I-PD controller as an structural alternative
to servo/regulation tradeoff tuning

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Abstract: One of the recurrent topics in the PI/PID literature of recent years is the incorporation
of the tradeoff between the two possible modes of closed-loop operation: servo and regulation.
Tuning rules are usually provided as for servo or regulation. Operator should choose which one
to apply depending on the most usual loop operation. As an alternative, the so called tradeoff
 alleges provide a controller tuning that is not optimal in either of the operation modes but aims
to provide a reasonable (in fact, the best) tradeoff among both in such a way that the loss of
performance is minimised with respect to the corresponding optimal tunings. In this paper the
use of the I-PD controller structure is proposed as an structural solution to the tradeoff tuning.
The proposal states that a direct, simple and efficient solution is found if the controller tuning
is addressed for the servo mode but using the I-PD controller structure. This is the feedback
error just drives the integral mode or, if preferred, a two degrees of freedom controller with the
set-point weight to zero.

1. INTRODUCTION

The wide and large literature on PID controllers includes
a wide variety of design and tuning methods based on
different performance criteria O’Dwyer (2009). A common
factor that all approaches face is the frequently referred
topic of set-point vs. disturbance rejection performance.
It is well known Alcantara et al. (2013) that there is an
inherent tradeoff between both, in addition to the also well
known performance/robustness tradeoff.
This distinction has made available in recent years a num-
ber of research works that analyse and provide tuning solu-
tions for each one of the operational modes under a variety
of performance indexes as well as control constraints. See
for example Alcantara et al. (2013); Alfaro and Vilanova
(2013). As a solution to the having to choose problem,
studies that suggest intermediate or tradeoff tunings have
appeared Arrieta and Vilanova (2007b) and Arrieta and
Vilanova (2007a). Also, another way of facing this problem
is the use of a Two-Degree-Of-Freedom (2-DoF) controller
Alfaro and Vilanova (2016). When available, a 2-DoF
controller introduces a set-point weight as an additional
tuning parameter that allows to smooth the response to a
change in the set-point it is too aggressive or even to speed
it up if necessary because of the required high robustness
of the control loop. However, no all vendors provide full
freedom with respect to this second additional parameter,
if available. In some cases it is limited to a value between 0
and 1 and in others it is not even available for free tuning.
Therefore, even in these days, the previously commented
solutions make sense.
However, it has also been recognised Shinskey (2002);
Vilanova et al. (2017) that disturbance rejection is much
more important than set-point tracking for many pro-
cess control applications, leading set-point tracking to a
secondary level of interest. Therefore a controller design
that emphasizes disturbance rejection rather than set-
point tracking is an important design problem that, even
if it has been the focus of research it may have not
received the appropriate attention. Indeed much of the
design approaches as well as application and/or simulation
examples provided in academic works almost concentrate
on set-point experiments for controller evaluation. In fact
set-point response can be further adjusted by the use of
set-point filters Hagglund (2013). A set-point filter can
be used to separate the design for set-point responses
from the design of responses to load disturbances and to
reduce the high-frequency variations in controller output
introduced by the set-point. Therefore there is the need to
focus attention on disturbance attenuation properties.
One way of making explicit such focus on load disturbance
is by appropriate selection of the controller equation. In
the ideal PID formulation, all the three modes process the
error signal, therefore both the reference as well as disturbance signals. However, industrial software packages, use to offer a choice-menu where different implementations are available on the basis of which ones of the controller modes are fed with the reference signal. On that respect we can go, for example, from the PID to the PI-D, where the derivative term just acts on the process output, or the I-PD where the error signal is just seen by the integral term.

In this work we will concentrate on the I-PD control system as the preferred PID control equations in industry. See for example Ogawa and Kano (2008) and Ang et al. (2005). The problem raised in this paper is very simple:

**as there are no specific tuning rules for the I-PD controller configuration, what are the implications of using the actual existing PID tuning rules?**

In fact, usually, the control algorithm implementation is manufacturer dependent and not all of its variations are available in the same controller. In addition, it would be the case that a tuning rule of interest had been obtained using a control algorithm different from the one implemented in the controller to tune. We are not referring in this work to the problem of not appropriately convert the tuning equations and make them consistent with the controller formulation. This has already been addressed by Alfaro and Vilanova (2012).

