $M_i$ and $N_i$ should be $M_{ij} = X_{33j}^T F_i$, and $N_{ij} = X_{33j}^T K_i$. For the case, even if the LMIs in Corollary 3 are feasible, $F_i$ and $K_i$ can not be uniquely determined from $M_{ij} = X_{33j}^T F_i$ and $N_{ij} = X_{33j}^T K_i$.

**Remark 5:** In [15], we derived stabilizing conditions for the ordinary Takagi-Sugeno fuzzy models using fuzzy Lyapunov functions. Unfortunately, the conditions were not LMIs. Therefore, in [15], the well-known completing square
technique was introduced to convert into LMIs. However, the conversion causes conservative results. On the one hand, Corollary 3 directly provides LMI conditions since $F_i$ and $B_i$ appear separately in the matrices $A_i^T$. This is an advantage of redundancy of descriptor representation.

When $K_i = I$ and $E = 0$, the LMI conditions can be simplified as follows.

**Corollary 4:** Assume that we use the matrix (20). When $K_i = I$ and $E = 0$, i.e., when (10) reduces to (5), the conditions (19), (17) and (18) reduce to

$$
\begin{align*}
\frac{1}{2} \begin{bmatrix}
X_{21i} + X_{21i}^T + X_{21j}^T \\
A_i^TX_j + A_i^TX_i - X_{21i} + X_{21j}^T - X_{21j} + X_{22j}^T \\
B_i^TX_j + B_i^TX_i + M_i + M_j + X_{23i}^T + X_{24j}^T
\end{bmatrix} &

- X_{22i} - X_{22j}^T - X_{22j}^T &

- X_{23i} - X_{23j}^T \\
2(X_{33}^T + X_{34}^T)
\end{bmatrix}

\end{align*}
$$

$$
\begin{align*}
\phi_k \begin{bmatrix}
X_k - X_r \\
0 \\
0
\end{bmatrix} < 0 & \quad i \leq j,
\end{align*}
$$

\begin{equation}
(25)
\end{equation}

$$
X_i = X_i^T \geq 0 \quad i = 1, 2, \ldots, r,
$$

\begin{equation}
(26)
\end{equation}

$$
X_k - X_r \geq 0 \quad k = 1, 2, \ldots, r - 1,
$$

\begin{equation}
(27)
\end{equation}

respectively, where $M_i = X_{34}^T F_i$.

**V. DESIGN EXAMPLE**

Consider the pendulum system (28) used in [19].

$$
\begin{align*}
x_1(t) &= x_2(t) \\
x_2(t) &= -a \sin x_1(t) - bx_2(t) + cu(t),
\end{align*}
$$

\begin{equation}
(28)
\end{equation}

where $a = \frac{Mg}{l} = \frac{9.8}{l}$, $b = \frac{\mu l}{M + l}$, and $c = \frac{1}{M + l}$. $M$, $I$, $\mu$ and $l$ denote mass, inertia, friction and distance to the center of gravity of the pendulum, respectively. $u(t)$ denotes the motor torque. In this simulation, $l = 1.0$, $g = 9.8$, $I = \frac{Ml^2}{4}$. Under $x_1(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, the system (28) can be converted into the following fuzzy model [12]:

$$
\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t)) \{ A_i x(t) + B_i u(t) \},
$$

\begin{equation}
(29)
\end{equation}

where $z(t) = \frac{\sin(x_1(t))}{x_1(t)}$,

$$
A_1 = \begin{bmatrix}
0 & 1 \\
-\alpha x_1 - \frac{\alpha l}{2} & -b
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
0 & 1 \\
-\alpha x_1 & -b
\end{bmatrix}
$$

$$
B_1 = \begin{bmatrix}
0 \\
c
\end{bmatrix}^T, \quad B_2 = \begin{bmatrix}
0 \\
c
\end{bmatrix}^T.
$$

The membership functions are obtained as

$$
h_1(z(t)) = \frac{1 - z(t)}{1 - \frac{\pi}{2}}, \quad h_2(z(t)) = \frac{z(t) - \frac{\pi}{2}}{1 - \frac{\pi}{2}}.
$$

We compare the fuzzy Lyapunov approach (Corollary 4) with the common Lyapunov approach (Corollary 2). First, we design stable controllers for several combinations of $M$ and $\mu$ using Corollary 2. Figure 1 shows the feasible area for the combinations, where the dotted area denotes the feasible area. The common $X$ for $M = 0.1$ and $\mu = 0.1$ is obtained as

$$
X = \begin{bmatrix}
1.7191 & 0.0092 \\
0.0092 & 0.2937
\end{bmatrix}.
$$

Next, we design stable controllers using Corollary 4. In Corollary 4, we need to select the values of $\phi_1$ and $\phi_2$. Since $r=2$, it is enough to consider only $\phi_1$. $h_1(z(t))$ and $h_2(z(t))$ are obtained as

$$
h_1(z(t)) = \frac{-x_1(t) \cos(x_1(t)) + \sin(x_1(t))}{(1 - \frac{\pi}{2})x_1^2(t)},
$$

$$
h_2(z(t)) = \frac{x_1(t) \cos(x_1(t)) - \sin(x_1(t))}{(1 - \frac{\pi}{2})x_1^2(t)}.
$$

