Fuzzy Gain Scheduling Attitude Control for Hydrofoil Catamaran

Junsheng Ren and Yansheng Yang

***Abstract*— A novel attitude control strategy for hydrofoil catamaran throughout its operating range has been proposed to overcome the drawback of the conventional linear quadratic regulator (LQR) strategy, via fuzzy model-based system in this paper. The Tagaki-Sugeno (T-S) fuzzy model equipped with parallel distributed compensation (PDC) scheme is first constructed. By means of Matlab LMI control toolbox, a common positive definite matrix is found to guarantee the global fuzzy system's stability of hydrofoil catamaran. Second, the nonlinear mathematical model for hydrofoil catamaran is established. After Jacobian linearization, the state-space model is proposed. Finally, based on hydrofoil catamaran "HB200B-A1", simulation researches demonstrate the reliability of the established nonlinear mathematical model and the effectiveness of our fuzzy controller.**

I. INTRODUCTION

Past several decades have witnessed a rapid development of fast, even super-fast passenger ships in marine transportation market. Hydrofoil catamaran is a kind of high-speed boat of excellent performance. Generally, from stop to underway sailing, the decrease of the boat's draft brings out the deterioration of the boat's self-stability, and the sensitiveness to the environmental disturbances, such as waves and winds. This ship type essentially needs attitude stabilization because it does not have enough restoring moment. A control system of a hydrofoil catamaran must perform three functions [1]. The first is to assure stability throughout its maneuvering envelope. The second is to attenuate wave-induced motions. And the third is to ensure the safety of the ship and its passengers all the time even if there is any failure in the control system. A major drawback of the conventional control strategies, such as LQR and proportional-derivative (P-D) controller, is that they are no longer efficient and feasible once out of the small neighborhood of the operating point, i.e. design speed.

Takagi and Sugeno's approach [2] makes it probable to have feedback control covering the entire operating envelope. In this approach, local dynamics in different state-space regions are represented by linear models, and the overall system is synthesized by the fuzzy interpolation of these local linear models. The so-called parallel distributed compensation (PDC) has been proposed and developed over the last few years [3], [4]. There exists voluminous literature on the subject of making use of various control techniques to ship motion control, from model based modern control to fuzzy-based adaptive robust control. However, very few papers are found that report the T-S fuzzy system and PDC control structure, which can be used to control uncertain nonlinear systems, for ship motion control. The objective of this paper is to present the pipeline of developing the attitude control strategy for hydrofoil catamaran over its whole operating range, based on T-S fuzzy system. Herein, we place our research emphasis on its attitude control. The purpose of attitude control is to stabilize the vessel to ensure passenger comfort by compensating wave-induced disturbances, and to control the vessel attitude. The most important control parameters that this paper concerns includes flight height and pitch angle.

This paper is organized as follows. In Section II, we combine the linear systems around several operating conditions. On the basis of T-S fuzzy system, a fuzzy gain-scheduling approach is developed for hydrofoil catamaran. Mathematical model has been established for ship heave and pitch motion in Section III. Then, linearization of the nonlinear mathematic model yields state-space model In Section IV. The results of simulation research are presented in Section V. Finally, Section VI gives some conclusions.

II. FUZZY ATTITUDE CONTROLLER DESIGN

In recent years, engineers have successfully utilized fuzzy logic in varieties of industrial control applications [3]. Among various fuzzy modelling themes, T-S model is one of the most popular frameworks. A general T-S model employs an affine model with a constant term in the consequent part of each rule, based on a fuzzy partition of input space. This is often referred to as an affine T-S model. However, what we are mostly interested in is another type of T-S fuzzy model, in which the consequent part for each rule is represented by a linear model (without a constant term). This type of T-S fuzzy model is called a linear T-S model. The appeal of a linear T-S model is that it renders itself naturally to Lyapunov based system analysis and design techniques.