This I-PD controller is the "Type C" referred in some vendors implementations, such as Honeywell, for example. For such situations the point we would like to analyse is the benefit you may get from using a tuning rule directly designed for the I-PD configuration instead of a tuning rule designed on the basis of the PID. On that respect it is worth to analyse the loss in performance that may be incurred when the applied tuning rule does not conform with the final PID implementation.

On the basis of the previous scenario, in this work, the I-PD controller is presented and an example of the use of usual PID tuning rules such as the IMC, showing the loss of performance incurred when deploying the tuning parameters into an I-PD controller instead of the PID the tuning is conceived for. Next, a specific tuning is proposed for the I-PD controller. The tuning is aimed at minimising an Integral Absolute Error (IAE) performance index subject to a robustness constraint. The tuning is provided in two versions: smooth and tight.

### 2. I-PD: THE INDUSTRIAL PID CONTROLLER

A standard PID controller is also known as the "three-term" controller, whose transfer function is generally written in the "parallel form" given by (1) or the "ideal form" given by (2)

\[ K(s) = K_p + K_i \frac{1}{s} + K_d s \]

\[ K(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \]

These forms, will results in a theoretically infinite high control signal when a step change of the reference or (output) disturbance occurs. In order to deal with this, different variations of the ideal PID formulations are available in industrial software packages. A common remedy is to cascade the pre differentiator with a low-pass filter so

\[ T_d s \rightarrow \frac{T_d s}{(\gamma T_d s + 1)} \]

where most industrial PID provide a setting of $1/\gamma$ from 1 to 33 and the majority falls between 8 and 16 Ang et al. (2005). This is the modification that leads to most usual and well known form, such as the ISA form.

However it is significant that in practice, the following variants of the original PID structure are of application. They are the so called "Type B" (or P-ID) and "Type C" (or I-PD) control structures reflected in equations (4) and (5) respectively.

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau - K_d \frac{d}{dt} y(t) \]

\[ u(t) = -K_p y(t) + K_i \int_0^t e(\tau) d\tau - K_D \frac{d}{dt} y(t) \]

For what matters to the effect of the derivative term with respect to changes in the reference signal, both structures offer the same advantage, being the response to a step change in the reference signal, smoother in the I-PD configuration. As a transition alternative between the PID and the I-PD configuration, there is the Two-Degrees-of-Freedom (2-DoF) PID, where a set-point weight is introduced in the proportional term:

\[ u(t) = K_p (\beta r(t) - y(t)) + K_i \int_0^t e(\tau) d\tau - K_D \frac{d}{dt} y(t) \]

However, this set-point weight, as an additional, fourth, PID parameter is not available in all industrial control software. Therefore, bearing in mind the preference for regulatory control operation and the additional smoothness provided by the I-PD configuration, the I-PD is the preferred one for industrial applications. See for example Ogawa and Kano (2008) and Ang et al. (2005).

Therefore, in this work we will concentrate on the I-PD control system as it is depicted in figure (1), where the error signal is just fed into the integral term. The point with the use the I-PD controller formulation is that as the regulatory relations are the same as the ones for the PID controller, the stability properties remain unaltered. If we test the performance of a tuning rule when applied to a PID that is implemented by using the PID or the I-PD equation, the results will be exactly the same as long as we remain on the regulatory behaviour. However the relations for an input reference change can be very different.

In the I-PD controller structure all three parameters contribute to the disturbance attenuation as all three param
rameters process the output signal. On the other hand, only the integral time constant contributes to the tracking performance. Therefore, the final allocation of the controller gains should result in different controller tunings depending on the use of a I-PD or of a PID. Questions such as the following ones naturally arise:

- Will the performance on the I-PD degrade significantly from the PID one?
- Will it be beneficial to elaborate tuning rules specifically for the I-PD configuration?

Next section will exemplify the use of the well known IMC tuning when applied to a controller implemented by using the I-PD form. It is not intended to provide a full answer to the first question but to show that, at least for a tuning that has wide acceptance in industry, that for certain first order plus time delay process models, the degradation may be highly significative. Second question is addressed in section 4 with the proposal of a robust IAE tuning.