Note that $h_1(z(t))$ is a function of $x_2(t) = x_1(t)$. We consider the range of $x_2(t)$ as $x_2(t) \in [-q, q]$. Then, the maximum value of $h_1(z(t))$ is dependent of $q$. We use the maximum value of $h_1(z(t))$ for $x_2(t) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $x_2(t) \in [-q, q]$ as $\phi_1$. We design stable controllers for several combinations of $d$ in addition to $(M, \mu)$ using Corollary 4. Figures 2-7 show the feasible area for each range selection of $x_2(t)$, where the values of $\phi_1$ for the given $q$ are shown.

For $q = 0.5, 1.0, 2.0, 3.0, 6.0$ and $100$, we obtain $X_1$ and $X_2$ ($M = 0.1$ and $\mu = 0.1$) as

$$
X_1 = \begin{bmatrix}
80.866 & 4.1755 \\
4.1755 & 14.250
\end{bmatrix}, \quad X_2 = \begin{bmatrix}
98.108 & 3.4829 \\
3.4829 & 15.076
\end{bmatrix}
$$

$$
X_1 = \begin{bmatrix}
671.12 & 35.412 \\
35.412 & 113.42
\end{bmatrix}, \quad X_2 = \begin{bmatrix}
762.51 & 31.520 \\
31.520 & 121.25
\end{bmatrix}
$$

$$
X_1 = \begin{bmatrix}
7.4810 & 0.3872 \\
0.3872 & 1.2566
\end{bmatrix}, \quad X_2 = \begin{bmatrix}
8.1189 & 0.3531 \\
0.3531 & 1.3417
\end{bmatrix}
$$

$$
X_1 = \begin{bmatrix}
3794.6 & 202.70 \\
202.70 & 635.40
\end{bmatrix}, \quad X_2 = \begin{bmatrix}
3990.0 & 189.20 \\
189.20 & 660.93
\end{bmatrix}
$$

![Fig. 1. Feasible area for common Lyapunov function.](image)
Fig. 2. Feasible area for fuzzy Lyapunov function \( q = 0.5 \) & \( \phi_1 = 0.5576 \).

Fig. 3. Feasible area for fuzzy Lyapunov function \( q = 1.0 \) & \( \phi_1 = 1.1153 \).

Fig. 4. Feasible area for fuzzy Lyapunov function \( q = 2.0 \) & \( \phi_1 = 2.2306 \).

Fig. 5. Feasible area for fuzzy Lyapunov function \( q = 3.0 \) & \( \phi_1 = 3.3460 \).

Fig. 6. Feasible area for fuzzy Lyapunov function \( q = 6.0 \) & \( \phi_1 = 6.6920 \).

Fig. 7. Feasible area for fuzzy Lyapunov function \( q = 100 \) & \( \phi_1 = 111.53 \).
The LMIs in Corollary 2 for a Lyapunov function. In fact, the LMIs in Corollary 4 respectively. It can be seen from the figures that the feasible areas for the fuzzy Lyapunov functions are wider than that of the common Lyapunov function when φ1 is small. Even when φ1 is quite large, the feasible areas for the fuzzy Lyapunov functions approach that of the common Lyapunov function. In fact, X1 is almost same as X2 when q = 100. If X* = X1 for all i, that is, if we consider the common Lyapunov function case, Theorem 2 reduces to Theorem 1. Thus, the fuzzy Lyapunov approach provides less conservative results. In fact, the LMIs in Corollary 4 for a = 1, b = 0.1, c = 1 and φ1 = 0.17 are feasible. Figure 8 shows the simulation result for x(0) = [1 0]. The designed fuzzy controller stabilizes the system (28). The LMIs in Corollary 2 for a = 1, b = 0.1 and c = 1 are infeasible. Therefore, any feedback gains cannot be obtained by Corollary 2.

X1 = \[
\begin{bmatrix}
199.32 & 10.177 \\
10.177 & 33.254 \\
\end{bmatrix}, \quad X2 = \begin{bmatrix}
204.39 & 9.7224 \\
9.7224 & 34.138 \\
\end{bmatrix}
\]

X1 = \[
\begin{bmatrix}
140.52 & 6.9660 \\
6.9660 & 23.760 \\
\end{bmatrix}, \quad X2 = \begin{bmatrix}
140.75 & 6.9491 \\
6.9491 & 23.792 \\
\end{bmatrix}
\]

VI. CONCLUSION

This paper has presented fuzzy control system designs using redundancy of descriptor representation. A wider class of Takagi-Sugeno fuzzy controllers using the redundancy has been employed to derive stabilization conditions for both common Lyapunov functions and fuzzy Lyapunov functions. We show that the fuzzy Lyapunov function approach is less conservative than the common Lyapunov function approach. A design example has illustrated the utility of the fuzzy Lyapunov function approach using redundancy of descriptor representation.

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