In this paper, only continuous fuzzy model-based fuzzy systems are used to describe attitude controller design for hydrofoil catamaran. In our fuzzy model-based system, the premise of fuzzy rule is the boat's sailing speed, since the feedback gain is needed to cover the whole operating range smoothly. Namely, the sailing speed $\hat{U}(t)$ is chosen as scheduling variable. The i-th rule of T-S fuzzy model is

$$\begin{aligned}&\text{IF} \quad \hat{U}(t) \textit{ is about } U_i,\\ &\text{THEN } \dot{x}_e(t) = A_i x_e(t) + B_i u_e(t), \quad i = 1, 2, \cdots, r,\end{aligned}$$

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where $\hat{U}(t)$ is the language variable of premise part, herein the boat's speed, U_i is the i-th operating point, r is the number of $IF-THEN$ rules. We employ the fuzzy inference method with a singleton fuzzifier, product inference and center average defuzzifiers. Then, the final output of T-S fuzzy model is obtained as

$$\dot{x}_e(t)=\sum_{i=1}^{r}h_i(\hat{U}(t))\{A_i x_e(t)+B_i u_e(t)\}, \tag{1}$$

where $h_i(\hat{U}(t))=\omega_i(\hat{U}(t))/\sum_{i=1}^{r}\omega_i(\hat{U}(t))$, $\omega_i(\hat{U}(t))$ denotes the degree of membership of $\hat{U}(t)$ on U_i. The degree of membership satisfies $\omega_i(\hat{U}(t))>0$, where $i=1,\ 2,\ \cdots,\ r$. Note that for all t, there exist $\sum_{i=1}^{r}h_i(\hat{U}(t))=1$, where $h_i(\hat{U}(t))\geq 0,\ i=1,2,\cdots r$, and $h_i(\hat{U}(t))$ can be taken as the weights of normalized $IF-THEN$ rules.

Regarding PDC approach, fuzzy controller and fuzzy model-based model share the same premise. So the i-th control rule is described as

$$\begin{aligned}&\text{IF} \quad \hat{U}(t)\ \textit{is about}\ U_i,\\ &\text{THEN}\ u_e(t)=-F_i x_e(t), \quad i=1,2,\cdots,r.\end{aligned}$$

At the consequent part, fuzzy control rule has linear state feedback. We assume that all the states are observable in this paper. The overall fuzzy controller can be represented as follows

$$u_e(t)=-\sum_{i=1}^{r}h_i(\hat{U}(t))F_i x_e(t). \tag{2}$$

Synthesizing (1) and (2) yields the closed-loop system as follows

$$\dot{x}_e(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\hat{U}(t))h_j(\hat{U}(t))(A_i-B_iF_j)x_e(t). \tag{3}$$

A stability condition for continuous fuzzy system has been well studied by [4].

Theorem 1 (Stability condition): The equilibrium of the fuzzy control system (3) is asymptotically stable in the large if there exists a common symmetric positive definite matrix P such that the following two conditions are satisfied

$$G_{ii}^T P+PG_{ii}<0 \tag{4}$$

and

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P+P\left(\frac{G_{ij}+G_{ji}}{2}\right)\leq 0 \tag{5}$$

where $1\leq i<j\leq r,\ i<j$ except the pairs (i,j) such that $h_i(\hat{U}(t))h_j(\hat{U}(t))=0$ for $\forall\ t$, where $G_{ij}=A_i-B_iF_j$.

Originally, the common positive definite matrix P can be found via trial-and-error method, which becomes extremely difficult when the number of the subsystems increases. Recently, this problem can resort to some efficient numerical method. In this approach, the stability conditions (4) and (5) are formulated into an LMI problem. Convex optimization techniques can be then applied to check the existence of the common P. Matlab LMI control toolbox can solve this LMI problem swiftly.