3. I-PD USE OF USUAL PID TUNING RULES

This section will motivate the specific development for an I-PD tuning. Will present an example of application of usual tunings that where conceived for a PID controller but applied by using an I-PD configuration. Of course, this could be worked out for a large number of tuning rules, but it is not the purpose of this paper to do an extensive summary but to put on the table the problem that may arise because of the controller equation mismatch.

From the technological survey by Ang et al. (2005), it turns out that the Lambda and the IMC tuning are the most common approaches used in industry. Let us assume the IMC tuning rule of Rivera et al. (1986) for a first order plus time delay process. The process model takes the form:

\[ P(s) = \frac{Ke^{-Ls}}{Ts + 1} \]  

(7)

where \( K, T \) and \( L \) are, respectively, the process gain, time constant and time delay. In addition, the normalised time constant \( \tau_o = L/T \) is defined. The associated IMC tuning equations are:

\[ K_p = \frac{2T + L}{K(2\lambda + L)}, \quad T_i = T + L/2, \quad T_d = \frac{TL}{2T + L}. \]  

(8)

where the value of \( \lambda \) can be adjusted to determine the aggressiveness of the tuning. A common compromise value of \( \lambda = L \) is used. In this case we apply this tuning to the following process example

\[ P(s) = \frac{1.2e^{3.6s}}{3s + 1} \]  

(9)

The corresponding time responses for the IMC controller implemented as a PI-D and as a I-PD are shown in figure (2). As expected, the mismatch in the controller equation has its effect on the reference tracking performance. It has been slowed down significantly. If we compute the IAE for the overall time response, whereas for the PI-D implementation it takes a value of 12.23, for the I-PD implementation it degrades to 16.68. Where all the IAE increase comes from the error with respect to the reference. If we apply the IMC tuning but with \( \lambda = 2L \) the IAE changes from 19.75 to 24.52, whereas for \( \lambda = L/2 \) the IAE changes from 12.13 to 13.3. The degradation in performance is lower with the more aggressive tuning. This may be logical because of the higher gains resulting from the tuning. If we extend this analysis in terms of \( \tau_o = L/T \), the performance loss is shown in figure (3). The degradation effect is evident for processes with \( \tau_o = L/T < 1 \), and even for those with \( \tau_o = L/T > 1 \) the degradation may be significative.
This analysis could also be shown for other tunings that of usual practice such as, for example, the optimal methods of López et al. (1967) and Rovira et al. (1969), the CHR form Chien et al. (1952) or even the more actual methods such as the S-IMC method from Skogestad (2003) or the robust methods from Alfaro et al. (2009) and? Vilanova et al. (2012). However due to space constraints, it is just exemplified here for the IMC method.

4. ROBUST I-PD TUNING RULE

In this section a proposal for a tuning rule that aims to minimise the IAE subjected to a robustness constraint will be worked out. In fact there do exist optimal IAE PID tuning rules. However they are intended to work for a PID controller. In this case, we will work out the derivation of the tuning bearing in mind that the controller will be implemented by using the "Type C" or I-PD formulation. As told, for what matters to the controller equation, the following I-PD, with filtered derivative term\(^2\), equation is considered:

\[
u(s) = \frac{K_p}{T_1s}c(s) - K_p(1 + \frac{T_d s}{\gamma T_d s + 1})y(s) = \frac{K_p}{T_1s}r(s) - K_p(1 + \frac{1}{T_1s} + \frac{T_d s}{\gamma T_d s + 1})y(s) = C_r(s)r(s) - C_y(s)y(s)
\]

In order to include robustness considerations, the well known Maximum sensitivity peak, \(M_S\), will be employed.

\[
M_S = \max_{\omega} |S(j\omega)| = \max_{\omega} \left| \frac{1}{1 + C_y(j\omega)P(j\omega)} \right|
\]

By considering the \(M_S\) value we are constraining the distance from the Nyquist locus to the critical point. Also, simultaneous gain and phase margin values are ensured Astrom and Hagglund (2006). On the basis of the constraint for \(M_S\) will define two levels of the tuning as tight, with a constraint of \(M_S = 2.0\) and smooth, with a constraint of \(M_S = 1.6\). In all the situations, the proposed tuning obeys to the following normalised controller equations:

\[
\begin{align*}
k_p &= K_p \frac{1}{1 + c_1} + c_2 \quad (14) \\
t_1 &= T_1 / T = a_2 \frac{1}{a_2} + c_2 \quad (15) \\
t_d &= T_d / T = a_3 \frac{1}{a_3} + c_3 \quad (16)
\end{align*}
\]

Table 1. I-PD tuning rule coefficients for tight and smooth control

<table>
<thead>
<tr>
<th>Tuning</th>
<th>tight</th>
<th>smooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_p)</td>
<td>2.752</td>
<td>-0.3882</td>
</tr>
<tr>
<td>(T_1)</td>
<td>1.452</td>
<td>0.4622</td>
</tr>
<tr>
<td>(T_d)</td>
<td>0.2895</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table (1) provides the values for the coefficients of the normalised controller parameters for the two considered situations. A first thing that is noticed is that here we do not present a distinction between the optimisation of the tuning for servo or regulation operation. It turns out that with the use of the I-PD equation, the performance of the control system operating on the mode it was not tuned for has practically no performance degradation with respect to the tuning specially designed for such operation. In order to exemplify this, figure (4) shows the IAE values for all the considered range for \(\tau_d\) where for the two considered cases (tight/Smooth). It can be seen that the performance in each case is essentially the same. Therefore, there is no practical need to distinguish between two different tunings. The tuning presented here in table (2) is the one corresponding to the optimisation for servo operation. The motivation for this is that, as commented in previous sections, the integral time constant is the only parameter that determines the set-point following performance. Therefore, servo performance optimisation is preferred even, as it can be seen, the results if we use a regulation tuning operating as a servo, provides quite similar results.

In order to exemplify this performance issue related to the operation/tuning mode, we consider here the same example as in the previous section and comparing the performance of both tunings. Also in figure (5) the similarity with respect to the time responses is also verified.

Table 2. Servo/Regulation tuning IAE performance for a Set-point (SP) and a Load Disturbance (LD) step change

<table>
<thead>
<tr>
<th>Tuning</th>
<th>SP</th>
<th>LD</th>
<th>SP</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servo</td>
<td>7.86</td>
<td>4.71</td>
<td>8.80</td>
<td>5.89</td>
</tr>
<tr>
<td>Regulation</td>
<td>7.90</td>
<td>4.56</td>
<td>8.83</td>
<td>5.79</td>
</tr>
</tbody>
</table>

5. EXAMPLES

In this section we will compare the application of the proposed tuning with other two existing tunings. On one side the IMC from Rivera et al. (1986) because of its wide industrial acceptance. The tuning equations are those reproduced in equation (8). In this case, in order to make a fair comparison, the tuning parameter \(\lambda\) has been selected such that the robustness of the resulting IMC control system matches that of the proposed tuning for both the smooth and tight cases. As for the process at hand (9) we have that the proposed tuning gets \(M_S \approx 1.67\) for the smooth tuning and \(M_S \approx 2.1\) for the tight tuning, the values of \(\lambda\) for the IMC have been chosen accordingly. With

\[
\begin{align*}
K_p &= 0.462 \frac{1}{a_2} + c_2 \\
T_1 &= 1.026 + c_3 \\
T_d &= 0.186 + c_3
\end{align*}
\]

\[
\begin{align*}
K_p &= 0.462 \frac{1}{a_2} + c_2 \\
T_1 &= 1.026 + c_3 \\
T_d &= 0.186 + c_3
\end{align*}
\]
As second tuning considered is the one proposed in Ogawa and Katayama (2001). It is worth to stress that this is the only robust tuning proposal found in the literature for the I-PD controller. This work proposes optimised I-PD settings based on the ISE (Integral of Squared Error). The normalized controller parameters obey to the following tuning relation,

\[
K_pK = \frac{p - 2q + 4}{p + 2q} \quad (17)
\]

\[
T_i = \frac{(p + 2q)(p - 2q + 4)}{2p + 4} \quad (18)
\]

\[
T_d = \frac{p(p + 4 + 2q - 2q^2)}{(p + 2q)(p - 2q + 4)} \quad (19)
\]

where \( p = L/T \) and \( q \) is a design parameter that expresses the relation between the open-loop and the desired closed-loop time constant. In order to provide the optimal ISE solution, the value of \( q \) should be selected according to:

\[
q^{opt} = -0.1902p^2 + 0.6974p + 0.007393 \quad (20)
\]