III. MECHANICAL MODELLING FOR HYDROFOIL CATAMARAN

In this part, we will establish nonlinear mathematical model in waves, based on the motion model in calm water [5]. This mathematical nonlinear model will act as a platform for further research. Take into consideration the effects of not only waves but also the trim of hull on the motions of hydrofoil catamaran. Along Z-axis, by use of Newton's Second Law of Motion, the mathematical model is obtained of heave motion and pitch motion for hydrofoil catamaran as follows

$$\begin{cases} m(\ddot{\xi}+U\dot{\theta})=\sum\limits_{i=1}^{2}(F_{fi}+F_{pi})+F_H+mg\cos\theta \\ I_{yy}\ddot{\theta}=-\sum\limits_{i=1}^{2}(x_{fi}-x_g)(F_{fi}+F_{pi}) \\ \qquad\quad -(x_b-x_g)\nabla\cos\theta-(x_H-x_g)L_H \end{cases} \tag{6}$$

In the model (6), m is boat's mass, U is the alongship velocity supposed to be a constant, F_{fi} are the lift forces arising from hydrofoil, F_{pi} are the lift forces from flaps, g is gravitational acceleration, i.e. 9.8 $\mathrm{m/s^2}$, F_H is the force relevant to hull, L_H is the lift force of the hull, L_H is hull's lift force, ∇ is buoyant force from boat's hull, and I_{yy} is inertia of moment around Y-axis. θ is the hull's angular displacement of X-axis from initial position, and ξ is the hull's vertical displacement of X-axis from initial position. $|x_{fi}|$, $|x_G|$, $|x_b|$ and $|x_H|$ are the distance between amidships and the acting points of forces, i.e. hydrofoil's lift forces, gravitational forces, buoyant force and hull's lift force, respectively.

A. Forces Relevant to Twin Hydrofoil

Hydrofoil produces two forces, i.e. its dynamic lift force L_{fi} along Z-axis and the inertia force F_{ai} of its added mass

$$F_{fi}=L_{fi}+F_{ai}. \tag{7}$$

The drag force and its moment from hydrofoil both are trivial relative to the lift force, so that they can be neglected in modelling. Then the dynamic lift force of hydrofoil is given by

$$L_{fi}=-\frac{1}{2}\rho U^2 S_i C_{Li}, \tag{8}$$

where ρ is the density of water, S_i is hydrofoil's projection area calculated by $S_i=l_i\cdot b_i$, in which l_i is the span of hydrofoil and b_i is the chord of hydrofoil.

The lift coefficient decreases dramatically as hydrofoil approaches to the free-water surface. Hence, in (8), Wadlin's method [6] is adopted to calculate C_{Li}, the lift coefficient of hydrofoil, as follows

$$C_{Li}=\frac{2K_{2i}\pi\lambda_i\alpha_i}{\lambda_i+2K_{2i}+1}+\frac{8}{3}(1+\frac{\lambda_i}{10})\sin^2\alpha_i\cos\alpha_i, \quad (9)$$

where λ_i is the aspect ratio of hydrofoil, K_{2i} is the 2-dimensional depth correction factor of the lifting surface, and the 3-dimensional depth correction factor K_{3i} is omitted in this paper.

In (8), α_i is the attack angle of hydrofoil at the hydrodynamic center of hydrofoil. In consideration of the vertical resultant motions of hull at the hydrofoil, the attack angle α_i is expressed as

$$\alpha_i=\alpha_{si}+\theta+\frac{\dot{\xi}_i-(x_{fi}-x_G)\dot{\theta}-\dot{\zeta}}{U}-\alpha_{0i}, \quad (10)$$

where α_{si} is the set angle of hydrofoil, α_{0i} is the zero-lift angle of hydrofoil, and $\dot{\xi}$ is the velocity of boat's heave motion at hydrofoil. In general body axes system, the waveform model of sub-wave surface at hydrofoil is described as

$$\zeta(t_i)=\zeta_a e^{-\kappa d_i}\cos(\kappa(x_{fi}-x_g)\cos\chi-\omega_e t). \quad (11)$$

where ζ_a is wave amplitude, κ is wave number, χ is encountering angle, ω_e is the angular freuency of the waves and t is the time. The establishment of waveform model (11) is based on the famous Froude-Krylov assumption that the existence of the boat's hull has no effect on the motions of water particle nearby.

Moreover, there exists inertia force F_{ai}, arising from added mass of hydrofoil. Herein, the hydrofoil is regarded as a rigid body. Therefore, the force from inertial of the added mass of hydrofoil is given by

$$F_{ai}=-m_{fi}(\ddot{\xi}+U\dot{\theta}-(x_{fi}-x_G)\ddot{\theta}-\ddot{\zeta}_i), \quad (12)$$

where m_{fi} is the added mass of hydrofoil and calculated by

$$m_{fi}=\frac{\pi}{2}\cdot\frac{b_i^2}{4}\cdot l_i\cdot\rho. \quad (13)$$

B. Forces Relevant to the Boat's Hull

The force arising from the hull consists of buoyant force and the lift force of hull, i.e.

$$F_H=\nabla\cos\theta+L_H. \quad (14)$$

Buoyant force ∇ is generated by the hydrostatic pressure. The V-type hull L_H can be seen as a foil of small aspect ratio. Then the lift force from V-type hull is calculated, similarly to hydrofoil, by

$$L_H=-\frac{1}{2}\rho U^2 A_w C_{LH}\theta, \quad (15)$$

where C_{LH} is the derivative of hull's lift coefficient with respect to the attack angle, i.e. trim angle.

$$C_{LH}=\frac{1}{2}\cdot\frac{\pi}{2}\cdot\lambda_H, \quad \lambda_H=\frac{B_{\max}^2}{A_w}, \quad (16)$$

where λ_H is the aspect ratio, A_w is the area of waterplane, and $B_{\max}$ is the maximum breadth of the waterplane.

IV. STATE-SPACE MODEL OF HYDROFOIL CATAMARAN

Varying parameters and uncertainty from sensor noise, and disturbances, ensure that the model is never perfect. It seems that the most useful way of dealing with nonlinearity of the model is to linearize it about some point in its operating range. If the model is "smooth", the linearized equation will accurately represent the true system in some "sufficient small" region about the equilibrium point in the parameter space.

Based on the nonlinear model (6), Jacobian linearization around equilibrium point is implemented. Let $F_{di}=1/2\rho U^2 S_i C_{Li}=C'_{fi}C_{Li}$, $x_{fGi}=x_{fi}-x_G$ and $\dot{U}_{ri}=(\ddot{\xi}+U\dot{\theta})-x_{fGi}\ddot{\theta}$. Then, the nonlinear mathematical model can be rewritten as

$$\begin{cases} Z(\ddot{\xi},\dot{\xi},\xi;\ddot{\theta},\dot{\theta},\theta)=m\ddot{\xi}+\sum_{i=1}^{2}C'_{fi}C_{Li}+\sum_{i=1}^{2}m_{fi}\dot{U}_{ri} \\ \qquad -\nabla-mg-F_H=0 \\ M(\ddot{\xi},\dot{\xi},\xi;\ddot{\theta},\dot{\theta},\theta)=-\sum_{i=1}^{2}C'_{fi}C_{Li}x_{fGi}-\sum_{i=1}^{2}m_{fi}\dot{U}_{ri}x_{fGi} \\ \qquad -(x_b-x_g)\nabla\cos\theta-(x_H-x_g)L_H+I_{yy}\ddot{\theta}=0 \end{cases} \quad (17)$$

Linearizing the formulae $Z(\ddot{\xi},\dot{\xi},\xi;\ddot{\theta},\dot{\theta},\theta)$ and $M(\ddot{\xi},\dot{\xi},\xi;\ddot{\theta},\dot{\theta},\theta)$ at a certain operating point results in the linear model of hydrofoil hydrofoil. Let $\ddot{\xi}_\delta=\ddot{\xi}-\ddot{\xi}_d$, $\dot{\xi}_\delta=\dot{\xi}-\dot{\xi}_d$, $\xi_\delta=\xi-\xi_d$, $\ddot{\theta}_\delta=\ddot{\theta}-\ddot{\theta}_d$, $\dot{\theta}_\delta=\dot{\theta}-\dot{\theta}_d$ and $\theta_\delta=\theta-\theta_d$, where $\ddot{\xi}_d$, $\dot{\xi}_d$, ξ_d, $\ddot{\theta}_d$, $\dot{\theta}_d$ and θ_d are the desired value of $\ddot{\xi}$, $\dot{\xi}$, ξ, $\ddot{\theta}$, $\dot{\theta}$ and θ, respectively. Then, we have