It is worth to say that as it is based on an ISE specification, the generated behaviour is quite oscillating. In fact, if we apply this controller as well as the two selections for the IMC tuning, the corresponding time responses can be observed in figure (6) for the tight tuning and in figure (7) for the smooth tuning.

In order to provide a more complete and quantitative view, table (3) provides the controller parameters as well as achieved \( M_S \) and IAE cost for the different operations. Even the similar values that both the IMC and the proposed tuning provide for the controller proportional gain, the difference in the integral time constants is what introduces the major difference. In fact, according to the observation made in Åström et al. (1998), the Integrated Error (IE) criterion is directly given by the inverse of the integrating gain \( (K_p/T_i) \) of the controller. Therefore, having almost equal proportional gains, as \( T_i \) is minimised we get better values for the IE, therefore also for the as IAE \( \approx \) IE when the error is positive, the system has a smooth, non-oscillating response.

**Table 3. Performance and controller values for the comparison example**

<table>
<thead>
<tr>
<th></th>
<th><strong>tight</strong></th>
<th><strong>smooth</strong></th>
<th><strong>Ogawa</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>0.80</td>
<td>0.60</td>
<td>0.64</td>
</tr>
<tr>
<td>( T_i )</td>
<td>4.09</td>
<td>5.00</td>
<td>3.56</td>
</tr>
<tr>
<td>( T_d )</td>
<td>1.10</td>
<td>1.20</td>
<td>1.32</td>
</tr>
<tr>
<td>( M_S )</td>
<td>2.11</td>
<td>2.10</td>
<td>1.67</td>
</tr>
<tr>
<td>( IAE_{sp} )</td>
<td>8.68</td>
<td>10.2</td>
<td>9.68</td>
</tr>
<tr>
<td>( IAE_{sd} )</td>
<td>5.40</td>
<td>6.24</td>
<td>6.76</td>
</tr>
</tbody>
</table>

As it is expected by the controller structure itself, an can be verified by the provided time responses, the generated process output and control signal are very smooth. This is an inherent facility of the I-PD controller that makes possible to provide not aggressive responses for a change in the set-point signal and, therefore, does not generates the unavoidable spikes (unless set-point weight is available) in the control signal.
6. CONCLUSIONS

This work has introduced the problem introduced because of the mismatch regarding the PID implementation equation and the PID tuning equation. With specific emphasis on the use of the so called "Type C" or I-PD controller equation because of its widespread use in the industrial case. It turns out that most of the industrial software packages do recommend this implementation because of its smoothness regarding step reference change without the need of introducing additional controller parameters usually linked to the reference set-point weight.

Most of the existing tuning rules are conceived for a PID controller implementation where the error signal is affected by either all the three controller modes or, at least, by the proportional and integral modes. However, with the I-PD equation, just the integral term processes the error. This slight mismatch may incur in a notable performance degradation with respect to the expected one from the original tuning.

The previous scenario, jointly with the fact that there is, to the knowledge of the authors, practically availability of tuning guidelines for I-PD controllers, leads this work to propose a Robust IAE tuning for I-PD controllers. The use of the well known Mz constraint as a robustness measure, allows to establish two levels of robustness therefore two tuning suggestions for smooth and tight control. The obtained tuning is evaluated and compared with other PID methods and with a (the only one found) proposal for robust optimal ISE tuning for I-PD controllers.

The effectiveness of the explicit use of the I-PD equation for deriving the tuning rules, suggests to expand the approach to the use of other process model dynamics as well as to other controller structures such as the I-P.

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