$$\begin{cases} Z_{\ddot{\xi}}\ddot{\xi}_\delta+Z_{\dot{\xi}}\dot{\xi}_\delta+Z_\xi\xi_\delta+Z_{\ddot{\theta}}\ddot{\theta}_\delta+Z_{\dot{\theta}}\dot{\theta}_\delta+Z_\theta\theta_\delta \\ \qquad =-Z_{\alpha1}\alpha_{1\delta}-Z_{\alpha2}\alpha_{2\delta} \\ M_{\ddot{\xi}}\ddot{\xi}_\delta+M_{\dot{\xi}}\dot{\xi}_\delta+M_\xi\xi_\delta+M_{\ddot{\theta}}\ddot{\theta}_\delta+M_{\dot{\theta}}\dot{\theta}_\delta+M_\theta\theta_\delta \\ \qquad =-M_{\alpha1}\alpha_{1\delta}-M_{\alpha2}\alpha_{2\delta} \end{cases} \quad (18)$$

where $\alpha_{1\delta}$ and $\alpha_{2\delta}$ are the variations of fore and aft controlled flap angle, respectively, i.e. control input. Then, the local state-space equation after linearization is proposed as follows

$$\dot{x}_e=E^{-1}\tilde{A}(\hat{U}(t))x_e+E^{-1}(\hat{U}(t))\tilde{B}(\hat{U}(t))u_e, \quad (19)$$

where $x_e=[\dot{\xi}_\delta,\xi_\delta,\dot{\theta}_\delta,\theta_\delta]$, and $u_e=[\alpha_{1\delta},\alpha_{2\delta}]$. The coefficient matrices for local state-space model (19), i.e. $\tilde{A}(U(t))$, $\tilde{B}(U(t))$ and $E(U(t))$, are given by

$$E(U(t)) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Z_{\ddot{\xi}} & 0 & Z_{\ddot{\theta}} \\ 0 & 0 & 1 & 0 \\ 0 & M_{\ddot{\xi}} & 0 & M_{\ddot{\theta}} \end{bmatrix},$$

$$\tilde{A}(U(t)) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -Z_{\xi} & -Z_{\dot{\xi}} & -Z_{\theta} & Z_{\dot{\theta}} \\ 0 & 0 & 0 & 1 \\ -M_{\xi} & -M_{\dot{\xi}} & -M_{\theta} & -M_{\dot{\theta}} \end{bmatrix},$$

$$\tilde{B}(U(t)) = \begin{bmatrix} 0 & 0 \\ -Z_{\alpha 1} & -Z_{\alpha 2} \\ 0 & 0 \\ -M_{\alpha 1} & -M_{\alpha 2} \end{bmatrix}.$$

The items contained in those coefficient matrices are formulated as follows

$$Z_{\ddot{\xi}} = m + \sum_{i=1}^{2} m_{fi},\ Z_{\dot{\xi}} = \sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \dot{\xi}},$$
$$Z_{\xi} = \sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \xi},\ Z_{\ddot{\theta}} = -\sum_{i=1}^{2} m_{fi} x_{fGi},$$
$$Z_{\dot{\theta}} = \sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \dot{\theta}} + \sum_{i=1}^{2} m_{fi} U,$$
$$Z_{\theta} = \sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \theta},\ Z_{\alpha i} = C'_{fi} \frac{\partial C_{Li}}{\partial \alpha_i};$$
$$M_{\ddot{\xi}} = \sum_{i=1}^{2} m_{fi} x_{fGi},\ M_{\dot{\xi}} = -\sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \dot{\xi}} x_{fGi},$$
$$M_{\xi} = -\sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \xi} x_{fGi},\ M_{\ddot{\theta}} = I_{yy} + \sum_{i=1}^{2} m_{fi} x_{fGi}^2,$$
$$M_{\dot{\theta}} = \sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \dot{\theta}} x_{fGi} - \sum_{i=1}^{2} m_{fi} U x_{fGi},$$
$$M_{\theta} = -\sum_{i=1}^{2} C'_{fi} \frac{\partial C_{Li}}{\partial \theta} x_{fGi}, \quad M_{\alpha i} = -C'_{fi} \frac{\partial C_{Li}}{\partial \alpha_i} x_{fGi}.$$

So far, the local linear model for hydrofoil catamaran can be represented by

$$\begin{cases} \dot{x}_e = A(U(t)) x_e + B(U(t)) u_e \\ y_e = C x_e \end{cases} \tag{20}$$

where $A(U(t)) = E^{-1}(U(t))\tilde{A}(U(t))$, $B(U(t)) = E^{-1}(U(t))\tilde{B}(U(t))$, $C = I_{4\times 4}$, input $u_e \in R^2$, state $x_e \in R^4$, and output $y_e \in R^4$.

V. HYDROFOIL CATAMARAN EXAMPLE

Based on a hydrofoil catamaran, HC200B-A1, the simulation researches are implemented. The principal particulars of HC200B-A1 are listed in Table I [7]. Foils with adjustable flap like airplanes can regulate boat's attitude in accordance with speed and sea condition. Provided that HC200B-A1 is equipped with flap on the fore and aft hydrofoil. The span and chord of the either flap are 0.33 meter and 4.53 meters, respectively.

Assuming no effect of waves, i.e. in calm water, Fig. 1 presents hydrofoil load i.e. $L_f = L_{f1} + L_{f2}$, and the boat's attitude from simulation results and experimental data. Fig. 1 has also exposed that the boat's attitude varies from its stop mode to foil-borne mode. $L_f/(1000 \cdot \Delta \cdot g)$ denotes the ratio of the total lift forces to boat's weight. It can also be concluded that more than 80 percent of the boat's weight is

TABLE I

HC200B-A1'S PRINCIPAL PARTICULARS

L.O.A **(m)**	*L.B.P* **(m)**	**breadth(m)**	d_0 **(m)**	Δ **(ton)**
38.08	35.84	11.584	3.84	200
speed (kn)	**chord (m)**	**span (m)**	α_1 **(°)**	α_2 **(°)**
40	0.96	8.32	2	2

supported by hydrofoils at the underway speed. According to 6 groups of wave conditions in [8], the simulation results of heave and pitch motion in ahead sea is presented in Fig. 2, at the design speed, 40 knots[1]. The experimental data are also presented, accordingly. In Fig. 2, the large amplitudes of heave and pitch motion have also proved that the boat is severely subject to the influences of external waves.

From Fig. 1 and Fig. 2, it's shown that simulation results and experimental data agree with each other well. Hence, the nonlinear mathematical model in Section III is fully competent in the further research. Jocobian linearization work is carried out on the nonlinear model of HB200B-A1 in Section III at 5 operating points all contained by the working envelope, which is shown in Fig. 1. In this way, the state-space model for HC200B-A1 is obtained. Next, Because LMI-based local feedback gain design introduces too much conservativeness, we prefer to employ LQR control strategy to design the local controllers. Feedback control gain at 5 operating points are obtained. Because of the space limit, these matrices are omitted. In this illustrative example, 5 fuzzy inference rules, equipped with triangular membership function, are employed to construct

[1] 1 knot = 1.852 km/h

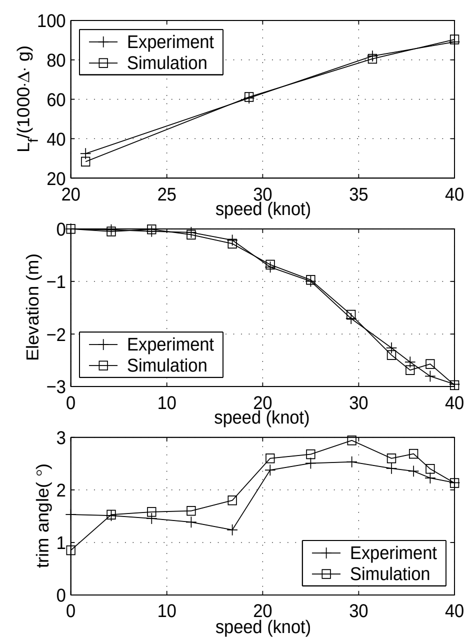

Fig. 1. Comparison of HC200B-A1's attitude between simulation results and experimental data in calm sea

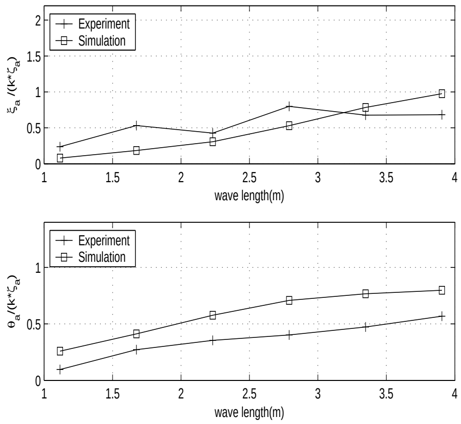

Fig. 2. Comparison of the amplitudes of heave motion and pitch motion between simulation results and experimental data in ahead sea

model-based system at 5 operating points. The membership functions are shown in Fig. 3.

To meet the requirement of seaworthiness throughout the working envelope, the interpolated fuzzy model-based system must be of global stability. According to the stability criterion in Section II, total 16 (for this fuzzy system, $r = 5$) LMIs should be solved to obtain the common matrix P. It almost seems impossible to solve such a large lump of matrices inequalities via trial-and-error method. Fortunately, this problem can be reduced to a series of LMIs. Matlab LMI toolbox can gives P conveniently as follows

$$P = \begin{bmatrix} 0.1822 & 0.0120 & 0.0227 & -0.0347 \\ 0.0120 & 0.0103 & -0.0055 & -0.0137 \\ 0.0227 & -0.0055 & 0.4730 & 0.1107 \\ -0.0347 & -0.0137 & 0.1107 & 0.1536 \end{bmatrix}.$$

The existence of matrix P makes the stability of global system guaranteed. So far, the whole fuzzy system has been constructed. To testify our proposed fuzzy system, a speed point 28 knots, (not one of the above 5 operating points) is selected, arbitrarily. Simulation research is implemented, by use of Matlab Simulink toolbox. Fig. 4 presents time response of HB200B-A1's attitude without control. And Fig. 5 exhibits time response of HB200B-A1's attitude and control efforts with controller on. During simulation study, wave parameters are that, wavelength is 120 meters, wave period 8.77 seconds, and wave amplitude 0.6250 meter. Fig. 5 manifests that, at the arbitrary speed, the wave-induced heave and pitch motion can also be attenuated greatly, which means that the feedback gain obtained from our T-S fuzzy system is efficient.

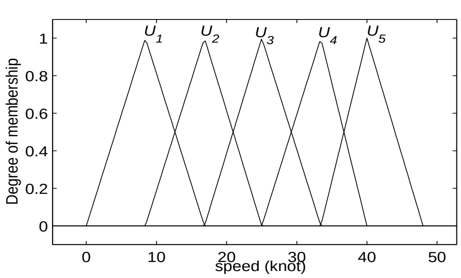

Fig. 3. Membership function

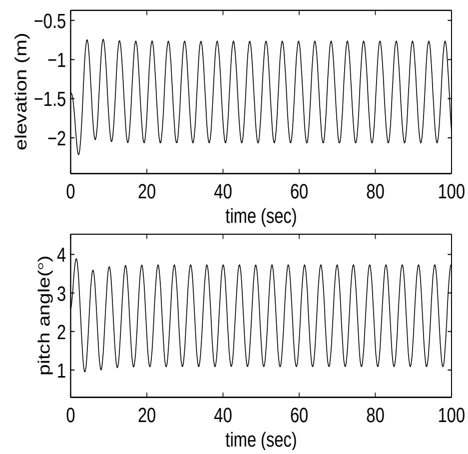

Fig. 4. Time response of HC200B-A1's attitude, i.e. elevation height (ξ) and pitch angle (θ) with controller off

To show further the effectiveness of our T-S fuzzy controller for hydrofoil catamaran, Fig. 6 compares the attitude control performance between traditional approach and the algorithm in this paper. It is shown that the latter's performance is much better than that of the former's. Hence, the drawback of traditional LQR method is successfully overcome, which is confined only in the vicinity of a limited number of operating points.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper has proposed a novel approach to meet the special requirement of attitude control for a type of fully-submerged hydrofoil catamaran, through fuzzy interpolation of a series of linear system. An integrated pipeline of developing such a control system is presented. The mathematical model of heaving motion and pitching motion is established, acting as simulation model for further research. This nonlinear model shows good performance both in calm water and in heavy seas. Through the above pipeline, a control strategy which satisfies many requirements for stabilization, maneuverability, and comfort for a fully submerged hydrofoil catamaran has been developed. At the end of the paper, numerical simulations illustrate our whole research procedures. The simulation results have shown that at any speed point over the working range, the obtained feedback gain can reject wave-induced harmful motion greatly. By use of our method, the main drawback of conventional LQR approach is eliminated. Simulation researches also have revealed the effectiveness and reliability of the established simulation model.

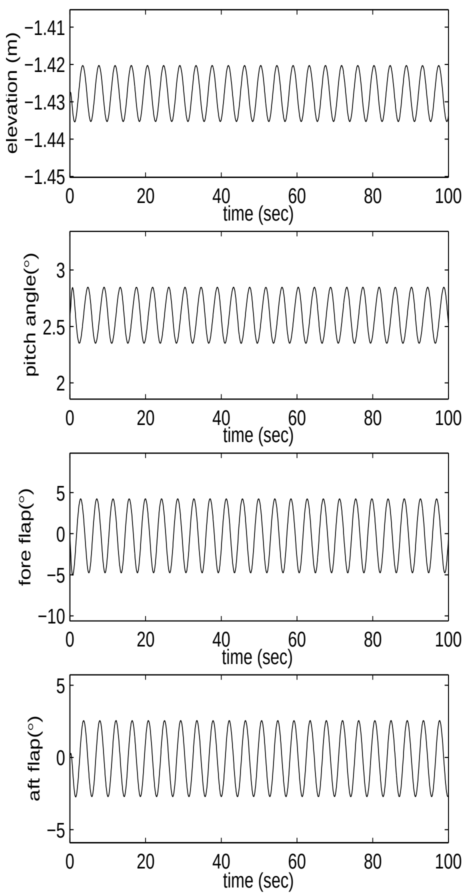

Fig. 5. Time response of HC200B-A1's attitude, i.e. elevation height (ξ) and pitch angle (θ), and control efforts, i.e. fore (α_{fp1}) and aft (α_{fp2}) flap control angles

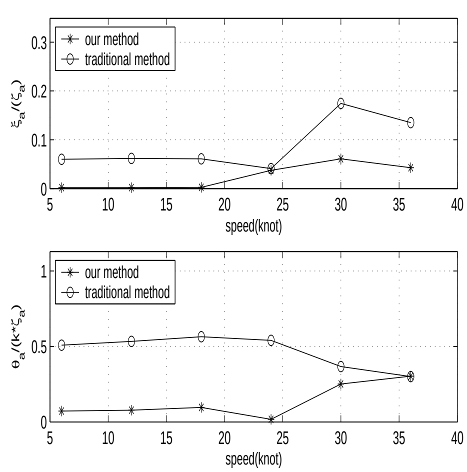

Fig. 6. Comparison of control performance between our method and traditional method

B. Future Works

However, there is still a lot of further work left to deal with in the future. E.g., the modelling of six degrees of freedom, how to suppress the roll motion of hydrofoil catamaran in rough seas, and so on, are needed.

VII. ACKNOWLEDGMENTS

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